Chapter 11 : Integration

End of topic task:
Write down a revision summary on the next page

Personal Notes

11.1 - Integrating standard functions

Standard Functions

y	$\int y\,dx$
x^n	$\frac{1}{n+1}x^{n+1} + C$
e^x	$e^x + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
$\sec^2 x$	$\tan x + C$
$cosec\ x \cot x$	$-cosec\ x + C$
$cosec^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$

Notes

-
-

Example

Find the following integrals.

a $\int\left(2\cos x + \frac{3}{x} - \sqrt{x}\right)dx$ **b** $\int\left(\frac{\cos x}{\sin^2 x} - 2e^x\right)dx$

11.1 - Integrating standard functions

Example

Given that a is a positive constant and $\int_a^{3a} \frac{2x+1}{x}\,dx = \ln 12$, find the exact value of a.

11.2 - Reverse Chain Rule 1

Starter :

$\frac{d}{dx}(\sin(3x+1)) =$

Try to answer this : $\int \cos(3x+1)\ dx =$

Reverse chain rule is about considering…

Example

$\int e^{4x+1}\,dx$

Example

Find the following integrals:

a $\int \frac{1}{3x+2}\,dx$ **b** $\int (2x+3)^4\,dx$

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11.2 - Reverse Chain Rule 1

Example

$\int \sec^2 3x \, dx$

Practice

1) $\int (3x+4)^3 \, dx$

2) $\int \sin(1-5x) \, dx$

3) $\int \frac{1}{3(4x-2)^2} \, dx$

4) $\int (10x+11)^{12} dx$

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11.3 - Integration with Trig Identities

Some expressions, such as $\sin^2 x$ and $\sin x \cos x$ can't be integrated directly, but we can use one of our trig identities to replace it with an expression we can easily integrate.

Example

Find $\int \sin^2 x \; dx$

Example

Find $\int \sin 3x \cos 3x \; dx$

11.3 - Integration with Trig Identities

Example

Find $\int (\sec x + \tan x)^2 \, dx$

Practice

Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^2 3x \, dx$

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11.4 - Reverse Chain Rule 2

Reverse Chain Rule…

1. Consider some expression that differentiate to something similar to the question
2. Differentiate and adjust/scale appropriately.

Example

$$\int x(x^2+5)^3\,dx$$

Example

$$\int \cos x \sin^2 x\;dx$$

Example

$$\int \frac{2x}{x^2+1}\,dx$$

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11.4 - Reverse Chain Rule 2

Practice

$$\int \frac{x^2}{x^3+1}\,dx$$

$$\int x\,e^{x^2+1}\,dx$$

Integrating $\sec^n x$

Work out $\frac{d}{dx}\left(\sin^5 x\right)$

Work out $\frac{d}{dx}(\sec^5 x)$

Example

$$\int \sec^4 x \tan x \;\, dx$$

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11.5 - Integration by *Substitution*

For some complicated expressions, we cannot use *"reverse chain rule"* to integrate. Instead, we use a clever substitution to make the expression simpler to integrate.

Notes

- The aim is to completely remove any reference to x, and replace with u.

Example

Use the substitution $u = 2x + 5$ to find $\int x\sqrt{2x+5}\ dx$

Example

Use the substitution $u = \sin x + 1$ to find $\int \cos x \sin x\,(1 + \sin x)^3\ dx$

11.5 - Integration by *Substitution*

Example (Definite Integration)

Use Integration by substitution to evaluate $\int_0^{\frac{\pi}{2}} \cos x \sqrt{1+\sin x}\ dx$

Practice

Edexcel C4 June 2011 Q4

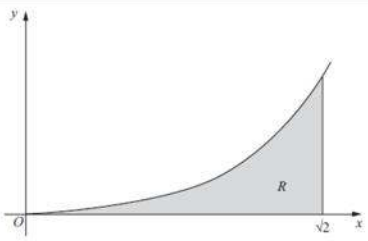

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2+2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

(*c*) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \ \mathrm{d}u.$$

11.5 - Integration by *Substitution*

**<u>One of a kind</u>

Prove that $\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \arcsin x + c$.

