Chapter 5 - Radians

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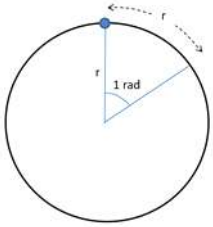
Personal notes:

5.1 - Radian measure

Notes

- Radian is another measure of size of angles, alternative to degrees.
- A degree is a 360^{th} of a rotation around a full circle.
- One radian however is the movement of one radius' worth around the circumference of the circle.



- Full circle =

Conversion between degree and radian

- Arc length =

5.1 - Radian measure

Example

Convert the following degree/rad into rad/degree:

a) 90^o
b) 135^o
c) $\frac{\pi}{3}$
d) $\frac{3\pi}{2}$

Practice

a) 45^o
b) 72^o
c) $\frac{\pi}{6}$
d) $\frac{7\pi}{5}$

Example (Calculator)

Work out

a) $\sin(0.3\ rad)$
b) $\cos(\pi\ rad)$
c) $\tan(2\pi\ rad)$

Graph sketching

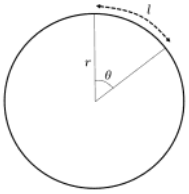
Example

Sketch the graph of $y = \cos(x + \pi)$ for $0 \leq x \leq 2\pi$

5.2 - Arc length

Notes

- Arc length (l) of a sector of a circle:



Example

Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.

Example

An arc AB of a circle with radius 7 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

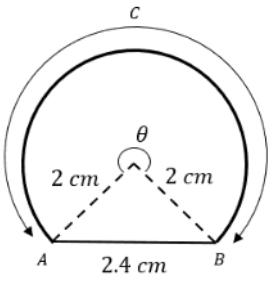
Example

An arc AB of a circle, with centre O and radius r cm, subtends an angle of θ radians at O. The perimeter of the sector AOB is P cm. Express r in terms of P and θ.

5.2 - Arc length

Example

The border of a garden pond consists of a straight edge AB of length 2.4m, and a curved part C, as shown in the diagram. The curve part is an arc of a circle, centre O and radius 2m.
Find the length of C.



Exam Practice

Edexcel C2 Jan 2005 Q7

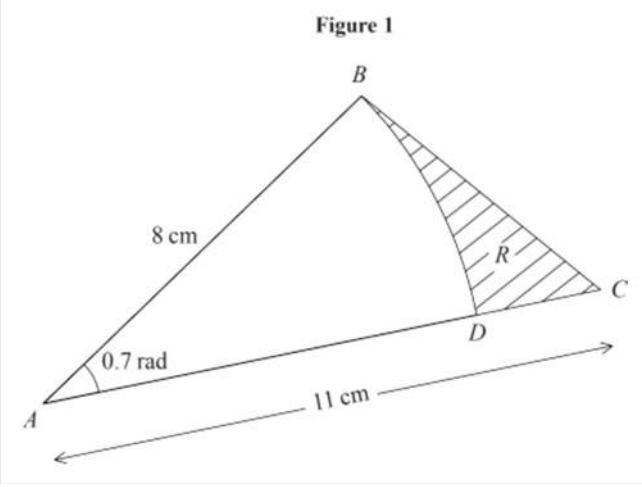
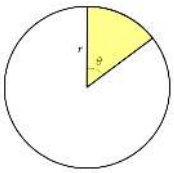


Figure 1 shows the triangle ABC, with $AB = 8\ cm$, $AC = 11\ cm$ and $\angle BAC = 0.7$ radians. The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 8 cm. The region R, shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD.
Find
(a) The length of the arc BD.
(b) The perimeter of R, giving your answer to 3 significant figures.

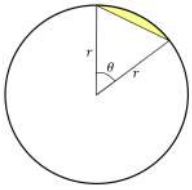
5.3 - Area of segments and sectors

Notes

- Area of a sector (radians):

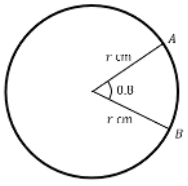


- Area of a segment (radians):



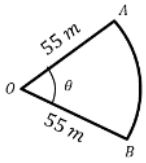
Example

In the diagram, the area of the minor sector AOB is 28.9 cm^2. Given that $\angle AOB = 0.8$ radians, calculate the value of r.



Example

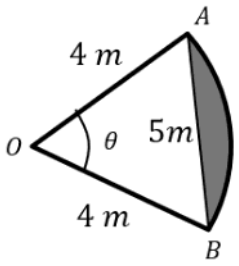
A plot of land is in the shape of a sector of a circle of radius 55 m. The length of fencing that is erected along the edge of the plot to enclose the land is 176 m. Calculate the area of the plot of land.



5.3 - Area of segments and sectors

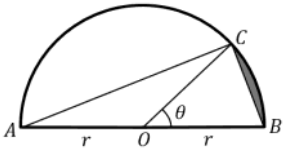
Example

In the diagram above, OAB is a sector of a circle, radius 4m. The chord AB is 5m long. Find the area of the shaded segment.



Example

In the diagram, AB is the diameter of a circle of radius $r$cm, and $\angle BOC = \theta$ radians. Given that the area of ΔAOC is three times that of the shaded segment, show that $3\theta - 4\sin\theta = 0$.



5.3 - Area of segments and sectors

Exam Practice

Edexcel C2

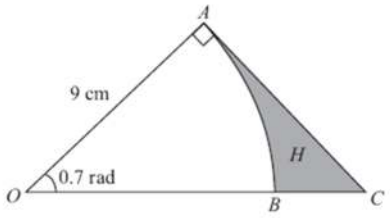


Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector OAB.

(2)

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places.

(2)

The region H is bounded by the arc AB and the lines AC and CB.

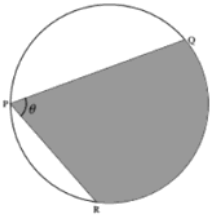
(d) Find the area of H, giving your answer to 2 decimal places.

(3)

Challenge

[MAT 2012 1J]

If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram on the left, then find the largest possible area of the shaded region RPQ, giving your answer in terms of θ.



5.4 - Solving trigonometric equations

Notes

Solving trigonometric equations in radians is not too different from year 1

a) Use _____

b) Use _____ instead of _____when finding other pairs of values.

Starter (Year 1 - Trig equations)

Solve $\cos 3\theta = \frac{\sqrt{3}}{2}$, in the interval $0 \leq \theta \leq 360°$.

Example

Solve the equation

$\sin 3\theta = \frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 2\pi$.

5.4 - Solving trigonometric equations

Example

Solve the equation $17\cos\theta + 3\sin^2\theta = 13$ in the interval $0 \leq \theta \leq 2\pi$.

Exam Practice

[Jan 07 Q6] Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π. **(6)**

5.5 - Small angle approximations

Notes

- We can use radians to find *approximations* for the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.
- When θ is small (think of values of like 0.0000001) and measured in radians:

Example

When θ is small, find the approximate value of:

a) $\frac{\sin 2\theta + \tan\theta}{2\theta}$

b) $\frac{\cos 4\theta - 1}{\theta \sin 2\theta}$

Example

a) Show that, when θ is small, $\sin 5\theta + \tan 2\theta - \cos 2\theta \approx 2\theta^2 + 7\theta - 1$.

b) Hence state the approximate value of $\sin 5\theta + \tan 2\theta - \cos 2\theta$ for small values of θ.