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#### 4.4 points of intersection

1 a.

$$y = x(x - 4) \quad \cup$$

y intercept set  $x = 0$

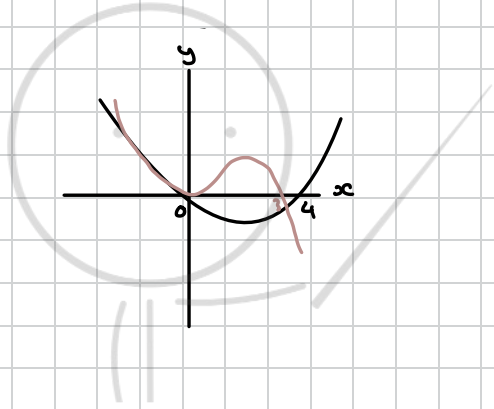
$$y = 0(0 - 4)$$

$$y = 0$$

x intercept set  $y = 0$

$$0 = x(x - 4)$$

$$x = 0 \quad x = 4$$



$$y = x^3(3 - x) \quad \cup$$
$$= x^4 + 3x^3$$

y intercept set  $x = 0$

$$y = 0(3 - 0)$$

$$y = 0 \quad (\text{double root})$$

x intercept set  $y = 0$

$$0 = x^3(3 - x)$$

$$x = 3$$

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1 b.

$$y = x^2 + x - 2 \quad \cup$$

$$(x + 2)(x - 1)$$

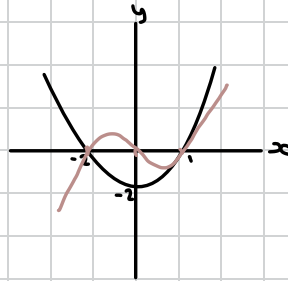
x intercept set  $y = 0$

$$x = -2 \quad x = 1$$

y intercept set  $x = 0$

$$(2) \times (-1)$$

$$y = -2$$



$$y = x^3 + x^2 - 2x \quad \cup$$
$$= x(x + 2)(x - 1)$$

x intercept set  $y = 0$

$$x = 0 \quad x = -2 \quad x = 1$$

y intercept set  $x = 0$

$$(0) (2) (-1)$$

$$y = 0$$

2 a.

$$y = x^3$$

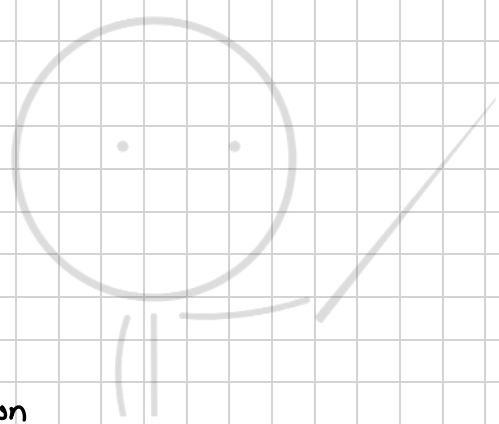
$$y = \frac{5}{x^2}$$

2 b.

$$x^3 = \frac{5}{x^2}$$

$$x^5 = 5$$

$$x^5 - 5 = 0$$



2 c. has 1 real solution

3 a.

$$y = -\frac{3}{x^2}$$

$y = -x^2(2x+5)$

x intercept  $y = 0$

$x = 0 \quad x = 0 \quad x = -5/2$

y intercept  $x = 0$

$(0) 2(0)$

$y = 0$

BF MATHS

3 b.

$$-\frac{3}{x^2} = -x^2(2x+5)$$

$$3 = x^4(2x+5)$$

$$x^4(2x+5) - 3 = 0$$

3 c. 3 as the two graphs intersect 3 times

4 a i.

$$f(x) = -\frac{1}{x}$$

$$g(x) = (x+1)(x-3)^2$$

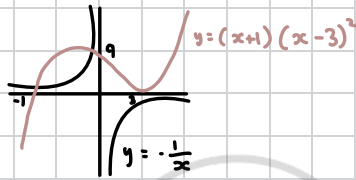
x intercept

$$x = -1 \quad x = 3$$

y intercept  $x = 0$

$$(1) (3)^2$$

$$y = 9$$



4 a ii.

2 intersections = 2 real solution

4 b i.

$$f(x) = \frac{7}{x^2}$$

$$g(x) = x^2(4x-9)$$

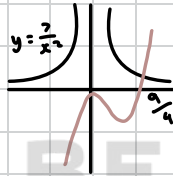
x intercept  $y = 0$

$$x = 0 \quad x = \frac{9}{4}$$

y intercept  $x = 0$

$$0(4(0)-9)$$

$$y = 0$$



4 b ii.

