

Chapter 7 - Statics and Inclined Plane

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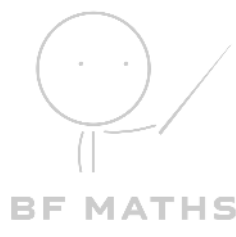
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Personal notes:



7.1 - Static Particles

Notes

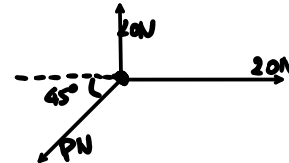
Static particles =

The 3 steps to work out questions related to forces in different directions:

- 1) Draw a force diagram that only includes the forces on x and y-directions
- 2) Resolve any force(s) that have not been included (Break the force into x and y-direction) and then include in step 1
- 3) Use $F=ma$ on both directions: Consider x-direction, then consider y-direction

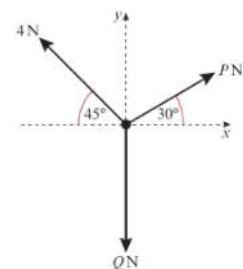
Example

Find P such that the particle is in equilibrium.



Example

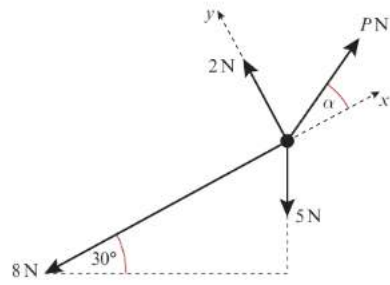
The diagram shows a particle in equilibrium under the forces shown. By resolving horizontally and vertically find the magnitudes of the forces P and Q .



7.1 - Static Particles

Example 3

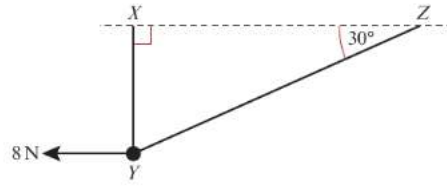
The diagram shows a particle in equilibrium on an inclined plane under the forces shown.
Find the magnitude of the force P and the size of the angle α .



7.2 - Modelling with Static Particles

Example 1

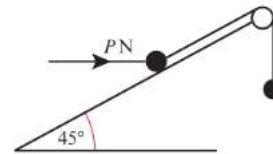
A smooth bead Y is threaded on a light inextensible string. The ends of the string are attached to two fixed points, X and Z , on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 8 N acting parallel to ZX . The bead Y is vertically below X and $\angle XZY = 30^\circ$ as shown in the diagram.



Find the tension in the string and the weight of the bead.

Example 2

A mass of 3 kg rests on the surface of a smooth plane which is inclined at an angle of 45° to the horizontal. The mass is attached to a cable which passes up the plane along the line of greatest slope and then passes over a smooth pulley at the top of the plane. The cable carries a mass of 1 kg freely suspended at the other end. The masses are modelled as particles, and the cable as a light inextensible string. There is a force of $P\text{ N}$ acting horizontally on the 3 kg mass and the system is in equilibrium.



- Calculate: **a** the magnitude of P **b** the normal reaction between the mass and the plane.
c State how you have used the assumption that the pulley is smooth in your calculations.



7.3, 7.5 - Inclined Plane (with Friction)

- On the point slipping = Limiting equilibrium (acceleration on both directions = 0)

Example 1

A box of mass 10 kg rests in limiting equilibrium on a rough plane inclined at 20° above the horizontal.

a Find the coefficient of friction between the box and the plane.

A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium,

b find the maximum possible value of P .



7.3, 7.5 - Inclined Plane (with Friction)

Example 2

A particle of mass 5 kg is projected up a rough slope at 16 m s^{-1} and comes to rest at a point P after 5 s. Given that the slope is inclined at 10° to the horizontal,

a work out the coefficient of friction μ . **(7 marks)**

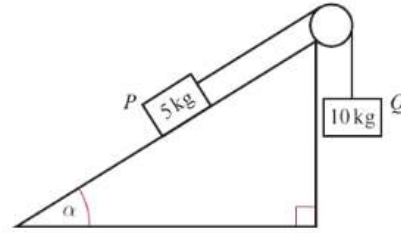
b State, with supporting calculations, whether the particle will remain at rest at P or will begin to slide back down the slope. **(2 marks)**



7.6 - Inclined Plane with Pulleys

Example 1

Two particles P and Q of masses 5 kg and 10 kg respectively are connected by a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. P rests on the inclined plane and Q hangs on the edge of the plane with the string vertical and taut. The plane is inclined to the horizontal at an angle α where $\tan \alpha = 0.75$, as shown in the diagram. The coefficient of friction between P and the plane is 0.2. The system is released from rest.



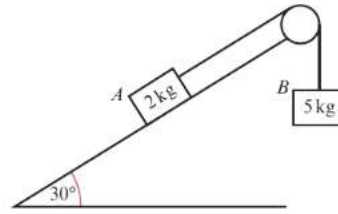
- a** Find the acceleration of the system. **b** Find the tension in the string.



