

Chapter 7 - Hypothesis Testing

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Personal notes:

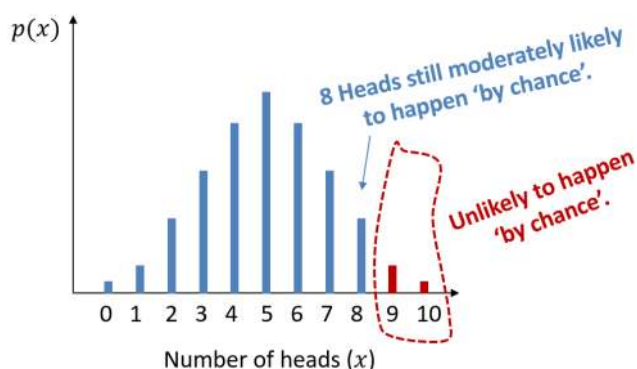


7.1 - Hypothesis testing

What is Hypothesis testing?

Consider the following situation:

"I throw a coin 10 times. For what numbers of heads might you conclude that the coin is biased towards heads? Why?"



Definitions	In this case:
<ul style="list-style-type: none"> • A hypothesis is a statement made about the value of a _____ that we wish to test by collecting evidence in the form of a sample. • The null hypothesis, H_0, is the default position, i.e. • The alternative hypothesis, H_1, is that there has been some change in the population parameter. • The test statistic, X, is the result of the experiment or the statistic that is calculated from the sample. 	<ul style="list-style-type: none"> • Statement to verify: "Is the coin biased towards heads?" -> Population parameter is • The "default position" is that the coin is fair. -> • The "alternative position" is that coin is biased towards heads. -> • X is the

- **One-tailed test** if H_1 is in the form $H_1: p < \dots$ or $H_1: p > \dots$
- **Two-tailed test** if H_1 is in the form $H_1: p \neq \dots$
- **Significance level** is the statistical threshold that tells if the null hypothesis being rejected or not. (You will be told which significance level to use in questions, 10%, 5% or 1%)

7.1 - Hypothesis testing

Example

John wants to see if a coin is unbiased or if it is biased towards coming down heads. He tosses the coin 8 times and counts the number of times, X , that it lands head uppermost.

- a) Describe the test statistic.
- b) Write down a suitable null hypothesis.
- c) Write down a suitable alternative hypothesis.

Example

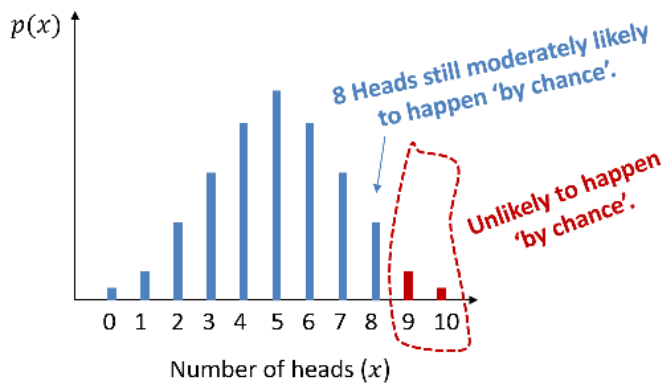
An election candidate believes she has the support of 40% of the residents in a particular town. A researcher wants to test, at the 5% significance level, whether the candidate is over-estimating her support. The researcher asks 20 people whether they support the candidate or not. 3 people say they do.

- a) Write down a suitable test statistic.
- b) Write down two suitable hypotheses.
- c) Explain the condition under which the null hypothesis would be rejected.



7.2 - Critical value and region

- **Critical region** is the region of the probability distribution which, if the _____ falls within it, would cause you to _____.



- We have learnt to work out the cumulative probabilities in previous chapter (e.g. $P(X \leq 5)$), this value is known as the **p-value**.
- If $p - value > sig. level$:
- If $p - value < sig. level$:
- The **actual significance level** is the actual probability of being in the critical region.

Example (the probability distribution graph above)



7.2 - Critical value and region

Example (1-tailed crit. region)

A single observation is taken from a binomial distribution $B(6, p)$. The observation is used to test $H_0: p = 0.35$ against $H_1: p > 0.35$.

- a) Using a 5% level of significance, find the critical region for this test.
- b) State the actual significance level of this test.

Example

A single observation is taken from a binomial distribution $B(15, p)$. The observation is used to test $H_0: p = 0.40$ against $H_1: p < 0.40$.

- a) Using a 10% level of significance, find the critical region for this test.
- b) State the actual significance level of this test.