11.6 - Integration by *parts*

$$\int x\cos x \ \ dx = ?$$

- Just as the Product Rule was used to **differentiate the product** of two expressions, we can often use 'Integration by Parts' to **integrate a product.**

Integration by parts:

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}\ dx$$

Example

$$\int x\cos x \ \ dx = ?$$

Example

Find $\int x^2 \ln x \ \ dx$

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11.6 - Integration by *parts*

Example (IBP Twice!!)

Find $\int x^2 e^x \, dx$

Practice

Find $\int x^2 \sin x \, \, dx$

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11.6 - Integration by *parts*

Example (Integrating $\ln x$ with finite integral)

Find $\int_1^2 \ln x \, dx$, leaving your answer in terms of natural logarithms.

One of a kind

Use integration by parts to integrate $\int e^x \sin x \ dx$

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11.7 - Integrating Partial Fractions

When to split into partial fractions ?
-> When the bottom denominator has the same or larger power than the numerator.

<u>Example</u>

Find $\int \frac{2}{x^2-1}\, dx$

<u>Example</u>

Find $\int \frac{x-5}{(x+1)(x-2)}\, dx$

11.7 - Integrating Partial Fractions

Example

Find $\int \frac{8x^2-19x+1}{(2x+1)(x-2)^2}\, dx$

Exam Practice

Edexcel C4 June 2009 Q3

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*. **(4)**

(*b*) (i) Hence find $\int f(x)\ dx$. **(3)**

(ii) Find $\int_0^2 f(x)\ dx$ in the form $\ln k$, where *k* is a constant. **(3)**

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11.7 - Integrating Partial Fractions

Example (Integrating Top-heavy fraction)

$$\int \frac{x^2}{x+1}\,dx = ?$$

Practice

$$\int \frac{x^3+2}{x+1}\,dx$$

11.7.5 - Integrating Parametric Equations

Notes

Suppose we have a set of parametric equations:
$x = f(t)$ and $y = g(t)$

To find the area of curve, we want to determine $\int y\, dx$.

- The problem is that y is in terms of t, not in terms of x.

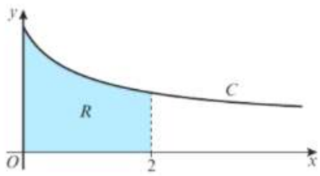
Area = $\int y\, dx =$

Example

Determine the area bound between the curve with parametric equations $x = t^2$ and $y = t + 1$, the x-axis, and the lines $x = 0$ and $x = 3$.

Example

The curve C has parametric equations $x = t(1 + t)$, $y = \frac{1}{1+t}$, $t \geq 0$. Find the exact area of region R, bounded by C, the $x - axis$ and the lines $x = 0$ and $x = 2$.



11.7.5 - Integrating Parametric Equations

Exam Practice

Edexcel C4 Jan 2013 Q5

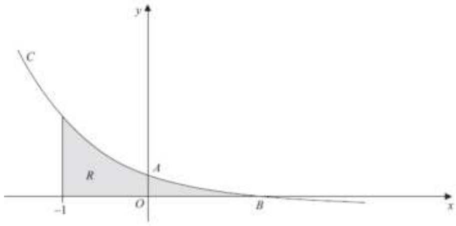


Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1.$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3). **(2)**

(b) Find the x-coordinate of the point B. **(2)**

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R. **(6)**

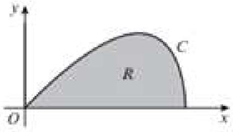
11.7.5 - Integrating Parametric Equations

(P) **1** The curve C has parametric equations $x = t^3$, $y = t^2$, $t \geqslant 0$. Show that the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$ is $k\sqrt[3]{2}$, where k is a rational constant to be found.

(E/P) **2** The curve C has parametric equations

$x = \sin t,\ y = \sin 2t,\ 0 \geqslant t \geqslant \frac{\pi}{2}$

The finite region R is bounded by the curve and the x-axis.
Find the exact area of R. **(6 marks)**



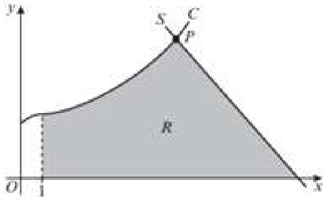
(E/P) **3** This graph shows part of the curve C with parametric equations $x = (t + 1)^2$, $y = \frac{1}{2}t^3 + 3$, $t \geqslant -1$

P is the point on the curve where $t = 2$.
The line S is the normal to C at P.

