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6.4 - Use tangent and chord properties

1a) $(x-3)^2-9+(y+2)^2-4-156=0$

$(x-3)^2+(y+2)^2=9+4+156$

$(x-3)^2+(y+2)^2=169$

$(-2-3)^2+(\overset{10}{y}+2)^2=169$

$(-5)^2+(\overset{12}{\cancel{y+2}})^2=169$

$25+144=169$

↳ As $25+144=169$, this proves point $(-2,10)$ lies on C

1b) centre $=(3,-2)$

$m=\frac{\Delta y}{\Delta x} \rightarrow \frac{-2-10}{3--2} \rightarrow -\frac{12}{5}$

↳ $=\frac{5}{12}$

$y=mx+c$

$10=\frac{5}{12}(-2)+c$

$10=-\frac{5}{6}+c$

$10+\frac{5}{6}=c$

$\frac{65}{6}$

$y=-\frac{5}{6}x+\cancel{86}\ \frac{65}{6}$

$6y=-5x+65$

$6\ 5x+6y-65=0$

2a) $(8x-8)^2+(y-2)^2=r^2$

$(4-8)^2+(-6-2)^2=r^2$

$(-4)^2+(-8)^2=r^2$

$16+64=80$

2b) $m=\frac{\Delta y}{\Delta x} \rightarrow \frac{2--6}{8-4} \rightarrow \frac{8}{4} \rightarrow 2$

$-\frac{1}{2}$

$-6=-\frac{1}{2}(4)+c$

$-6=-2+c$

$-6+2=c$

$-4=c$

$\therefore y=-\frac{1}{2}x-4$

3a) equation of circle $=$

$(x+3)^2+(y-4)^2=9+16+16$

$(x+3)^2+(y-4)^2=41$

↳ midpoint of A + B $=$

$\frac{2+-7}{2}=-2.5$

$\rightarrow (-2.5, 8.5)$

$\frac{8+9}{2}=8.5$

$m=\frac{\Delta y}{\Delta x} \rightarrow \frac{9-8}{-7-2} \rightarrow -\frac{1}{9}=9$

$y=mx+c$

$8.5=9(-2.5)+c$

$8.5=-22.5+c$

$8.5+22.5=c$

$31=c$

$y=9x+31$

3b) $y = 9x + 31$

$\cancel{y = 9}$ centre of circle $= (-3, 4)$

$y = 9(-3) + 31$

$y = -27 + 31$

$y = 4$

As $y = 4$, the point $(-3, 4)$ passes through perpendicluar bisector of AB.

4a) value of p:

$\cancel{(x} \; (-2-p)^2 + (7-3)^2 = 20$

$p^2 + 4p + 4 + 16 = 20$

$p^2 + 4p + 20 = 20$

$p^2 + 4p = 0$

$p = -4$

possible values of k:

$\cancel{(-4} \; (x+4)^2 + (y-3)^2 = 20$

$(-8+4)^2 + (y-3)^2 = 20$

$(-4)^2 + (y-3)^2 = 20$

$16 + y^2 - 6y + 9 = 20$

$y^2 - 6y + 9 + 16 - 20 = 0$

$y^2 - 6y + 5 = 0$

$y = 5$ or $y = 1$

$k = 5$ and $k = 1$

4b) greater value of k: 5

$P(-2, 7) \quad Q(-8, 5)$

midpoint of PQ:

$\frac{-2 + -8}{2} = -5$

$\frac{7+5}{2} = 6 \rightarrow (-5, 6)$

$m = \frac{\Delta y}{\Delta x} \rightarrow \frac{5-7}{-8--2} \rightarrow \frac{1}{3} \rightarrow -3$

