

Chapter 8 - Binomial Expansion

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Personal notes:



8.1 - Pascal's Triangle

Starter

a) Expand and simplify $(a + b)^0 =$

b) Expand and simplify $(a + b)^1 =$

c) Expand and simplify $(a + b)^2 =$

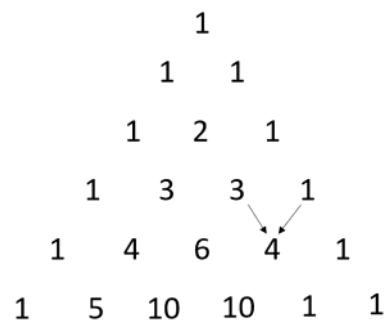
d) Expand and simplify $(a + b)^3 =$

e) Expand and simplify $(a + b)^4 =$

What do you notice about the coefficients?

What do you notice about the powers of a and b ?

Notes (Pascal's Triangle)



****Tips:** I highly recommend memorising each row up to what you see here.

Example

Use the pascal triangle to find the expansion of $(x + 2y)^3$



8.1 - Pascal's Triangle

Example

Use the pascal triangle to find the expansion of $(1 - 2x)^4$

Practice Q1

- a) Expand $(2x - 3)^3$
- b) Expand $(3x + 4)^4$

Example

The coefficient of x^2 in the expansion of $(2 - cx)^5$ is 720. Find the possible value(s) of the constant c .

Practice Q2

- a) Find the coefficient of x^3 in the expansion of $(x + 2)^5$.
- b) Find the coefficient of x^3 in the expansion of $\left(4x - \frac{1}{2}\right)^4$.



8.2 - Factorial Notation!

Notes

- $n! =$

Said "n factorial", is the number of ways of arranging n objects in a line

- E.g. Suppose you want to arrange letters A, B and C in a line to make a word (ABC, ACB...)
 - There are 3 choices for the first letter
 - There are 2 choices for the second letter
 - There is 1 choice for the third letter
- => Altogether there are $3 \times 2 \times 1 = 3! = 6$ possible combinations

- **By definition, $0! = 1$**

- $nCr = \binom{n}{r} =$

Said "n choose r", is the number of ways of "choosing" r things from n , such that the order does not matter.

These are always known as **binomial coefficients**.

- E.g. Possible selections of choosing 4 people out of 10
=

Example/Practice Q1

Calculate the value of the following. You may use the factorial button, but not the nCr button.

a) $5!$

b) $\binom{5}{3}$

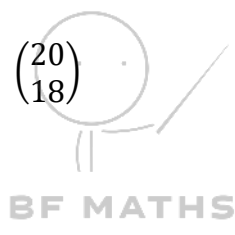
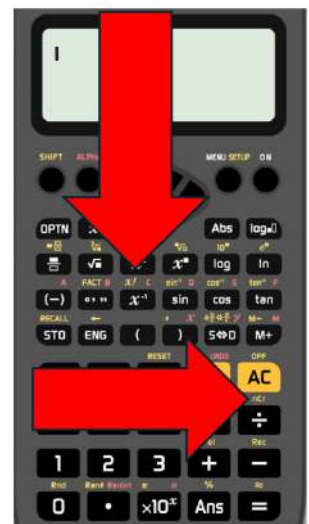
c) $0!$

d) $\binom{20}{1}$

e) $\binom{20}{0}$

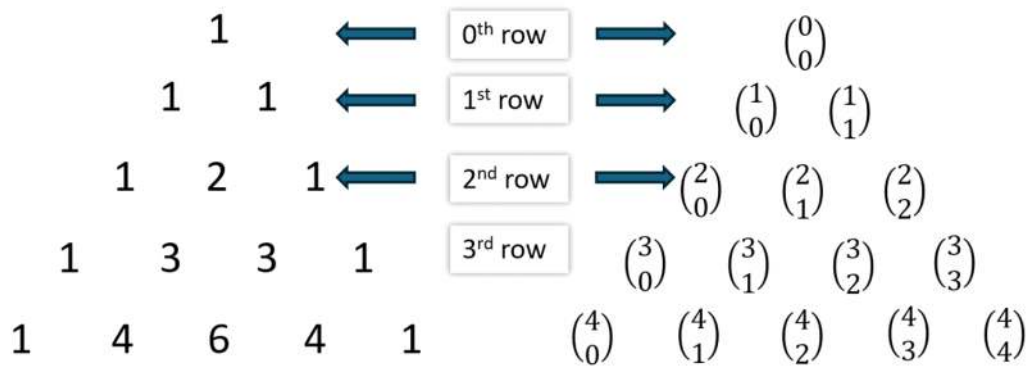
f) $\binom{20}{2}$

g) $\binom{20}{18}$



8.2 - Factorial Notation!

nCr and Pascal Triangle



Example

Use the nCr button to work out the 7th row of the Pascal Triangle.

