Chapter 12 - Vectors

12.1, 12.2 - 3D Distance and 3D Vectors - Pg. 2 - 4

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Personal notes:

12.1, 12.2 - 3D Distance and Vectors

Example 1 :

Find the distance between the points $A(1, 3, 4)$ and $B(8, 6, -5)$, giving your answer to 1 d.p.

Example 2

The coordinates of A and B are $(5, 0, 3)$ and $(4, 2, k)$ respectively.

Given that the distance from A to B is 3 units, find the possible values of k.

12.1, 12.2 - 3D Distance and Vectors

Example 3

Consider the points $A(1, 5, -2)$ and $B(0, -3, 7)$.

a Find the position vectors of A and B in **ijk** notation.

b Find the vector $\overrightarrow{AB}$ as a column vector.

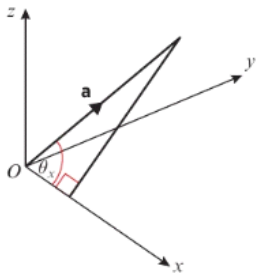
Example 4

Find the magnitude of $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and hence find $\hat{\mathbf{a}}$, the unit vector in the direction of $\mathbf{a}$.

12.1, 12.2 - 3D Distance and Vectors

Notes

- **If the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ makes an angle θ_x with the positive x-axis then $\cos\theta_x = \frac{x}{|\mathbf{a}|}$ and similarly for the angles θ_y and θ_z**



Example 5

Find the angles that the vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ makes with each of the positive coordinate axes to 1 d.p.

12.3 - Geometric Problems

Example 1

A, B, C and D are the points $(2, -5, -8)$, $(1, -7, -3)$, $(0, 15, -10)$ and $(2, 19, -20)$ respectively.

a Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$, giving your answers in the form $p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$.

b Show that the lines AB and DC are parallel and that $\overrightarrow{DC} = 2\overrightarrow{AB}$.

Example 2

P, Q and R are the points $(4, -9, -3)$, $(7, -7, -7)$ and $(8, -2, -0)$ respectively. Find the coordinates of the point S so that $PQRS$ forms a parallelogram.

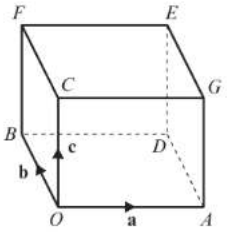
Example 3

Given that $3\mathbf{i} + (p + 2)\mathbf{j} + 120\mathbf{k} = p\mathbf{i} - q\mathbf{j} + 4pqr\mathbf{k}$, find the values of p, q and r.

12.3 - Geometric Problems

Example 4

The diagram shows a cuboid whose vertices are O, A, B, C, D, E, F and G. Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of the vertices A, B and C respectively. Prove that the diagonals OE and BG bisect each other.



12.4 - Application to mechanics

Example 1

A particle of mass 0.5 kg is acted on by three forces:

$\mathbf{F}_1 = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N

$\mathbf{F}_2 = (-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ N

$\mathbf{F}_3 = (4\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$ N

a Find the resultant force **R** acting on the particle.

b Find the acceleration of the particle, giving your answer in the form $(p\mathbf{i} + q\mathbf{j} + r\mathbf{k})\,\text{m s}^{-2}$.

c Find the magnitude of the acceleration.

Given that the particle starts at rest,

d find the distance travelled by the particle in the first 6 seconds of its motion.

Example 2: (From Ex 12D Q6)

6 In this question **i** and **j** are the unit vectors due east and north, and **k** is the unit vector vertically upwards. An aeroplane of mass 1200 kg is initially in level flight. The forces of thrust **T**, lift **L**, and the combined forces of wind and air resistance **F**, acting on the aeroplane are modelled as:

$\mathbf{T} = 2800\mathbf{i} - 1800\mathbf{j} + 300\mathbf{k}$

$\mathbf{L} = 11\,000\mathbf{k}$

$\mathbf{F} = -900\mathbf{i} + 500\mathbf{j}$

a Taking $g = 9.8\,\text{m s}^{-2}$, find the magnitude of the acceleration of the aeroplane.

b Determine whether the aeroplane is ascending or descending, and find the size of the obtuse angle its acceleration makes with the vector **k**.