a Find an equation of S. **(5 marks)**

The shaded region R is bounded by C, S, the x-axis and the line with equation $x = 1$.

b Using integration, find the area of R. **(5 marks)**

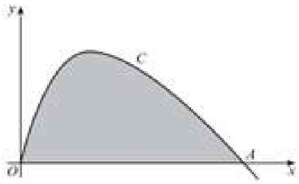


(E/P) **4** The diagram shows the curve C with parametric equations

$x = 3t^2,\ y = \sin 2t,\ t \geqslant 0.$

a Write down the value of t at the point A where the curve crosses the x-axis. **(1 mark)**

b Find, in terms of π, the exact area of the shaded region bounded by C and the x-axis. **(6 marks)**

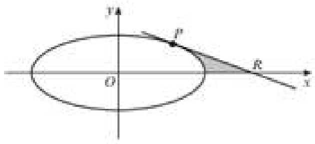


(E/P) **5** The curve shown has parametric equations

$x = 5\cos\theta,\ y = 4\sin\theta,\ 0 \leqslant \theta < 2\pi$

a Find the gradient of the curve at the point P at which $\theta = \frac{\pi}{4}$ **(3 marks)**

b Find an equation of the tangent to the curve at the point P. **(3 marks)**

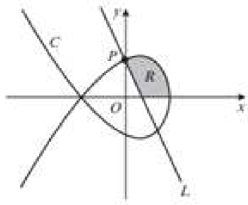


c Find the exact area of the shaded region bounded by the tangent PR, the curve and the x-axis. **(6 marks)**

(E/P) **6** The curve C has parametric equations

$x = 1 - t^2,\ y = 2t - t^3,\ t \in \mathbb{R}$

The line L is a normal to the curve at the point P where the curve intersects the positive y-axis. Find the exact area of the region R bounded by the curve C, the line L and the x-axis, as shown on the diagram. **(7 marks)**



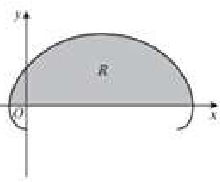
(E/P) **7** The curve shown in the diagram has parametric equations

$x = t - 2\sin t,\ y = 1 - 2\cos t,\ 0 \leqslant t \leqslant 2\pi$

a Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$ **(3 marks)**

The finite region R is enclosed by the curve and the x-axis, as shown shaded in the diagram.

b Show that the area R is given by $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2\,dt$ **(3 marks)**



c Use this integral to find the exact value of the shaded area. **(4 marks)**

ANSWERS

1 Area $= \int y\frac{dx}{dt}\,dt = \int_0^{\sqrt[3]{4}} t^2(3t^2)\,dt = \frac{3}{5}\left(\sqrt[3]{4}\right)^5 = \frac{3}{5}2^{\frac{10}{3}}$

$= \frac{3}{5}(2^3)(2^{\frac{1}{3}}) = \frac{24}{5}\sqrt[3]{2}$

2 $\frac{2}{3}$

3 a $x + y = 16$ b 61.85

4 a $\frac{\pi}{2}$ b $\frac{3\pi}{2}$

5 a $-\frac{4}{5}$ b $y - 2\sqrt{2} = -\frac{4}{5}\left(x - \frac{5}{\sqrt{2}}\right)$

c $10 - \frac{5\pi}{2}$

6 $\frac{41}{60}$

7 a $2\cos t = 1 \Rightarrow \cos t = \frac{1}{2} \Rightarrow t = \frac{\pi}{3}$ or $t = \frac{5\pi}{3}$

b $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} y\frac{dx}{dt}\,dt = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)(1 - 2\cos t)\,dt$

$= \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2\,dt$

c $4\pi + 3\sqrt{3}$

11.8 - Finding Areas

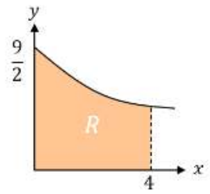
Notes

- You should be familiar that definite integration gives you the *area bound between the curve and the* $x-axis$.
- You need to be able to use expanded repertoire of integration skills to find the area under a great variety of curves.

Example

The diagram shows part of the curve $y = \frac{9}{\sqrt{4+3x}}$

The region R is bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, as shown in the diagram. Use integration to find the area of R.



