Chapter 1 - Algebraic Methods

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Personal notes:

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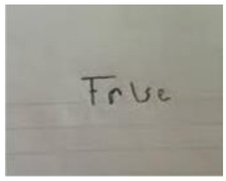
1.1 - Proof by Contradiction



Concept
Imagine we have to prove a statement that is true…

- Assume that the statement is ___________.

- Prove that the statement ____.

- Therefore it is wrong to conclude the statement is false, hence the statement must be true.

Structure of proof
1.

2.

3.

Example
Prove, by contradiction, that there is no greatest odd integer.

Negating the original statement
The first part of proof by contradiction requires you to negate the original statement.
Think and discuss which statement should be the negation statement of each scenario.

“There are infinitely many prime numbers.”	“All Santa Claus are costumed by men over 50 years old.	“If it is drawn by Mr. Fan, it is a masterpiece.”
“There are infinitely many non-prime (i.e. composite) numbers.”	“There exists a Santa Claus who is less than 50 yo.”	“If it is not drawn by Mr. Fan, it is not a masterpiece.”
“There are finitely many prime numbers.”	“No Santa Claus is costumed by men over 50 years old.”	“If it is not drawn by Mr. Fan, it is a masterpiece.”
“There are finitely many non-composite numbers.”	“There is no such thing known as Santa Claus.”	“If it is drawn by Mr. Fan, it is not a masterpiece.”

1.1 - Proof by Contradiction

<u>Example</u>
Prove by contradiction that if n^2 is even, then n must be even.

<u>Classic Proof 1:</u>
Prove by contradiction that there are infinite amount of prime numbers.

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1.1 - Proof by Contradiction

Classic Proof 2:
Prove $\sqrt{2}$ is irrational.

1.2 - Algebraic fractions

<u>Recap: Multiplying & Dividing fractions</u>

- Cancel factors as much as you can before multiplying/dividing.

$$\frac{a}{b} \times \frac{c}{a} =$$

$$\frac{x+1}{2} \times \frac{3}{x^2-1} =$$

$$\frac{a}{b} \div \frac{a}{c} =$$

$$\frac{x+2}{x+4} \div \frac{3x+6}{x^2-16} =$$

<u>Practice</u>

$$\frac{x+3}{5} \times \frac{10}{x^2-9} =$$

$$\frac{x^2+x}{y} \div \frac{x^2-x-2}{y^2} =$$

<u>Common Mistake</u>

$$\frac{x^2+\cancel{y}}{2\cancel{y}} = \frac{x^2}{2}$$

<u>Recap: Adding & Subtracting fractions</u>

- Cross multiply the numerators and multiply denominators for common denominators.

$$\frac{3}{x+1} - \frac{1}{x+2} =$$

$$\frac{2}{x+1} - \frac{4x}{x^2-1} =$$

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1.2 - Algebraic fractions

<u>Practice</u>

[Edexcel C3 June 2013(R) Q1]

Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

Express the following as a single fraction, giving your answer in its simplest form.

$$\frac{10x+4}{3x^2+4x+1} - \frac{3}{x+1}$$

1.3 - Partial Fractions

Notes

- If a fraction has **a product of linear factors** as denominators, it can be split into ***"partial fractions"***, where each denominator is a single linear factor.

$$\frac{5}{(x+1)(x+4)} \equiv$$

Example

Split $\frac{6x-2}{(x-3)(x+1)}$ into partial fractions.

1.3 - Partial Fractions

Example

Given that $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$. Find the values of the constants A, B and C.

Exam Practice

C4 June 2005 Q3a

Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

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1.4 - Repeated Factors

Starter:
Think and discuss what is wrong with the following working out:
Since $\frac{2x-1}{(x+1)^2} \equiv \frac{2x-1}{(x+1)(x+1)}$

Hence, we can split the fraction $\frac{2x-1}{(x+1)^2}$ into $\frac{A}{x+1} + \frac{B}{x+1}$.

Notes (repeated factor)

- To split a fraction with repeated factors in denominator,

Example
Split $\frac{11x^2+14x+5}{(x+1)^2(2x+1)}$ into partial fractions.

Exam Practice

C4 June 2011 Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C. **(4)**

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1.5 - Improper Fractions

Notes

- An algebraic fraction is **improper** if the degree of the numerator is **at least** the degree of the denominator.

  e.g. $\frac{x^2-3}{x+2}$, $\frac{x+1}{x-1}$, $\frac{x^3-2x}{x-2}$

- When dealing with improper fractions, we can split into partial fractions by using ____________________________.

Example

Given that $\frac{x^3+x^2-7}{x-3} \equiv Ax^2 + Bx + C + \frac{D}{x-3}$, determine the values of A, B and C.

Example

Given that $x^2 + 5x + 8 \equiv (Ax + B)(x - 2) + C$. Find the values of A, B and C.

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1.5 - Improper Fractions

<u>Exam Practice</u>

Edexcel C4 June 2013 Q1

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

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