Practice Q2

A row of Pascal's triangle is shown below

1, 17, 136, ...

- a) Find the new two values in the row.
- b) Hence find the coefficient of x^4 in the expansion of $(1 + 2x)^{17}$



8.3 - The Binomial Expansion

Notes

The binomial expansion:

Example

Find the first 4 terms in the expansion of $(2x + 1)^{10}$, in ascending powers of x .

Example

Find the first 5 terms in the expansion of $\left(\frac{1}{3}x - 2\right)^7$, in descending powers of x .



8.3 - The Binomial Expansion

Practice Q1

Write down the expansion of the following:

a) $(1 + 2x)^4$

b) $(2 + 5x)^6$

Practice Q2

Find the first 4 terms in the expansion of $\left(10 - \frac{1}{2}x\right)^{10}$, in ascending powers of x .



8.4 - Solving Binomial problems

Starter

Expand and simplify $(3 + x)^5$

Example

Find the term in the expansion of the following expressions which contains:

a) x^3 in $(2 + x)^{10}$

b) x^7 in $(3 - x)^{12}$

c) x^5 in $(3x + 4)^9$

Practice Q1

Find the term in the expansion of the following expressions which contains:

a) x^2 in $(6 + x)^5$

b) x^4 in $(2x + 3)^8$

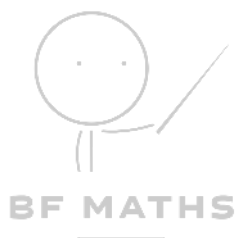
c) x^7 in $\left(\frac{1}{2}x - 3\right)^{10}$

Example

Find the coefficient of x^3 in the binomial expansion of $(2 + x)(3 - 2x)^7$.

Practice Q2

Find the coefficient of x^4 in the binomial expansion of $(x + 1)(2 - 3x)^6$.



8.4 - Solving Binomial problems

Example

$g(x) = (1 + kx)^{10}$, where k is a constant.

Given that the coefficient of x^3 in the binomial expansion of $g(x)$ is 15, find the value of k .

Practice Q3

The coefficient of x^4 in the expansion of $(1 + qx)^{10}$ is 3360. Find the possible value(s) of the constant q .

Practice Q4

- Write down the first three terms, in ascending power of x , of the binomial expansion of $(1 + qx)^8$, where q is a non-zero constant.
- Given that, in the expansion of $(1 + qx)^8$, the coefficient of x is $-r$ and the coefficient of x^2 is $7r$, find the value of q and the value of r .



8.5 - Binomial Estimation

Notes

- In engineering and science, it is often useful to find the simple **approximations** for complicated functions.
- To work out binomial estimation questions, the most important thing is to find

Example

- Find the first four terms of the binomial expansion, in ascending powers of x , of $\left(1 - \frac{x}{4}\right)^{10}$.
- Use your expansion to estimate the value of 0.975^{10} , giving your answer to 4 decimal places.
- Use your answer to part (b) to work out the percentage error compared to the actual calculator display value of 0.975^{10} .

Practice Q1

- Find the first four terms of the binomial expansion, in ascending power of x , of $\left(1 - \frac{x}{10}\right)^6$.
- By substituting an appropriate value of x , find an approximate value of 0.99^6 .



8.5 - Binomial Estimation

Exam Practice (Jan 2013 C2 Q3)

a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of $\left(1 + \frac{x}{4}\right)^8$, giving each term in its simplest form. **(4 marks)**

b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places. **(3 marks)**

Exam Practice (Jan 2008 C2 Q3)

a) Find the first 4 terms of the expansion of $\left(1 + \frac{x}{2}\right)^{10}$ in ascending powers of x , giving each term in its simplest form. **(4 marks)**

b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places. **(3 marks)**