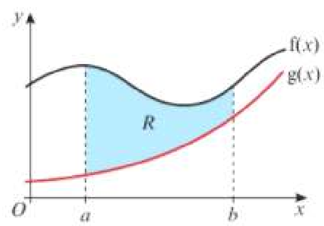
Notes (Area between two curves)

The areas under the two curves are $\int_a^b f(x)\,dx$ and $\int_a^b g(x)\,dx$.

It therefore follows the area between them (provided the curves don't overlap) is:

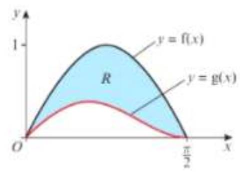
$$R = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b \big(f(x) - g(x)\big)\,dx$$



Example

The diagram shows part of the curves $f(x) = \sin 2x$ and $g(x) = \sin x \cos^2 x$ where $0 \le x \le \frac{\pi}{2}$. The region R is bounded by the two curves. Use integration to find the area of R.



11.8 - Finding Areas

Exam Practice

Edexcel C4 Jan 2009 Q2

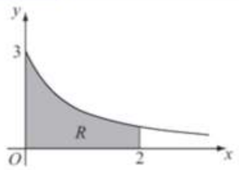


Figure 1

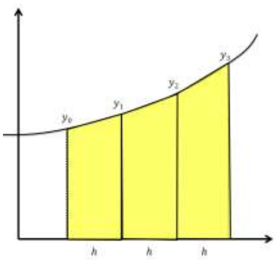
Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R. **(4)**

11.9 - Trapezium Rule

<u>Notes</u>

- Sometimes you cannot integrate a function algebraically, you can use a numerical method to ***<u>approximate</u>*** the area under a curve.
- Trapezium Rule = Dividing the area into trapeziums of equal width.



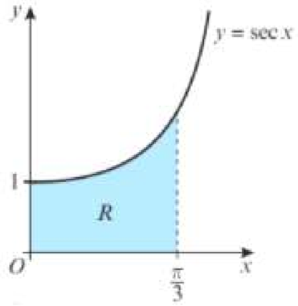
<u>Example</u>

Use trapezium rule to approximate the region bounded between $x = 1$, $x = 3$, the x-axis and the curve $y = x^2$, using 4 strips.

11.9 - Trapezium Rule

<u>Example</u>

The diagram shows a sketch of $y = \sec x$. The finite region R is bounded by the curve, the $x - axis$, the $y - axis$ and the line $x = \frac{\pi}{3}$.



a) Create a table with values of y correspondingly, giving your answers to 3 decimal places.
b) Use the trapezium rule to obtain an estimate for the area of R, giving your answer to 2 decimal places.
c) Explain with a reason whether your estimate in part **b** will be an underestimate or an overestimate.
d) Use integration to find the actual area in exact value.
e) Find the percentage error of the estimated error found in part **b**.

<u>Exam Practice</u>

Edexcel C2 May 2013 (R) Q2

$$y = \frac{x}{\sqrt{(1+x)}}$$

(*a*) Complete the table below with the value of y corresponding to $x = 1.3$, giving your answer to 4 decimal places.

(1)

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\,dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

11.9 - Trapezium Rule

Exam Practice

Edexcel C4 June 2014(R) Q2

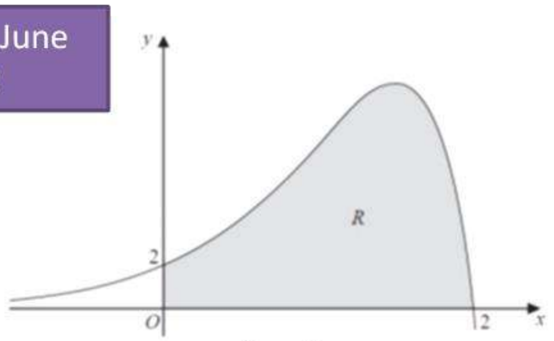


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x},$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$.

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(*a*) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.

(3)

(*b*) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.

(1)

(*c*) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.

(5)

11.10 - Solving Differential Equations

Recall

- A differential equation is an equation that consists of a mix of variables and derivatives, e.g. y, x and $\frac{dy}{dx}$.
- You have learnt how to construct a differential equations (Chp 9.10)
- Examples of differential equation:

  $$\left(1+x^2\right)\frac{dy}{dx} = x\tan y$$

  $$\frac{dy}{dx} = -\frac{3\left(y-2\right)}{(2x+1)(x+2)}$$

Notes

- "Solving" these equations means to get y in terms of x (with no $\frac{dy}{dx}$) through **separation of variables**.