$y = mx + c$

$6 = -3(-5) + c$

$6 = 15 + c$

$6 - 15 = c$

$-9 = c$

$y = -3x - 9$

passes through centre:

centre $= (-4, 3)$

$y = -3(-4) - 9$

$y = 12 - 9$

$y = 3$

5a) $\frac{4+q}{2} = 1 \qquad \frac{p-2}{2} = -4$

$4 + q = 2 \qquad p - 2 = -8$

$q = 2 - 4 \qquad p = -8 + 2$

$q = -2 \qquad p = -6$

5b) $(4, -6)$ A $\quad (-2, -2)$ Q

$\sqrt{(4--2)^2 + (-6--2)^2}$

$\sqrt{(4+2)^2 + (-6+2)^2}$

$\sqrt{(6)^2 + (-4)^2}$

$\sqrt{36 + 16}$

$\sqrt{52}$

radius $= \frac{\sqrt{52}}{2} = \sqrt{13}$

5c) centre of circle: $(1, -4)$

$(x-1)^2 + (y+4)^2 = 13$

5d) $m = \frac{\Delta y}{\Delta x} \rightarrow \frac{-4 - -9}{1-3} \rightarrow -\frac{5}{2} = \frac{2}{5}$

$y = mx + c$

$y = \frac{2}{5}x + c$

$-9 = \frac{2}{5}(3) + c$

$-9 = \frac{6}{5} + c$

$-9 - \frac{6}{5} = c$

$-\frac{51}{5} = c$

$y = \frac{2}{5}x + c$

6a) $(x+5)^2 + (y-9)^2 = r^2$

$(8+5)^2 + (14-9)^2 = r^2$

$(13)^2 + (5)^2 = r^2$

$169 + 25 = r^2$

$194 = r^2$

$(x+5)^2 + (y-9)^2 = 194$

↳ equation of circle

$(8+5)^2 + (\cancel{-9}^{4} - 9)^2$

$(\cancel{8}^{13})^2 + (-\cancel{18}^{5})^2$

$169 + \cancel{324}^{25} = 194\ (r^2)$

6b) Tangent equation:

$m = \frac{\Delta y}{\Delta x} \rightarrow \frac{9-4}{-5-8} = \frac{5}{-13}$

$\downarrow$

$\frac{13}{5}$

$y = mx + c$

$\cancel{1}4 = \frac{13}{5}(8) + c$

$\cancel{1}4 = \frac{104}{5} + c$

$\cancel{1}4 - \frac{104}{5} = c$

$-\frac{84}{5}$

$y = \frac{13}{5}x - \frac{84}{5}$

$5y = 13x - 84 \rightarrow$ y axis $= y = 0$

$5(0) = 13x - 84$

$0 = 13x - 84$

$84 = 13x$

$x = \frac{84}{\cancel{13}\ 5}$, $B = (0; \frac{84}{\cancel{13}\ 5})$

~~xn~~ $m = \frac{14-9}{8 - -5} \rightarrow \frac{5}{13} \rightarrow -\frac{13}{5}$

$y = mx + c$

$14 = -\frac{13}{5}(8) + c$

$14 = -\frac{104}{5} + c$

$14 + \frac{104}{5} = \frac{174}{5}$

$y = -\frac{13}{5}x + \frac{174}{5}$

$5y = -13x + 17\cancel{8}^{4} \rightarrow$ y axis $= y = 0$

$5(0) = -13x + 17\cancel{8}4$

$0 = -13x + 17\cancel{8}4$

$-17\cancel{8}^{4} = -13x$

$\frac{17\cancel{8}^{4}}{\cancel{13}\ 5} = x \rightarrow B = (0, \frac{174}{\cancel{13}\ 5})$

distance :

$$\sqrt{\left(\frac{174}{5\times3} - \frac{84}{5\times3}\right)^2 + (0-0)^2}$$

$$\sqrt{\cancel{115976334} + 0}$$

$\frac{258}{5}$

$= \frac{258}{5} = 51.6$

7a) $m = \frac{\Delta y}{\Delta x} \rightarrow \frac{8-10}{7-1} = \frac{-2}{6} = -\frac{1}{3} = 3$

midpoint of P and Q :

$\left(\frac{1+7}{2}\right) = 4 \quad \left(\frac{10+8}{2}\right) = 9 \quad (4, 9)$

