Chapter 6 - Statistical Distribution

Personal notes:

BF MATHS

6.1 - Probability distributions

Notes

- A **random variable** is a variable whose
- A **probability distribution** fully describes
- Take rolling a fair die as an example, a probability distribution could be presented in the following ways:

Example

The random variable X represents the **number of tails when three coins are tossed**.

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6.1 - Probability distributions

Example

A biased four-sided dice with faces numbered 1, 2, 3 and 4 is rolled. The number of the bottom-most face is modelled as a random variable X. Given that $P(X = x) = \frac{k}{x}$,

a) Find the value of k

b) Give the probability distribution of X in table form

c) Find the probability that

i) $X > 2$

ii) $1 < X < 4$

iii) $X > 4$

Exam Practice

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1. A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{1}{6}$.

(3)

Example

$\boldsymbol{x}$	2	3	4	5
$P(X = x)$	x	$3x$	$3x - 0.1$	$x + 0.3$

Determine

a) $P(X > 3)$, b) $P(2 \leq X < 4)$, c) $P(2X + 1 \geq 6)$

6.2 - Binomial distribution

Notes
You can model a random variable X with a *binomial distrubution*, $B(n, p)$, if:
- There are a fixed number of trials, n
- There are two possible outcomes (success and failure)
- There is a fixed probability of success, p
- The trials are independent of each other

If a random variable X has the binomial distribution $B(n, p)$ - notated as ,
then its probablity mass function is given by

Example
The random variable $X \sim B(12, \frac{1}{6})$. Find
a) $P(X = 2)$
b) $P(X = 9)$
c) $P(X \leq 1)$

Practice
Given $X \sim B(10, 0.3)$. Find
a) $P(X = 3)$
b) $P(X = 7)$
c) $P(X \leq 2)$
d) $P(X \geq 2)$
e) $P(X > 2)$

6.2 - Binomial distribution

<u>Example</u>

There are 60 students in a school and 20 of them are studying maths. A random student is picked in each trial. The trial takes place 50 times.

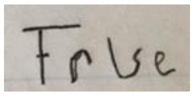
a) How is X distributed?
b) Determine the probability that a maths student was chosen 40 times.

<u>Example</u>

The probability that a randomly chosen member of a reading group is left-handed is 0.15. A random sample of 20 members of the group is taken.

a) Suggest a suitable model for the random variable X, the number of members in the sample who are left-handed. Justify your choice.
b) Use your model to calculate the probability that
   i) exactly 7 of the members in the sample are left-handed
   ii) fewer than two of the members in the sample are left-handed

6.3 - Cumulative probabilities



Word of art

Phrase	Meaning	Calculation...
...greater than 5...		
...no more than 3...		
...at least 7...		
...fewer than 10...		
...at most 8...		
...no more than 20 but no less than 10...		
...no more than 20 but more than 10		

Example

Given a random variable $X \sim B(20, 0.4)$. Find

a) $P(X \leq 7)$

b) $P(X < 6)$

c) $P(X \geq 15)$

Example

Given that $X \sim B(25, 0.25)$. Find the probability such that...

a) $X = 6$

b) X *is no less than 19*

c) X is at most 10 but greater than 6

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6.3 - Cumulative probabilities

Practice

First write the following in terms of cumulative probabilities, e.g. $P(X > 5) = 1 - P(X \leq 5)$, then work out the probability when $X \sim B(30, 0.6)$

a) $P(X < 4)$
b) $P(X \geq 4)$
c) $P(X > 2)$
d) $P(5 \leq X \leq 17)$
e) $P(5 < X \leq 17)$
f) $P(5 \leq X < 17)$
g) $P(5 < X < 17)$
h) $P(X = 30)$

Example (Reverse - find x given p-value)

$X \sim B(25, 0.4)$

a) Given $P(X \leq x) = 0.5858$, work out x.
b) Given $P(X < x) = 0.2735$, work out x.
c) Given $P(X > x) = 0.0778$, work out x.
d) Given $P(X \geq x) = 0.0043$, work out x.

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$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 25, x = 0$	0.2774	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000
3	0.9659	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001
4	0.9928	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005
5	0.9988	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020
6	0.9998	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073
7	1.0000	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216
8	1.0000	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539
9	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148
10	1.0000	1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122
11	1.0000	1.0000	0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450
12	1.0000	1.0000	1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878
15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9971	0.9868	0.9560	0.8852
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9957	0.9826	0.9461
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9942	0.9784
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Practice

$X \sim B(40, 0.3)$

a) Given $P(X \leq x) = 0.5772$, work out x.
b) Given $P(X < x) = 0.9367$, work out x.
c) Given $P(X > x) = 0.0024$, work out x.
d) Given $P(X \geq x) = 0.9762$, work out x.

6.3 - Cumulative probabilities

<u>Example (statistical table not usable)</u>
What if $X \sim B(10, 0.26)$, find x when $P(X \le x) = 0.7521$

- Statistical table does not show values when p = 0.26

<u>Practice</u>
$X \sim B(15, 0.41)$, find x when $P(X \le x) = 0.5785$

<u>Example</u>
A spinner is designed so that the probability it lands on *RED* is 0.3. Jane has 12 spins. Find the probability that Jane obtains:

a) No more than 2 *RED*s
b) At least 5 *REDs*

Jane decides to use this spinner for a class competition. To win a prize, the player has to land on at least a certain number of *RED*s. Jane wants the probability of winning a prize to be <0.05. Each player has 12 spins.

a) Find how many *RED*s are needed to win a prize.

$p =$	0.05	0.10	0.15	0.20	0.25	0.30
$n = 12, x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

<u>Example</u>
At Hogwarts Quidditch cup, each team has roughly 30 shots on goal. Each shot on goal has 0.45 chance of scoring. The team is likely to win if they shoot a certain minimum number on goal out of the 30.
What is the minimum number if the team has at least 88% chance of winning?