Author: Tharaniya Rasakulasingham

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Exercise 6.1 : Midpoints and perpendicular bisectors

1) a) $(2, -5)\ (-4, 7)$

$\left(\frac{2-4}{2}\right), \left(\frac{-5+7}{2}\right)$

$= \left(\frac{-2}{2}, \frac{2}{2}\right)$

$= (-1, 1)$

b) $(-3, 9)(-1, -5)$

$\left(\frac{-3-1}{2}\right), \left(\frac{9-5}{2}\right)$

$= \frac{-4}{2}, \frac{4}{2}$

$(-2, 2)$

c) $\left(\frac{9}{5}, -\frac{3}{8}\right), \left(-\frac{3}{5}, -\frac{1}{8}\right)$

$\left(\frac{\frac{9}{5} - \frac{3}{5}}{2}\right), \left(\frac{-\frac{3}{8} - \frac{1}{8}}{2}\right)$

$= \left(\frac{\frac{6}{8}}{2}, \frac{-\frac{4}{8}}{2}\right)$

$= \left(\frac{3}{5}, -\frac{1}{4}\right)$

d) $(2\sqrt{3} - \sqrt{5}), (-\sqrt{3}, -3\sqrt{5})$

$\left(\frac{2\sqrt{3} - \sqrt{3}}{2}, \frac{-\sqrt{5} - 3\sqrt{5}}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -\frac{4\sqrt{5}}{2}\right)$

$\left(\frac{\sqrt{3}}{2}, -2\sqrt{5}\right)$

e) $(1 + 2\sqrt{2}, 3 - \sqrt{2})\ (3 + 2\sqrt{2}, 1 - 3\sqrt{2})$

$\left(\frac{1 + 2\sqrt{2} + 3 + 2\sqrt{2}}{2}, \frac{3 - \sqrt{2} + 1 - 3\sqrt{2}}{2}\right)$

$\left(\frac{4 + 4\sqrt{2}}{2}, \frac{4 - 4\sqrt{2}}{2}\right)$

$(2 + 2\sqrt{2}, 2 - 2\sqrt{2})$

2) a) $(-3, 4)(1, -8)$

$\left(\frac{-3+1}{2}, \frac{4-8}{2}\right)$

$= \left(\frac{-2}{2}, \frac{-4}{2}\right)$

$= (-1, -2)$

gradient $= \frac{-8-4}{1+3} = \frac{-12}{4} = -3$

$m_2 = \frac{1}{3}$

$y + 2 = \frac{1}{3}(x + 1) \quad y + 2 = \frac{1}{3}x + \frac{1}{3} \rightarrow y = \frac{1}{3}x - \frac{5}{3} \rightarrow x - 3y - 5 = 0$

b) $(3,-5)\ (-7,15)$

midpoint: $\left(\frac{3-7}{2}, \frac{-5+15}{2}\right) = \left(\frac{-4}{2}, \frac{10}{2}\right) = (-2,5)$

gradient: $\frac{15+5}{-7-3} = \frac{20}{-10} = \underline{-2}$.

$m_2 = \frac{1}{2}$

$y-5 = \frac{1}{2}(x+2) \rightarrow y-5 = \frac{1}{2}x+1 \rightarrow y = \frac{1}{2}x+6 \Rightarrow x-2y+12=0$

c) $(2,-3)\ (-6,-9)$

midpoint: $\left(\frac{2-6}{2}, \frac{-3-9}{2}\right) = \left(\frac{-4}{2}, \frac{-12}{2}\right) = (-2,-6)$

gradient: $\frac{-9+3}{-6-2} = \frac{-6}{-8} = \frac{3}{4}$

$m_2 \rightarrow -\frac{4}{3}$

$y+6 = -\frac{4}{3}(x+2) \Rightarrow y+6 = -\frac{4}{3}x - \frac{8}{3} \rightarrow y = -\frac{4}{3}x - \frac{10}{3} \Rightarrow 4x+3y+26=0$

3) a) $(-3,4)\ (5,8)$

midpoint: $\left(\frac{-3+5}{2}, \frac{4+8}{2}\right) = \left(\frac{2}{2}, \frac{12}{2}\right)\ (1,6)$

b) $(\underset{x}{1},\underset{y}{6})\ (\underset{x}{5},\underset{y}{8})$

$d = \sqrt{(5-1)^2+(8-6)^2} = \sqrt{(4)^2+(2)^2} = \sqrt{16+4} \rightarrow \sqrt{20} \rightarrow \sqrt{4}\sqrt{5} \rightarrow 2\sqrt{5}$ $\underline{k=2}$.