$9 = 3(4) + c \qquad y = mx + c$

$9 = 12 + c \qquad \underline{y = 3x - 3}$

$9 - 12 = c$

$-3 = c$

7b) $3(3) - 3$

$9 - 3 = 6\ (y)$

$k = 6$

7c) $(x-3)^2 + (y-6)^2 = r^2$

$(1-3)^2 + (10-6)^2 = r^2$

$(-2)^2 + (4)^2 = r^2$

$4 + 16 = 20$

$(x-3)^2 + (y-6)^2 = 20$

7d) $(x-3)^2 + (3x-3-6)^2 = 20$

$(x-3)^2 + (3x-9)^2 = 20$

$x^2 - 6x + 9 + 9x^2 - 54x + 81 = 20$

$10x^2 - 60x + 90 = 20$

$10x^2 - 60x + 70 = 0$

$x = 3 + \sqrt{2} \quad x = 3 - \sqrt{2}$

$y = 3(3+\sqrt{2}) - 3 = 6 + 3\sqrt{2}$

$y = 3(3-\sqrt{2}) - 3 = 6 - 3\sqrt{2}$

$(3+\sqrt{2},\ 6+3\sqrt{2})$

$(3-\sqrt{2},\ 6-3\sqrt{2})$

8a) $(x-4)^2 + (y+1)^2 = r^2$

$(-2-4)^2 + (3+1)^2 = r^2$

$(-6)^2 + (4)^2 = r^2$

$36 + 216 = r^2$

$52 = r^2$

$(x-4)^2 + (y+1)^2 = 52$

8b) midpoint of PQ

$= \frac{2+8}{2} = 3 \quad \frac{3+5}{2} = 4 \quad (3, 4)$

$(4, -1) \rightarrow$ midpoint of circle

gradient $= \frac{\Delta y}{\Delta x} \rightarrow \frac{4 - -1}{3-4} = -5$

$y = mx + c \qquad y = -5x + 19$

$4 = -5(3) + c$

$4 = -15 + c$

$4 + 15 = c$

$19 = c$

8b) A = let $y = 0$ B = let $x = 0$

$0 = -5x + 19 \qquad y = -5(0) + 19$

$-19 = -5x \qquad y = 0 + 19$

$x = \frac{19}{5} \qquad y = 19$

$\frac{\text{Base} \times \text{height}}{2} = \text{Area}$

$\frac{\frac{19}{5} \times 19}{2} = \frac{361}{10} = 36.1$

9a) $(x-1)^2-1+(y+3)^2-9-30=0$

$(x-1)^2+(y+3)^2=1+9+30$

$(x-1)^2+(y+3)^2=40$

centre = $(1,-3)$

gradient = $\frac{\Delta y}{\Delta x} \rightarrow \frac{-5--3}{7-1} = \frac{-2}{6} = \frac{-1}{3}$

$\downarrow$

3

$y=mx+c$ $\quad$ $y=3x-26$

$-5=3(7)+c$ $\quad$ $\cancel{6\ 3x-y+26=0}$

$-5=21+c$ $\quad$ $\cancel{3y\ y+26=0}$

$-21-5=c$ $\quad$ $3x-y-26=0$

$-26=c$ $\quad$ or $-3x+y+26=0$

9b) midpoint of PQ

$\frac{3+7}{2}=5$ $\quad$ $\frac{3+-5}{2}=-1$

$(5,-1)$

$m=\frac{\Delta y}{\Delta x} \rightarrow \frac{-5-3}{7-3} = \frac{-8}{4} = -2$

$\downarrow$

$\frac{1}{2}$

$y=mx+c$

$-1=\frac{1}{2}(5)+c$

$-1=\frac{5}{2}+c$

$-1-\frac{5}{2}=c$

$-\frac{7}{2}=c$

$y=\frac{1}{2}x-\frac{7}{2}$

$2y=x-7$

$x-2y-7=0$

9c) $\cancel{6x-2y-52=0}$ $3x-y-26=0$

$x-2y-7=0$

use calculator and put equation in this form.

$3x-y=26$

$x-2y=7$

$x=9$, $y=1$

$(9,1)$