Chapter 10 - Forces and Motion

Personal notes:

BF MATHS

10.1, 10.2 - Force Diagrams and Force Vectors

Types of forces

- **Newton's 1st Law of Motion** states than an object at rest will stay at rest and that an object moving with constant velocity will remain at that velocity unless an unbalanced force acts on the object.

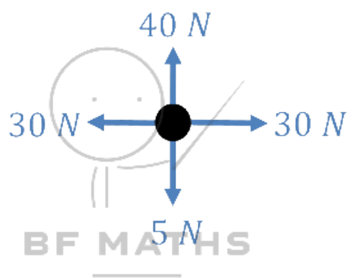
- The '**resultant force**' is the overall force acting on the object. An object will accelerate in the direction of the resultant force.

- If the forces on an object are balanced,

- We can resolve resultant force in two directions:

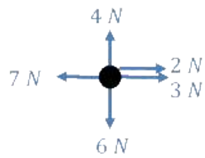
Example

Four forces have exerted to an object, as shown in the diagram.
Work out the resultant force
a) in the horizontal direction
b) in the vertical direction

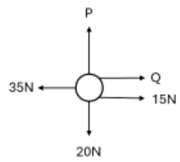


10.1, 10.2 - Force Diagrams and Force Vectors

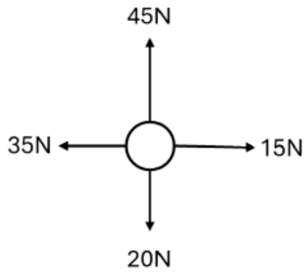
<u>Example</u>
a) Find the resultant force in the two directions
b) Find the size and direction of the resultant force



<u>Practice Q1</u>
Given that this particle is stationary (i.e. all forces are balanced), work out the value of *P* and *Q*.



<u>Practice Q2</u>
a) Work out the resultant force in the x-direction
b) Work out the resultant force in the y-direction
c) Work out the size and direction of the *overall* resultant force



<u>Notes</u>

- Forces have direction, and therefore we can naturally write them as vectors, either in $\boldsymbol{i}$-$\boldsymbol{j}$ notation or as column vectors.
- You can find the resultant of two or more forces given as vectors by adding the vectors.

<u>Example</u>
The forces $(2\boldsymbol{i} + 3\boldsymbol{j})$N , $(4\boldsymbol{i} - \boldsymbol{j})$N , $(-3\boldsymbol{i} + 2\boldsymbol{j})$N and $(a\boldsymbol{i} + b\boldsymbol{j})$N act on an object which is in equilibrium. Find the values of a and b.

10.1, 10.2 - Force Diagrams and Force Vectors

Example

The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

b) Work out the magnitude and bearing of the resultant force.

Practice Q3

Work out the resultant force acting on this particle:

$(2i - 4j)N$ and $(-i + 5j)$ N.

Practice Q4

A particle is acted upon by three forces: $(5i - 3j)$N , $(3i + 4j)N$ and $(-2i + 3j)N$. Work out:

a) the resultant vector

b) the magnitude of the resultant vector

c) the bearing of the resultant vector

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10.3 - Forces and acceleration

Notes

- Recall: We can work out resultant forces in two directions:

i) Parallel to motion ($x - direction$)

ii) Perpendicular to motion ($y - direction$)

- Newton's 2nd law:

Example

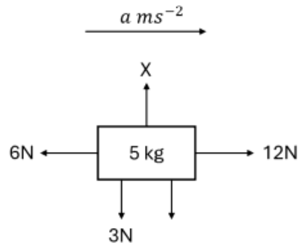
A car of 3000kg has a driving force of 900N and forces of 250N resisting its motion. Work out its acceleration.

Example

A child has a mass of 50kg. What is the gravitational force acting on the child? (i.e. its weight)

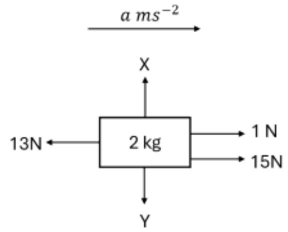
Example

Work out the missing forces on the diagram and the acceleration of the particle.

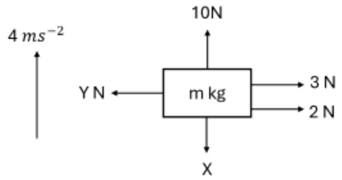


10.3 - Forces and acceleration

Practice Q1
Work out the values of X,Y and a.



Practice Q2
Work out the values of X,Y and m.



Example/Practice Q3
A body of mass 5kg is pulled along a rough horizontal table by a horizontal force of magnitude 20N against a constant friction force of magnitude 4N. Given that the body is initially at rest, find

a) the acceleration of the body
b) the distance travelled by the body in the first 4 seconds
c) the magnitude of the normal reaction between the body and the table

10.4 - Motions in 2 dimensions

Notes

- Forces have magnitude and direction. We can represent forces as vectors.