Example

Find the general solution to $\frac{dy}{dx} = xy + y$

Example

Find the general solution to the differential equation $\left(1+x^2\right)\frac{dy}{dx} = x\tan y$

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11.10 - Solving Differential Equations

- This is not a specific solution. Sometimes we are interested in finding a **particular solution** at a specific point and this point is also known as **boundary condition**.

Example

Find the general solution to $\frac{dy}{dx} = -\frac{3(y-2)}{(2x+1)(x+2)}$

Given that $x = 1$ when $y = 4$. Leave your answer in the form $y = f(x)$

Exam Practice

Edexcel C4 Jan 2012 Q4

Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{y\cos^2 x}.$$ **(5)**

BF MATHS

11.11 - Modelling with Differential Equations

Notes

- Differential equations can be used to model real-life situations.
  - we have learnt to construct differential equations (Chp 9.10)
  - we have learnt to solve differential equations (Chp 11.10)

Example

The rate of increase of a population P of micro-organisms at time t, in hours, is given by $\frac{dP}{dt} = 3P$.
Initially the population was of size 8.

a) Find a model for P in the form $P = Ae^{3t}$, stating the value of A.
b) Find, to the nearest hundred, the size of the population at time $t = 2$.
c) Find the time at which the population will be 1000 times its starting value.
d) State one limitation to the model.

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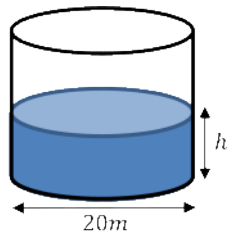
11.11 - Modelling with Differential Equations

<u>Example</u>

Water in a manufacturing plant is held in a large cylindrical tank of diameter 20m. Water flows out of the bottom of the tank through a tap at a rate proportional to the cube root of the volume.



a) Show that t minutes after the tap is opened, $\frac{dh}{dt} = -k\sqrt[3]{h}$ for some constant k.

b) Show that the general solution of this differential equation may be written $h = (P - Qt)^{\frac{3}{2}}$, where P and Q are constants. Initially the height of the water is 27m. 10 minutes later, the height is 8m.

c) Find the values of the constants P and Q.

d) Find the time in minutes when the water is at a depth of 1m.

11.11 - Modelling with Differential Equations

Exam Practice

Edexcel C4 June 2005 Q8

Liquid is pouring into a container at a constant rate of 20 $cm^3 s^{-1}$ and is leaking out at a rate proportional to the volume of the liquid already in the container.

(*a*) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where k is a positive constant. **(2)**

The container is initially empty.

(*b*) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k. **(6)**

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(*c*) find the volume of liquid in the container at 10 s after the start. **(5)**

BF MATHS

11.12 - Integration as the limit of sum

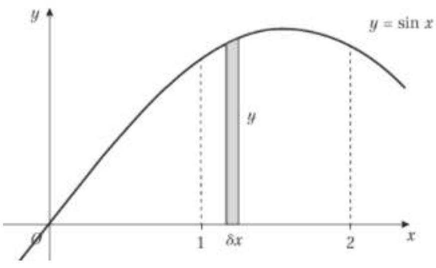
Notes

Example

The diagram shows a sketch of the curve with equation $y = \sin x$.

The area under the curve between $x = 1$ and $x = 2$ can be thought of a series of thin strips of height y and width δx.

Calculate $\lim_{\delta x \to 0} \sum_{x=1}^{2} \sin x \, \delta x$, giving your answer correct to 4 s.f.



