

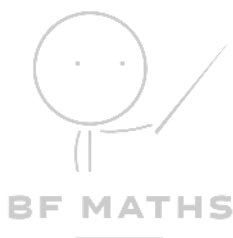
Chapter 4 - Binomial Expansion

4.1 - Expanding $(1 + x)^n$ - Pg. 2 - 5

4.2 - Expanding $(a + bx)^n$ - Pg. 6 - 7

4.3 - Using partial fractions - Pg. 8 - 9

Personal notes:



4.1 - Expanding $(1 + x)^n$

Recap from Year 1 Binomial

- Pascal triangles (up to x^5)
- $n! =$
- n choose $r = nCr = \binom{n}{r} =$
- $\binom{n}{1} =$
- $\binom{n}{2} =$
- $\binom{n}{3} =$
- $\binom{n}{4} =$
- $(1 + x)^5$
- $(1 - 3x)^3$



4.1 - Expanding $(1 + x)^n$

Notes

- When the power, n , of a binomial expansion is a *fraction* or *negative number*, use this version of binomial expansion:

- This expansion is valid when $|x| < 1$, so that the expansion is a _____.

Example

Use the binomial expansion to find the first four terms of $\frac{1}{1+x}$ and state the range of values of x for it to be valid.

Example/Practice

Use binomial expansion to find the first four terms of $\sqrt{1 - 3x}$ state the range of values of x for it to be valid.



4.1 - Expanding $(1 + x)^n$

Quickfire examples (validity)

State the range of values of x for each expansion to be valid:

a) $(1 + 4x)^{-3}$

b) $(1 - 2x)^{\frac{1}{2}}$

c) $\left(1 + \frac{2}{3}x\right)^{-5}$

Example

- Find the expansion of $\sqrt{1 - 2x}$ up to and including the term x^3 .
- By substituting $x = 0.01$, find an approximation of $\sqrt{2}$.
- Without further calculation, state how the accuracy of the approximation found in (b) could be improved.



4.1 - Expanding $(1 + x)^n$

Example

Given $f(x) = \sqrt{\frac{1+x}{1-5x}}$,

- a) Use binomial expansion to find up to and including x^2 term.
- b) State the range of values of x for which the expansion is valid.

Exam Practice

C4 Edexcel Jan 2010

1. (a) Find the binomial expansion of

$$\sqrt[3]{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3 , simplifying each term. (6)

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt[3]{(1-8x)}$ is $\frac{\sqrt[3]{23}}{5}$. (2)

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt[3]{23}$. Give your answer to 5 decimal places. (3)



4.2 - Expanding $(a + bx)^n$

Notes

- $(a + bx)^n =$

- For $(a + bx)^n$ to be valid when n is negative or fractional,

Example

Find the first four terms in the binomial expansion and state the validity of

a) $\sqrt{4 + x}$

b) $\frac{1}{(2 + 3x)^2}$



4.2 - Expanding $(a + bx)^n$

Practice (Validity)

State the validity of each of the following binomial expansions:

- a) $(2 + x)^{-3}$
- b) $(5 - 2x)^{\frac{1}{2}}$
- c) $(3 - x)^{-2}$
- d) $(16 + 5x)^{-1}$

Exam Practice

Edexcel C4 June 2013 (Withdrawn) Q1

(a) Find the binomial expansion of

$$\sqrt{9 + 8x}, \quad |x| < \frac{9}{8}$$

in ascending powers of x , up to and including the term in x^2 .
Give each coefficient as a simplified fraction.

(5)

(b) Use your expansion to estimate the value of $\sqrt{11}$, giving your answer as a single fraction.

(3)



4.3 - Using partial fractions

Notes

- Use partial fraction when question asks so.



Example

- a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions.
- b) Hence show that the cubic approximation $\frac{4-5x}{(1+x)(2-x)}$ is $2 - \frac{7}{2}x + \frac{11}{4}x^2 - \frac{25}{8}x^3$.
- c) State the range of values of x for which the expansion is valid.



4.3 - Using partial fractions

Exam Practice

[C4 June 2010 Q5]

10.

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

- (a) Find the values of the constants A , B and C . (4)
- (b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x , as far as the term in x^2 . Give each coefficient as a simplified fraction. (7)