Example

Find the magnitude of acceleration of a 3kg block under 3 forces: $F_1 = (2i + 4j)N$, $F_2 = (-5i + 4j)N$ and $F_3 = (6i - 5j)N$.

Example

A 60kg boat is accelerating at rate of $\begin{pmatrix} 0.8 \\ 1.5 \end{pmatrix} ms^{-2}$ by forces $F_1 = (80i + 50j)N,\ F_2 = (10pi + 20qj)N\ and\ F_3 = (-75i + 100j)N$. Find the values of p and q.

Practice Q1

Three forces act upon a particle of mass 0.5kg. The three forces are $(4i - 2j)N$, $(3i + j)N$, and $(-8i + 4j)N$.

a) Find the acceleration of the particle in the form $(pi + qj)ms^{-2}$.

b) Find the magnitude and bearing of the acceleration of the particle.

10.5 - Connected Particles

Notes

- When we have multiple *connected* objects moving in the *same direction*:
  - Consider both particles as a *single system*
  - Consider individual particle separately

Example

Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N.

a) Find the acceleration of the particles.
b) Find the tension in the string.
c) Explain how the modelling assumptions that the string is light and inextensible have been used.

Practice Q1

Two particles *P* and *Q* of masses 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude *F* is applied to *P* in a direction away from *Q* and when the string is taut the particles move with acceleration $0.4\ ms^{-2}$.

a) Draw a diagram that includes all the forces to illustrate
b) Find the value of *F*
c) Find the tension in the string.
c) Explain how the two modelling assumptions that the string is light and inextensible are used.

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10.5 - Connected Particles

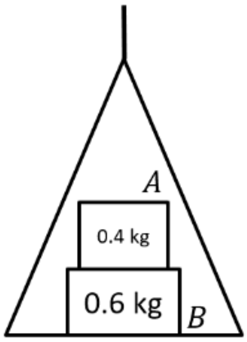
- Newton's 3^{rd} law: For every action there is an equal and opposite reaction

  Therefore when two bodies A and B are in contact, if body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and acts in the opposite direction.

<u>Example</u>

A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B, as shown in the diagram.
The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

a) Find the tension in the string.
b) Find the force exerted on mass B by mass A.
c) Find the force exerted on mass B by the scale-pan.



<u>Exam Practice</u>

Edexcel M1 May 2013 Q2

A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable, **(3)**

(b) the magnitude of the force exerted on the woman by the floor of the lift. **(3)**

10.6 - Pulleys

Notes

- A pulley is a wheel which a cord/string/rope/cable passes over.
- Why can't we just model both particles as a single particle as before?

- Under what condition is the tension in each part of the string the same?

Example

Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a) Write down an equation of motion for P and for Q.
b) Find the acceleration of each mass.
c) Find the tension in the string in terms of m.
d) Find the force exerted on the pulley by the string.
e) Find the distance moved by Q in the first 4 s, assuming that P does not reach the pulley.

Practice Q1

Two particles *A* and *B* of masses 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.

a) Find the tension in the string.
b) Find the acceleration.
Given *A* has travelled a distance of 2 m it strikes the ground,
c) Find the speed that *A* when it hits the ground.

10.6 - Pulleys

Example**

Two particles A and B of masses 0.4kg and 0.8kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5m above horizontal ground. A frictional force of magnitude 0.08g opposes the motion of particle A. The system is released from rest. Find:

a) The acceleration of the system
b) The time taken for B to reach the ground
c) The total distance travelled by A before it first comes to rest.
(This sub-question could worth up to 7 marks on its own)

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10.6 - Pulleys

<u>Practice Q2**</u>

A box *A* of mass 0.8 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley fixed at the edge of the table. The other end of the string is attached to a box *B* of mass 1.2 kg, which hangs freely below the pulley. The magnitude of the frictional force between *A* and the table is *f* N. The system is released from rest with the string taut. After release, *B* descends a distance of 0.9 m in 0.8 s to the ground. Modelling *A* and *B* as particles, calculate

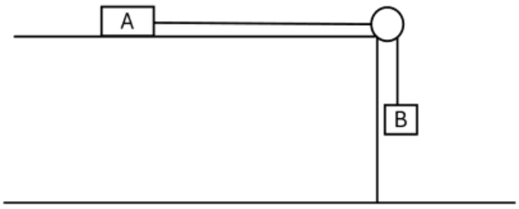
a) the acceleration of the particles **(2 marks)**

b) the tension in the string **(3 marks)**

c) the value of frictional force *F* **(3 marks)**

Given that *A* does not reach the pulley and the frictional force remains constant throughout,

d) find the total distance travelled by *A*. **(7 marks)**



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