11.12 - Integration as the limit of sum

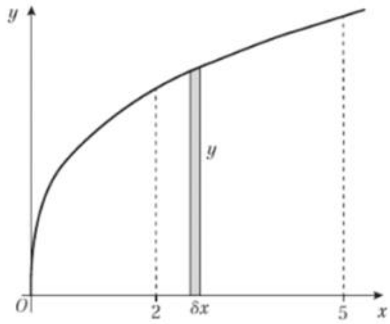
Exercise 11L

1 Find the exact value of:

a $\lim_{\delta x \to 0} \sum_{x=3}^{12} x^2 \, \delta x$ b $\lim_{\delta x \to 0} \sum_{x=9}^{25} \sqrt{x} \, \delta x$ c $\lim_{\delta x \to 0} \sum_{x=5}^{10} \left(x\sqrt{x-1}\right) \delta x$

E/P 2 Calculate $\lim_{\delta x \to 0} \sum_{x=2}^{3} \ln x \, \delta x$, giving your answer in the form $p + \ln q$, where p and q are rational numbers to be found. **(4 marks)**

E/P 3 The diagram shows a sketch of the curve with equation $y = \sqrt[3]{x}$, $x > 0$.



The area under the curve between $x = 2$ and $x = 5$ can bethought of as a series of thin strips of height y and width δx.

Calculate $\lim_{\delta x \to 0} \sum_{x=2}^{5} \sqrt[3]{x} \, \delta x$, giving your answer correct to 4 significant figures. **(3 marks)**

Integration 11L

1 a $\lim_{\delta x \to 0} \sum_{x=3}^{12} x^2 \, \delta x = \int_3^{12} x^2 \, dx$

This is the limit as the width of the strip tends towards 0.

$$\int_3^{12} x^2 \, dx = \left[\frac{x^3}{3}\right]_3^{12}$$
$$= 576 - 9$$
$$= 567$$

b $\lim_{\delta x \to 0} \sum_{x=9}^{25} \sqrt{x} \, \delta x = \int_9^{25} \sqrt{x} \, dx$

This is the limit as the width of the strip tends towards 0.

$$\int_9^{25} \sqrt{x} \, dx = \int_9^{25} x^{\frac{1}{2}} \, dx$$
$$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_9^{25}$$
$$= \frac{250}{3} - \frac{54}{3}$$
$$= \frac{196}{3}$$

c $\lim_{\delta x \to 0} \sum_{x=5}^{10} \left(x\sqrt{x-1}\right) \delta x = \int_5^{10} \left(x\sqrt{x-1}\right) dx$

This is the limit as the width of the strip tends towards 0.

Let $I = \int_5^{10} \left(x\sqrt{x-1}\right) dx$

Let $u = x - 1$

$\frac{du}{dx} = 1$

So replace dx with du
and replace $\sqrt{x-1}$ with u
and replace x with $u + 1$

Change the limits:

x	u
10	9
5	4

$$I = \int_4^9 \left((u+1)u^{\frac{1}{2}}\right) du$$
$$= \int_4^9 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$$
$$= \left[\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}\right]_4^9$$
$$= \frac{2 \times 9^{\frac{5}{2}}}{5} + \frac{2 \times 9^{\frac{3}{2}}}{3} - \frac{2 \times 4^{\frac{5}{2}}}{5} - \frac{2 \times 4^{\frac{3}{2}}}{3}$$
$$= \frac{486}{5} + \frac{54}{3} - \frac{64}{5} - \frac{16}{3}$$
$$= \frac{1456}{15}$$

2 $\lim_{\delta x \to 0} \sum_{x=2}^{3} \ln x \, \delta x = \int_2^3 \ln x \, dx$

This is the limit as the width of the strip tends towards 0.

Let $I = \int_2^3 \ln x \, dx$

Let $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\frac{dv}{dx} = 1 \Rightarrow v = x$

$$I = [x \ln x]_2^3 - \int_2^3 x \times \frac{1}{x} \, dx$$
$$= 3\ln 3 - 2\ln 2 - \int_2^3 1 \, dx$$
$$= \ln 3^3 - \ln 2^2 - [x]_2^3$$
$$= \ln\left(\frac{27}{4}\right) - 3 + 2$$
$$= -1 + \ln\left(\frac{27}{4}\right)$$

3 $\lim_{\delta x \to 0} \sum_{x=2}^{5} \sqrt[3]{x} \, \delta x = \int_2^5 \sqrt[3]{x} \, dx$

This is the limit as the width of the strip tends towards 0.

$$\int_2^5 \sqrt[3]{x} \, dx = \int_2^5 x^{\frac{1}{3}} \, dx$$
$$= \left[\frac{3x^{\frac{4}{3}}}{4}\right]_2^5$$
$$= 6.41240... - 1.88988...$$
$$= 4.523 \text{ (4 s.f.)}$$