

Chapter 2 - Conditional Probability

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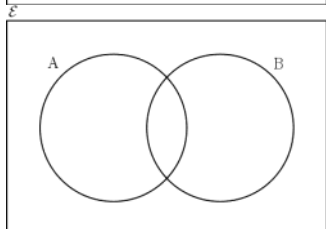
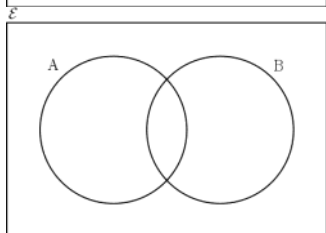
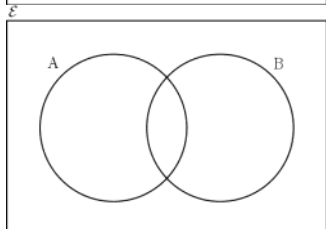
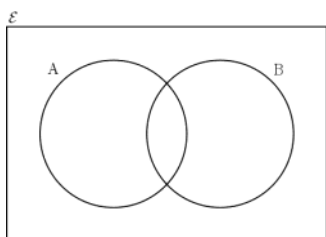
Personal notes:



2.1 - Set notation

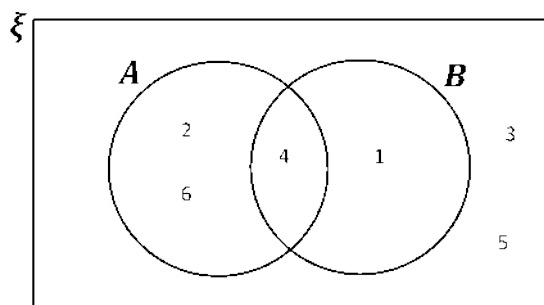
Recap from Year 1 Stats

- Venn diagrams are named after the English mathematician John Venn (1834 - 1923)



Intersection of "not A" and "not B"

Example



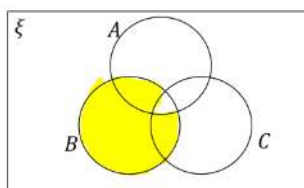
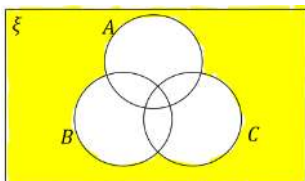
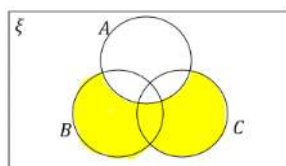
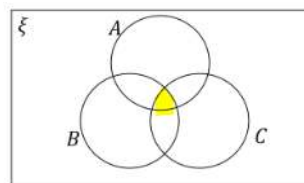
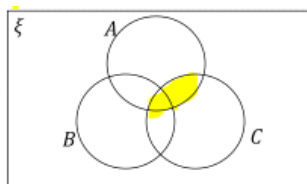
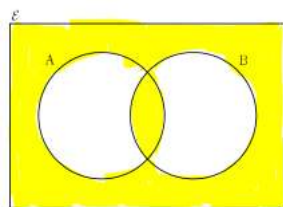
ξ = set of all outcomes in throwing a die
 A = set of even numbers
 B = set of square numbers

- A' =
- $A \cup B$ =
- $A \cap B$ =
- $A \cap B'$ =
- $(A \cup B)'$ =
- $(A \cap B)'$ =

2.1 - Set notation

Example

Use set notation to describe the shaded area of the following



Example

A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace and D the event that the card is a diamond. Find

- a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$

Example

Given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$,

- a) explain why events A and B are not independent.

Given also that $P(C) = 0.2$, that events A and C are mutually exclusive and events B and C are independent,

- b) draw a Venn diagram to illustrate the events A , B and C , showing the probabilities for each region.

- c) Find $P((A \cap B') \cup C)$.