4) $(3,-5)\ (a,b)$

midpoint: $\left(\frac{3+a}{2}, \frac{-5+b}{2}\right) = (6,-3)$

$\frac{3+a}{2} = 6$ $\qquad$ $\frac{-5+b}{2} = -3$

$3+a = 12$ $\qquad$ $-5+b = -6$

$a = 9$ $\qquad$ $b = -1$

5) a) PQ gradient: $\frac{7+3}{5+1} = \frac{10}{6}$

$\perp m = -\frac{6}{10}$

midpoint: $\left(\frac{-1+5}{2}, \frac{-3+7}{2}\right)$

$= \left(\frac{4}{2}, \frac{4}{2}\right)$

$= (2,2)$

$y-y_1 = m(x-x_1)$

$y-2 = \frac{-6}{10}(x-2)$

$y-2 = -\frac{6}{10}x + \frac{6}{5}$

$y = -\frac{6}{10}x + \frac{16}{5}$

$\downarrow$

$5y = 16-3x$

b) QR gradient: ~~$\frac{-5-7}{7-5}$~~ $\frac{-5-7}{7-5} = \frac{-12}{2} = -6$

$\perp m = \frac{1}{6}$

midpoint: $\left(\frac{5+7}{2}, \frac{7-5}{2}\right) = \frac{12}{2}, \frac{2}{2} = (6,1)$

$y-y_1 = m(x-x_1)$

$y-1 = \frac{1}{6}(x-6)$

$y-1 = \frac{1}{6}x-1$

$y = \frac{1}{6}x \rightarrow 6y = x$

5) c) $5y = 16 - 3x$

$6y = x$

$5y = 16 - 3(6y)$

$5y = 16 - 18y$

$5y + 18y = 16$

$23y = 16$

$y = \frac{16}{23}$

$6y = x$

$6\left(\frac{16}{23}\right) = x$

$x = \frac{96}{23}$

$\left(\frac{96}{23}, \frac{16}{23}\right)$

6) midpoint: $\left(\frac{3-1}{2}, \frac{8-4}{2}\right)$

$= \frac{2}{2}, \frac{4}{2}$

$= (1, 2)$

PQ $m = \frac{-4-8}{-1-3} = \frac{-12}{-4} = 3$

$lm = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x - 1)$

$y - 2 = -\frac{1}{3}x + \frac{1}{3}$

$y = -\frac{1}{3}x + \frac{7}{3}$

$x + 3y - 7 = 0$

7) a) midpoint: $\left(\frac{-3+5}{2}, \frac{6-2}{2}\right)$

$= \left(\frac{2}{2}, \frac{4}{2}\right)$

$= (1, 2)$

b) $(1, 2)$ $(-3, 6)$

$\sqrt{(-3-1)^2 + (6-2)^2}$

$= 4\sqrt{2}$

$k = 4$

c) PQRS

PRQS is a square

Length: $\sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$

$= \underline{8}$

$PR = 8$ $PR = RQ = QS = SP = 8$
$RQ = 8$
$QS = 8$
$SP = 8$

8) a) $(2k-1, -3)(3, 3k+7)$

$\left(\frac{2k-1+3}{2}, \frac{-3+3k+7}{2}\right)$

$\left(\frac{2k+2}{2}, \frac{3k+4}{2}\right)$

$(k+1, \frac{3}{2}k+2)$

$= (5, p)$

$k+1 = 5$
$k = 4$

$\frac{3}{2}(4) + 2$

$= 6 + 2$
$= 8 = p$

a) $k = 4$

b) $p = 8$

c) midpoint $= (5, 8)$

gradient: $(7, -3)(3, 19)$

$\frac{19+3}{3-7} = \frac{22}{-4}$

$= -\frac{11}{2}$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{11}{2}(x - 5)$

$y - 8 = -\frac{11}{2}x + \frac{55}{2}$

$y = -\frac{11}{2}x + \frac{78}{2}$

$2x - 11y + 78 = 0$