7.2 - Critical value and region

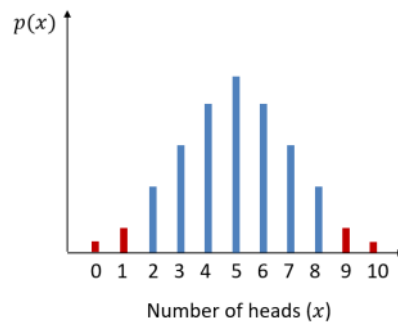
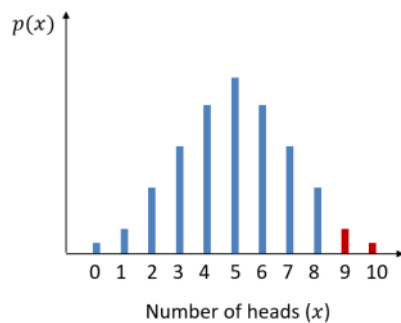
Practice

- A test statistic has a distribution $B(10, p)$. Given that $H_0: p = 0.3, H_1: p > 0.3$. Find the critical region, critical value and the actual significance level using 5% level of significance.
- A test statistic has a distribution $B(20, p)$. Given that $H_0: p = 0.2, H_1: p < 0.2$. Find the critical region, critical value and the actual significance level using 5% level of significance.

Know the difference: One-tailed vs Two-tailed

One-tailed: Toss a coin, check if the coin is biased on Head only (or Tail only)

Two-tailed: Toss a coin, check if the coin is biased on *either* Head *or* Tail



7.2 - Critical value and region

Example (2-tailed crit. regions)

A random variable X has binomial distribution $B(40, p)$. A single observation is used to test $H_0: p = 0.25$ against $H_1: p \neq 0.25$.

- Use a 2% level of significance, find the critical regions of this test.
- Write down the actual significance level of this test.

Practice

A random variable has distribution $B(30, p)$. A single observation is used to test $H_0: p = 0.25$ against $H_1: p \neq 0.25$.

- Using a 5% level of significance, find the critical region of this test.
- Write down the actual significance level of the test

Practice (Stats table not usable)

A random variable has distribution $B(13, p)$. A single observation is used to test $H_0: p = 0.44$ against $H_1: p \neq 0.44$.

- Using a 5% level of significance, find the critical region of this test.
- Write down the actual significance level of the test



7.3 - One-tailed test

Steps to carry out a 1-tailed hypothesis test

1. Define the test statistic, X , and the parameter, p .
2. Write the null (H_0) and alternate (H_1) hypotheses
3. Work out the p - *value*
4. Compare with the given significance level
5. Conclusion

Criteria to use Binomial model

- Fixed number of trials
- Outcomes are independent
- There are two outcomes - success and failure
- Probability of success is constant

Example

A coin was tossed 15 times and it came up heads 12 times. One claims the coin is biased towards heads. Use a one-tailed test with 5% significance level to test the claim.

Example

The standard treatment for a particular disease has a $\frac{2}{5}$ probability of success. A certain doctor has undertake research in this area and has produced a new drug which has been successful with 11 out of 20 patients. The doctor claims the new drug represents an improvement on the standard treatment. Test, at the 5% significance level, the claim made by the doctor.



7.3 - One-tailed test

Practice

The success rate of the standard treatment for patients suffering from a particular skin disease is claimed to be 68%. A random sample of 10 patients receives the standard treatment and in only 3 cases was the treatment successful. It is thought that the standard treatment was not as effective as it is claimed.

- Test the claim at 5% significance level.
- Find the critical region using significance level of 1%.

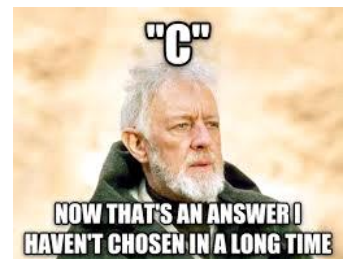
Exam Practice

Edexcel S2 Jan 2011 Q2

A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher's claim.

State your hypotheses clearly.

(6)



7.4 - Two-tailed test

Notes

- We have seen the use of two-tailed test that we are interested in bias "either way".
- Hence, we split the critical region by _____.

Example

A hypothesis that the probability that Gordon Ramsay swears in his TV shows is 65% is proposed by his fans. One then watched 30 random episodes of his shows and found in 26 of them, he swore. Test, at 5% significance level, whether or not there is evidence that the hypothesis stands.



Example

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-veg to veg meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal. Using a 5% level of significance, test whether or not the proportion of people eating veg meals in Manuel's restaurant is different to that in Enrico's restaurant.



7.4 - Two-tailed test

Practice

A machine makes glass bowls and it is observed that one in ten of the bowls have hairline cracks in them. The production process is modified and a sample of 20 bowls is taken. 1 of the bowls is cracked. Test, at the 10% level of significance, the hypothesis that the proportion of cracked bowls has changed as a result of the change in the production process. State your hypotheses clearly. **(7 marks)**

Exam Practice

Edexcel S2 Jan 2006 Q7a

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read the Deano.

- (a) (i) Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read the Deano is different from 20%. State your hypotheses clearly.
- (ii) State all the possible numbers of pupils that read the Deano from a sample of size 20 that will make the test in part (a)(i) significant at the 5% level. **(9)**

