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6.3 Binomial Expansion

1) a) $(1+x)^5$

$1^5 + {}^5C_1 1^4 x + {}^5C_2 1^3 x^2 + {}^5C_3 1^2 x^3 + {}^5C_4 1^1 x^4 + {}^5C_5 1^0 x^5$

$= 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

b) $(2+x)^6$

$2^6 + {}^6C_1 2^5 x + {}^6C_2 2^4 x^2 + {}^6C_3 2^3 x^3 + {}^6C_4 2^2 x^4 + {}^6C_5 2^1 x^5 + {}^6C_6 2^0 x^6$

$= 64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

c) $(10+x)^4$

$10^4 + {}^4C_1 10^3 x + {}^4C_2 10^2 x^2 + {}^4C_3 10^1 x^3 + {}^4C_4 10^0 x^4$

$= 10000 + 4000x + 600x^2 + 4x^3 + x^4$

2) a) $(3-2x)^4$

$3^4 + {}^4C_1 3^3(-2x) + {}^4C_2 3^2(-2x)^2 + {}^4C_3 3^1(-2x)^3 + {}^4C_4 3^0 (-2x)^4$

$= 81 - 216x + 216x^2 - 96x^3 + 16x^4$

b) $(4+\frac{1}{2}x)^5$

$4^5 + {}^5C_1 4^4(\frac{1}{2}x) + {}^5C_2 4^3(\frac{1}{2}x)^2 + {}^5C_3 4^2(\frac{1}{2}x)^3 + {}^5C_4 4^1(\frac{1}{2}x)^4 + {}^5C_5 4^0(\frac{1}{2}x)^5$

$= 1024 + 640x + 160x^2 + 20x^3 + 5/4\,x^4 + 1/32\,x^5$

c) $(x-2)^6$

$(-2)^6 + {}^6C_1(-2)^5 x + {}^6C_2(-2)^4 x^2 + {}^6C_3(-2)^3 x^3 + {}^6C_4(-2)^2 x^4 + {}^6C_5(-2)^1 x^5 + {}^6C_6(-2)^0 x^6$

$= 64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$

3) a) $(1+x)^{10}$

$1^{10} + {}^{10}C_1 1^9 x + {}^{10}C_2 1^8 x^2 + {}^{10}C_3 1^7 x^3$

$= 1 + 10x + 45x^2 + 120x^3$

b) $(1+x)^{15}$

$1^{15} + {}^{15}C_1 1^{14} x + {}^{15}C_2 1^{13} x^2 + {}^{15}C_3 1^{12} x^3$

$= 1 + 15x + 105x^2 + 455x^3$

c) $(2+x)^9$

$2^9 + {}^9C_1 2^8 x + {}^9C_2 2^7 x^2 + {}^9C_3 2^6 x^3$

$= 512 + 2304x + 4608x^2 + 5376x^3$

4) a) $(1-\frac{x}{2})^{10}$

$1^{10} + {}^{10}C_1 1^9 (-\frac{1}{2}x) + {}^{10}C_2 1^8 (-\frac{1}{2}x)^2 + {}^{10}C_3 1^7 (-\frac{1}{2}x)^3$

$= 1 - 5x + \frac{45}{4}x^2 - 15x^3$

b) $(2-\frac{1}{3}x)^9$

$2^9 + {}^9C_1 2^8 (-\frac{1}{3}x) + {}^9C_2 2^7 (-\frac{1}{3}x)^2 + {}^9C_3 2^6 (-\frac{1}{3}x)^3$

$= 512 - 768x + 512x^2 - \frac{1792}{9}x^3$

c) $(3+2x)^8$

$3^8 + {}^8C_1 3^7 (2x) + {}^8C_2 3^6 (2x)^2 + {}^8C_3 3^5 (2x)^3$

$= 6561 + 34992x + 81648x^2 + 108864x^3$

5) $(2-3x)^5$

$2^5 + {}^5C_1 2^4 (-3x) + {}^5C_2 2^3 (-3x)^2 + {}^5C_3 2^2 (-3x)^3$

$= 32 - 240x + 720x^2 - 1080x^3$

6) $(3-x)^{12} \equiv A + Bx + Cx^2$

$3^{12} + {}^{12}C_1 3^{11} (-x) + {}^{12}C_2 3^{10} (-x)^2$

$= 531441 - 2125764x + 3897234x^2$

$A = 531441$

$B = -2125764$

$C = 3897234$

7) $(1-\frac{2}{3}x)^9$

$$1^9 + ({}^9C_1)1^8(-\tfrac{2}{3}x) + ({}^9C_2)1^7(-\tfrac{2}{3}x)^2 + ({}^9C_3)1^6(-\tfrac{2}{3}x)^3$$
$$= 1 - 6x + 16x^2 - \frac{224}{9}x^3$$

8) $(x-\frac{2}{x})^5$

$$x^5 + ({}^5C_1)x^4(-\tfrac{2}{x}) + ({}^5C_2)x^3(-\tfrac{2}{x})^2 + ({}^5C_3)x^2(-\tfrac{2}{x})^3$$
$$+ ({}^5C_4)x^1(-\tfrac{2}{x})^4 + ({}^5C_5)x^0(-\tfrac{2}{x})^5$$
$$= x^5 - 10x^3 + 40x - \frac{80}{x} + \frac{80}{x^3} - \frac{32}{x^5}$$