Chapter 8 - Further Kinematics

Personal notes:

8.1 - Vectors in Kinematics

Notes

Example

A particle starts from the point with position vector $(3\boldsymbol{i} + 7\boldsymbol{j})$ m and moves with constant velocity $(2\boldsymbol{i} - \boldsymbol{j})ms^{-1}$.

a) Find the position vector of the particle 4 seconds later.

b) Find the time at which the particle is due east of the origin.

8.1 - Vectors in Kinematics

Example

An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point *O* and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})ms^{-1}$.

At time $t = 20$ s the skater the travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})ms^{-1}$.

Relative to *O*, the skater has position vector **s** at time t seconds.

Modelling the ice skater as a particle with constant acceleration, find:

a) the acceleration of the ice skater

b) an expression for **s** in terms of t

c) the time at which the skater is directly north-east of *O*.

A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to *O* at time t.

d) Show that the two skaters will meet.

8.2 - Vectors in Projectiles

Recall the vector version of suvat formulae:

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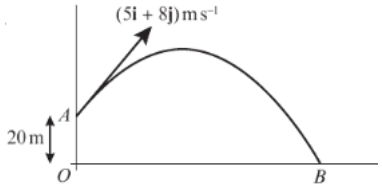
Example

A ball is struck by a racket from a point *A* which has position vector 20**j** m relative to a fixed origin *O*. Immediately after being struck, the ball has velocity $(5\boldsymbol{i} + 8\boldsymbol{j})ms^{-1}$, where **i** and **j** are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point *B.*

a) Find the speed of the ball 1.5 seconds after being struck.

b) Find an expression for the position vector, **r**, of the ball relative to *O* at time *t* seconds.

c) Hence determine the distance *OB*.



8.2 - Vectors in Projectiles

Example

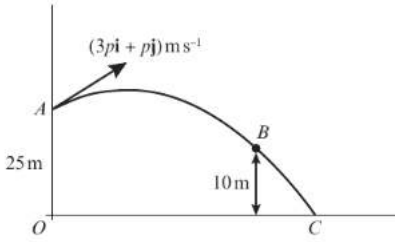
A ball is projected from a point *A* at the top of a cliff, with position vector $25\boldsymbol{j}$ m relative to the base of the cliff *O*. The base of the cliff is at sea level. The velocity of projection is $(3p\boldsymbol{i} + p\boldsymbol{j})ms^{-1}$, where *p* is a constant. After 2 seconds, the ball passes a point *B* with position vector $(q\boldsymbol{i} + 10\boldsymbol{j})$ m, where *q* is a constant, before hitting the sea at point *C*. The ball is modelled as a particle moving freely under gravity and the sea is modelled as a horizontal plane.

a) Suggest, with reasons, which of these two modelling assumptions is most realistic. **(2 marks)**

b) Find the velocity vector of the ball at point *B*. **(6 marks)**

A remote-control boat leaves *O* at the same time the ball is projected, and travels in a straight line towards *C* with constant acceleration. Given that the ball lands on the boat,

c) find the acceleration of the boat. **(6 marks)**



8.3 - Variable Acceleration in 1-D

Recall:

Example

A particle is moving in a straight line with acceleration at time t seconds given by $a = \cos 2\pi t \ ms^{-2}$, where $t \geq 0$

The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi} \ ms^{-1}$. Find:

a) an expression for the velocity at time t seconds

b) the maximum speed

c) the distance travelled in the first 3 seconds.

8.3 - Variable Acceleration in 1-D

Example

A particle of mass 6 kg is moving on the positive x-axis. At time *t* seconds the displacement, *s*, of the particle from the origin is given by

$s = 2t^{\frac{3}{2}} + \frac{e^{-2t}}{3}$ m, where $t \geq 0$

a) Find the velocity of the particle when $t = 1.5$.

b) Given that the particle is acted on by a single force of variable magnitude *F* N which acts in the direction of the positive x-axis, find the value of *F* when $t = 2$.

Example

A particle *P* moves on the x-axis. At time *t* seconds the velocity of *P* is $v\ ms^{-1}$ in the direction of x increasing, where v is given by

$$v = \begin{cases} 10t - 2t^{\frac{3}{2}}, & 0 \leq t \leq 4 \\ 24 - \left(\frac{t-4}{2}\right)^4, & t > 4 \end{cases}$$

When $t = 0$, *P* is at the origin *O*. Find

a) the greatest speed of *P* in the interval $0 \leq t \leq 4$ **(4 marks)**

b) the distance of *P* from *O* when t = 4 **(3 marks)**

c) the time at which *P* is instantaneously at rest for $t > 4$ **(1 mark)**

d) the total distance travelled by *P* in the first 10 seconds of its motion. **(7 marks)**

8.4, 8.5 - Vectors with Calculus (Last lesson ☹)

Notes

Example

A particle *P* of mass 0.8 kg is acted on by a single force **F** N. Relative to a fixed origin *O*, the position vector of *P* at time *t* seconds is **r** metres, where

$r = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$

Find:

a) the speed of *P* when $t = 4$

b) the acceleration of *P* as a vector when $t = 2$

c) **F** when $t = 2$

8.4, 8.5 - Vectors with Calculus

Example

A particle *P* moves in a horizontal plane. At time *t* seconds, the position vector of *P* is **r** metres relative to a fixed origin *O*, and **r** is given by $\boldsymbol{r} = (20t - 2t^3)\boldsymbol{i} + kt^2\boldsymbol{j}$, $t \geq 0$ where *k* is a positive constant. When $t = 2$, the speed *of P* is $16ms^{-1}$. Find:

a) the value *of k* **(6 marks)**

b) the acceleration of *P* at the instant when it is moving parallel to **j**. **(4 marks)**

Example

A particle *P* is moving in a plane. At time *t* seconds, its velocity **v** ms^{-1} is given by $\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}$, $t \geq 0$

When $t = 0$, the position vector *of P* with respect to a fixed *origin O* is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector *of P* at *time t* seconds.

8.4, 8.5 - Vectors with Calculus

Example

The velocity of a particle *P* at time *t* seconds is $\left(\left(3t^2 - 8\right)\boldsymbol{i} + 5\boldsymbol{j}\right) ms^{-1}$. When $t = 0$, the position vector of *P* with respect to a fixed origin *O* is $(2\boldsymbol{i} - 4\boldsymbol{j})$ m.

a) Find the position vector of *P* after *t* seconds.

A second particle *Q* moves with constant velocity $\left(8\boldsymbol{i} + 4\boldsymbol{j}\right) ms^{-1}$. When $t = 0$, the position vector of *Q* with respect to the fixed origin *O* is 2**i** m.

b) Prove that *P* and *Q* collide.