7.6 - Inclined Plane with Pulleys

Example 2

One end of a light inextensible string is attached to a block A of mass 2 kg . The block A is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle of 30° . The string lies along the line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a block B of mass 5 kg . The system is released from rest. By modelling the blocks as particles and ignoring air resistance,



- a i show that the acceleration of block B is $\frac{4}{7}g$
- ii find the tension in the string.
- b State how you have used the fact that the string is inextensible in your calculations.
- c Calculate the magnitude of the force exerted on the pulley by the string.



7.4 - Static Rigid Bodies

Notes

For a rigid **body** (not particle) in static equilibrium:

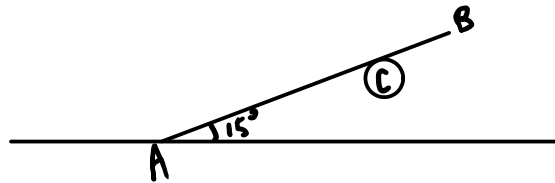
- Resultant Force in x and y-directions is 0N (or acceleration is 0ms^{-2})
Set up a $F = ma$ equation for each x and y-direction
- Resultant Moment is 0Nm.
3 steps for moments in equilibrium:
 - Set up 2 equations :
Vertical forces equal : Upward forces = Downward forces
Horizontal forces equal : Leftward forces = rightward forces
 - Take moments at a point (choose wisely), determine direction and work out the moment of each force
 - Sum of CW Moments = Sum of ACW moments

***Note : This is not inclined plane

Example (Scenario 1 - rod on a peg)

A uniform rod AB of mass 40 kg and length 10 m rests with the end A on rough horizontal ground. The rod rests against a smooth peg C where $AC = 8$ m. The rod is in limiting equilibrium at an angle of 15° to the horizontal. Find:

- the magnitude of the reaction at C
- the coefficient of friction between the rod and the ground.

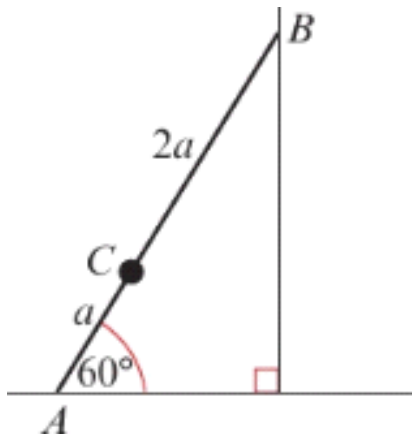


7.4 - Static Rigid Bodies

Example (Scenario 2 - Ladder on a wall)

A ladder AB , of mass m and length $3a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $2m$ is fixed on the ladder at the point C , where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as particle. The ladder rests in limiting equilibrium at an angle of 60° with the ground.

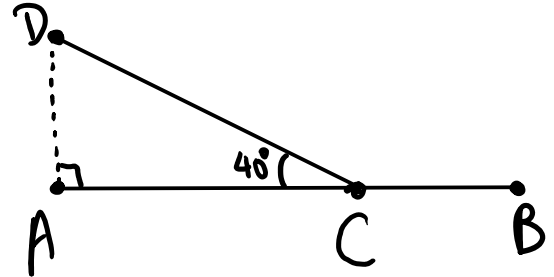
- Find the coefficient of friction between the ladder and the ground.
- State how you have used the assumption that the ladder is uniform in your calculations.



7.4 - Static Rigid Bodies

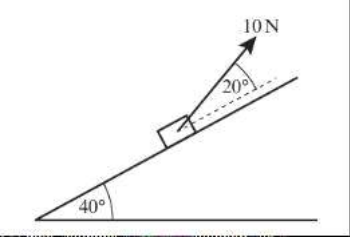
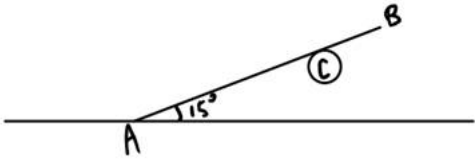
Example (Scenario 3 - Drawbridge)

A uniform rod AB , of mass 6 kg and length 4 m, is smoothly hinged at A . A light inextensible string is attached to the rod at point C , where $AC = 3$ m, and to a point D , vertically above A . If the string keeps the rod in equilibrium in a horizontal position and the angle between the string and the rod is 40° , calculate a) the tension in the string, and b) the magnitude and direction of the force exerted on the rod by the wall.



Particles vs Bodies

Summary : Solving Static Forces Questions between Particles and Bodies

Particles (block of mass, particle)	Bodies (Ladder, rod)
Example Diagram given in question: 	Example Diagram given in question: 
Steps to solve (Refer to 7.1 Attachments): 1. Draw a force diagram that only includes the forces on x and y-directions 2. Resolve any force(s) that have not been included (Break the force into x and y-direction) and then include in step 1 3. Use $F = ma$ on both directions: Consider x-direction, then consider y-direction	Steps to solve (Refer to 7.4 Attachments, very similar to moments apart from Step 1): 1. Resolve any forces that are not horizontal or vertical 2. Set up two equations: - Vertical Forces equal (Upward forces = downward forces) - Horizontal Forces equal (Leftward forces = rightward forces) 3. Take moments at a point (determine direction and work out moment of each force) 4. CW Moments = ACW Moments