2.1 - Set notation

Practice

A , B and C are three events with $P(A) = 0.47$, $P(B) = 0.25$ and $P(C) = 0.36$. $P(A \cap C) = 0.16$. Given that A and B are mutually exclusive and B and C are independent,

a) draw a Venn diagram to illustrate the probabilities

bi) $P(A' \cap B')$

bii) $P(A \cup (B \cap C'))$

biii) $P((A \cap C') \cup B')$



2.2, 2.3 - Conditional probability

Starter

Akshaya loves playing chess online. Among the many games she has played, the website listed her chance of playing white was 0.55. Akshaya's probability of winning when she plays white is 0.6 and the probability of winning with black is 0.4. Draw a tree diagram to illustrate and work out the probabilities of all outcomes.

Set notation version

Notes

- Probability towards any potential outcome could change easily based on different condition.
E.g. the chance of getting to school on time depends on if there is traffic on the way / bus delay...etc
- We can express this type of probability as conditional probability.
- The probability that B occurs given that A has already occurred is written as $P(B|A)$.
- For independent events, $P(A|B) = P(A|B') = P(A)$ because independence means the probability of one event happening does not affect the other happening.
- Alternatively $P(B|A) = \frac{P(A \cap B)}{P(A)}$



2.2, 2.3 - Conditional probability

Example

A school has 80 students in year 10. Of all the students, 50 say they like Maths (M) only and 12 say they like English (E) only. 6 say they like both Maths and English.

a) Draw a two-way table to show this information.

b) Draw a Venn diagram

ci) find $P(M|E)$

cii) find $P(M|E')$

ciii) find $P(E|M')$

Practice

75 students take part in a sports day. 40 students are girls. Of all the girls, 18 do 800m run and the rest do high jump. There are 25 boys running the 800m and the rest do high jump.

a) Draw a two-way table to illustrate the situation.

bi) find $P(\text{Boy}|\text{800m})$

bii) find $P(\text{Girl}|\text{high jump})$

biii) find $P(\text{high jump}|\text{Girl})$



2.2, 2.3 - Conditional probability

Example

Given that $P(A) = 0.4$ and $P(A \cap B) = 0.2$, what is $P(B|A)$?

Example

Given that $P(A) = 0.7$ and $P(A \cap B) = 0.15$, what is $P(A|B')$?

Drawing a Venn diagram might help.

Practice

Given that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.25$, what is $P(B|A')$?

Practice

Given that $P(A) = 0.3$, $P(A \cap B) = 0.19$, $P(B) = 0.47$

Find

- $P(A \cup B)$
- $P(B')$
- $P(A|B)$
- $P(A|B')$
- $P(B|A')$
- $P(B'|A')$
- $P((A \cup B)|B')$
- $P(B|(A' \cap B'))$



2.2, 2.3 - Conditional probability

Exam Practice (9MA0/31 Oct 2021, Q4)

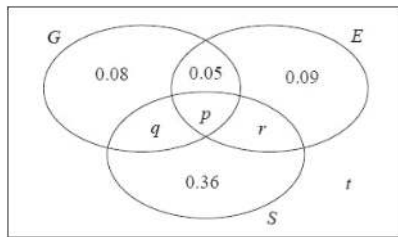
A large college produces three magazines.

One magazine is about green issues, one is about equality and one is about sports.

A student at the college is selected at random and the events G , E and S are defined as follows:

- G is the event that the student reads the magazine about green issues
- E is the event that the student reads the magazine about equality
- S is the event that the student reads the magazine about sports

The Venn diagram, where p , q , r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in college who read exactly one of these magazines.

(1)

No students read all three magazines and $P(G) = 0.25$

(b) Find

- the value of p
- the value of q

(3)

Given that $P(S|E) = \frac{5}{12}$

(c) find

- the value of r
- the value of t

(4)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.

Show your working clearly.

