Chapter 4 - Binomial Expansion

Personal notes:

4.1 - Expanding $\left(1+x\right)^n$

Recap from Year 1 Binomial

- Pascal triangles (up to x^5)

- $n! =$
- n choose r = nCr = $\binom{n}{r} =$
- $\binom{n}{1} =$
- $\binom{n}{2} =$
- $\binom{n}{3} =$
- $\binom{n}{4} =$

- $(1+x)^5$
- $(1-3x)^3$

4.1 - Expanding $(1+x)^n$

Notes

- When the power, n, of a binomial expansion is a *fraction* or *negative number*, use this version of binomial expansion:

- This expansion is valid when $|x| < 1$, so that the expansion is a _____.

Example

Use the binomial expansion to find the first four terms of $\frac{1}{1+x}$ and state the range of values of x for it to be valid.

Example/Practice

Use binomial expansion to find the first four terms of $\sqrt{1-3x}$ state the range of values of x for it to be valid.

4.1 - Expanding $(1+x)^n$

Quickfire examples (validity)

State the range of values of x for each expansion to be valid:

a) $(1+4x)^{-3}$

b) $(1-2x)^{\frac{1}{2}}$

c) $\left(1+\frac{2}{3}x\right)^{-5}$

Example

a) Find the expansion of $\sqrt{1-2x}$ up to and including the term x^3.

b) By substituting $x = 0.01$, find an approximation of $\sqrt{2}$.

c) Without further calculation, state how the accuracy of the approximation found in (b) could be improved.

4.1 - Expanding $(1+x)^n$

Example

Given $f(x) = \sqrt{\frac{1+x}{1-5x}}$,

a) Use binomial expansion to find up to and including x^2 term.
b) State the range of values of x for which the expansion is valid.

Exam Practice

C4 Edexcel Jan 2010

1. (*a*) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(6)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

4.2 - Expanding $\left(a + bx\right)^n$

Notes

- $(a + bx)^n =$

- For $(a + bx)^n$ to be valid when n is negative or fractional,

Example

Find the first four terms in the binomial expansion and state the validity of

a) $\sqrt{4 + x}$

b) $\dfrac{1}{(2 + 3x)^2}$

4.2 - Expanding $(a + bx)^n$

Practice (Validity)

State the validity of each of the following binomial expansions:

a) $(2 + x)^{-3}$

b) $(5 - 2x)^{\frac{1}{2}}$

c) $(3 - x)^{-2}$

d) $(16 + 5x)^{-1}$

Exam Practice

Edexcel C4 June 2013 (Withdrawn) Q1

(a) Find the binomial expansion of

$$\sqrt{(9 + 8x)}, \quad |x| < \frac{9}{8}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient as a simplified fraction.

(5)

(b) Use your expansion to estimate the value of $\sqrt{(11)}$, giving your answer as a single fraction.

(3)

4.3 - Using partial fractions

Notes

- Use partial fraction when question asks so.

Example

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions.

b) Hence show that the cubic approximation $\frac{4-5x}{(1+x)(2-x)}$ is $2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$.

c) State the range of values of x for which the expansion is valid.

4.3 - Using partial fractions

Exam Practice

[C4 June 2010 Q5]

10. $$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants A, B and C. **(4)**

(*b*) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction. **(7)**