1 intersection = 1 real solution

5.

$$\frac{2}{x} - 5 = 2x - 5$$

$$\frac{2}{x} = 2x$$

$$2 = 2x^2$$

$$1 = x^2$$

$$x = 1 \quad x = -1$$

Sub  $x = 1$  into  $y = 2x - 5$

$$y = 2(1) - 5 = -3$$

$$(1, -3)$$

Sub  $x = -1$  into  $y = 2x - 5$

$$y = 2(-1) - 5 = -7$$

$$(-1, -7)$$

Coordinates of intersection

$$(1, -3) \quad (-1, -7)$$

6 a.  $y = -\frac{1}{4}x^3$

$$y = x^2(x^2 - x - 2)$$

$$x^2(x+1)(x-2)$$

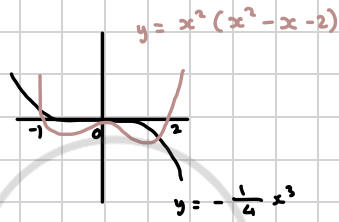
x intercept set  $y = 0$

$$x = 0 \quad x = -1 \quad x = 2$$

y intercept set  $x = 0$

$$0 \quad (1) \quad (-2)$$

$$y = 0$$



6 b. 3 intersection = 3 real solution

6 c.  $-4x^2(x^2 - x - 2) = x^3$

$$-4x^2(x^2 - x - 2) - x^3 = 0$$

$$-4x^4 + 4x^3 + 8x^2 - x^3 = 0$$

$$-4x^4 + 3x^3 + 8x^2 = 0$$

$$-x^2(4x^2 - 3x - 8) = 0$$

$$x^2(4x^2 - 3x - 8) = 0$$

6 d.  $x^2(4x^2 - 3x - 8) = 0$

$$x^2 = 0 : x = 0$$

$$4x^2 - 3x - 8 = 0$$

$$a = 4 \quad b = -3 \quad c = -8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = 0 \text{ or } x = \frac{3 \pm \sqrt{137}}{8}$$

7 a i. x intercept set  $y = 0$

$$0 = \frac{1}{x} + 2$$

$$-2 = \frac{1}{x}$$

$$x = -\frac{1}{2} = -0.5$$

$$(-0.5, 0)$$

ii.  $y = \frac{1}{x} + 2$

asymptotes verticle

$$x = 0$$

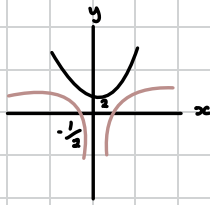
asymptotes horizon

$$y = 2$$

7 b.

$$y = \frac{1}{x} + 2$$

$$y = x^2 + 2$$



7 c.

$$\frac{1}{x} + 2 = x^2 + 2$$

$$\frac{1}{x} = x^2$$

$$1 = x^3$$

$$x = 1$$

Sub  $x = 1$  into  $x^2 + 2$

$$y = 3$$

Points of intersection (1, 3)

8 a.

$$y = -\frac{4}{x}$$

$$y = x(x+3)(2-x)$$

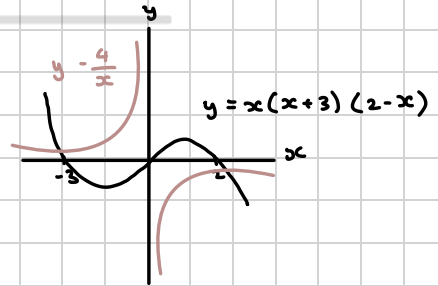
x intercept set  $y = 0$

$$x = 0 \quad x = -3 \quad x = 2$$

y intercept set  $x = 0$

$$(0)(3)(2)$$

$$y = 0$$

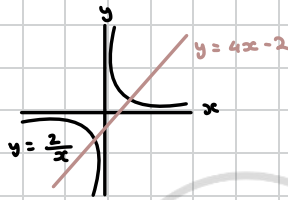


b: 2 intersections = 2 real solutions

9 a.

$$y = \frac{2}{x}$$

$$y = 4x - 2$$



9 b.

$$\frac{2}{x} = 4x - 2$$

$$2 = x(4x - 2)$$

$$2 = 4x^2 - 2x$$

$$4x^2 - 2x - 2 = 0$$

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

$$x = -\frac{1}{2} \quad x = 1$$

Sub  $x = -\frac{1}{2}$  into  $4x - 2$

$$y = -4$$

Sub  $x = 1$  into  $4x - 2$

$$y = 2$$

intersections =  $(-0.5, -4)$   
 $(1, 2)$

10.

$$x^2 - 10x - 15 = 10 - 10x$$

$$x^2 = 25$$

$$x = 5 \quad \text{or} \quad x = -5$$

Sub  $x = 5$  into  $10 - 10x$

$$y = -40$$

Sub  $x = -5$  into  $10 - 10x$

$$y = 60$$

intersections:  $(5, -40)$   
 $(-5, 60)$

$$11a. f(x) = g(x)$$

$$x^2 - 9x + 24 = 2x + a$$

$$x^2 - 11x + (24 - a) = 0$$

$$a = 1 \quad b = -11 \quad c = (24 - a)$$

$$D = b^2 - 4ac$$

$$(-11)^2 - 4(1)(24 - a) > 0$$

$$121 - 96 + 4a > 0$$

$$25 + 4a > 0$$

$$4a > -25$$

$$a > -\frac{25}{4}$$

$$a > -6.25$$

$$11b. \quad x^2 - 9x + 24 = 2x + a$$

$$x^2 - 9x + 24 = 2x + 5$$

$$x^2 - 11x + 19 = 0$$

$$\text{Sub into } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1 \quad b = -11 \quad c = 19$$

$$x = 8.86 \text{ (3 s.f.)} \quad x = 2.15 \text{ (3 s.f.)}$$

$$\text{Sub } 8.86 \text{ into } g(x) = 2x + 5$$

$$y = 22.72$$

$$\text{Sub } 2.15 \text{ into } g(x) = 2x + 5$$

$$y = 9.3$$

$$A = (2.15, 9.3)$$

$$B = (8.86, 22.7)$$