(3)

Total 11 marks



2.4 - Probability formulae

Notes

There are two useful formulae when it comes to calculating probabilities in Venn diagrams:

- **Addition formula:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
why? Because the two circles overlap in Venn diagram
- **Multiplication formula:** $P(B \cap A) = P(B|A) \times P(A)$
why? Because $P(B|A) = \frac{P(B \cap A)}{P(A)}$, then re-arrange

Example

A and B are two events such that $P(A) = 0.3$, $P(B) = 0.7$ and $P(A|B) = 0.2$. Find:

- a) $P(A \cap B)$ b) $P(B|A)$ c) $P(A \cup B)$

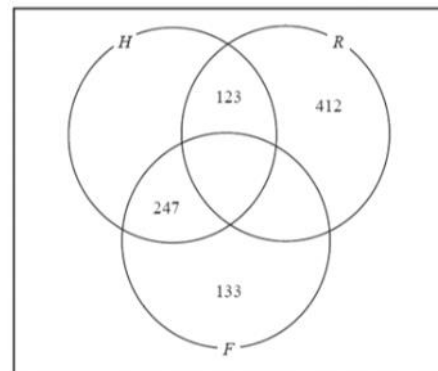
Exam Practice (9MA0/31, June 2022 Q6)

A company has 1825 employees.
The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, A or B , where the employees live

	A	B
Professional	740	380
Skilled	275	90
Elementary	260	80



An employee is chosen at random.

Find the probability that this employee

- (a) is skilled,

(1)

- (b) lives in area B and is not a professional.

(1)

Some classifications of employees are more likely to work from home.

- 65% of professional employees in both area A and area B work from home
- 40% of skilled employees in both area A and area B work from home
- 5% of elementary employees in both area A and area B work from home
- Event F is that the employee is a professional
- Event H is that the employee works from home
- Event R is that the employee is from area A

- (c) Using this information, complete the Venn diagram below.

(4)

- (d) Find $P(R' \cap F)$

(1)

- (e) Find $P((H \cup R)')$

(1)

- (f) Find $P(F|H)$

(2)

2.5 - Tree diagrams

Notes

- You have seen tree diagrams before, here the questions about tree diagrams are mixing with conditional probability.
- Key word is: **Given**

Example

A box contains two colours of marbles - red and blue. There are 8 red marbles and 5 blue marbles. A marble is taken from the box at random and the marble will not be replaced, another marble is then taken.

- Draw a tree diagram to illustrate
- Find the probability that a blue marble is taken at least once
- Find the probability that both marbles picked are red given that they are both same colour.

Example (9MA0/31 June 2019 Q1)

Three bags, *A*, *B* and *C*, each contain 1 red marble and some green marbles.

Bag *A* contains 1 red marble and 9 green marbles only

Bag *B* contains 1 red marble and 4 green marbles only

Bag *C* contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from bag *A*.

If he selects a red marble, he stops selecting.

If the marble is green, he continues by selecting at random one marble from bag *B*.

If he selects a red marble, he stops selecting.

If the marble is green, he continues by selecting at random one marble from bag *C*.

- Draw a tree diagram to represent this information. (2)
- Find the probability that Sasha selects 3 green marbles. (2)
- Find the probability that Sasha selects at least 1 marble of each colour. (2)
- Given that Sasha selects a red marble, find the probability that he selects it from bag *B*. (2)

(Total for question = 8 marks)



2.5 - Tree diagrams

Exam Practice (S1 June 2014 Q11)

In a factory, three machines, J , K and L , are used to make biscuits.

Machine J makes 25% of the biscuits.

Machine K makes 45% of the biscuits.

The rest of the biscuits are made by machine L .

It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made by machine L are broken.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities.

(2)

A biscuit is selected at random.

(b) Calculate the probability that the biscuit is made by machine J and is not broken.

(2)

(c) Calculate the probability that the biscuit is broken.

(2)

(d) Given that the biscuit is broken, find the probability that it was not made by machine K .

(3)

(Total 9 marks)

