Chapter 2 - Functions and Graph

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2.1 - The Modulus Function

Notes

Example 1

Given $f(x) = |5x + 3| - 7$

a) Find the value of $f(2)$
b) Find the value of $f(-3)$

Practice (Ex. 2A)

2 $f(x) = |7 - 5x| + 3$. Write down the values of:

a $f(1)$ **b** $f(10)$ **c** $f(-6)$

3 $g(x) = |x^2 - 8x|$. Write down the values of:

a $g(4)$ **b** $g(-5)$ **c** $g(8)$

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2.1 - The Modulus Function

Sketching graph of $y = |ax + b|$

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Example 2

Sketch the graph of $y = |3x - 2|$.

Practice (Ex. 2A Question 4c)

c $y = |4x - 7|$

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2.1 - The Modulus Function

Example 3

Solve the equation $|3x - 5| = 2 - \frac{1}{2}x$

Practice (Ex. 2A Q5)

5 $g(x) = \left|4 - \frac{3}{2}x\right|$ and $h(x) = 5$

a On the same axes, sketch the graphs of $y = g(x)$ and $y = h(x)$.

b Hence solve the equation $\left|4 - \frac{3}{2}x\right| = 5$.

2.1 - The Modulus Function

Example 4

Solve the inequality $|5x - 1| > 3x$ and write your answer in set notation.

2.2 - Functions and Mappings

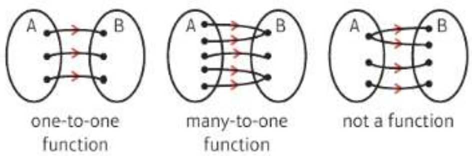
Notes

Express the function machine as:

a) An algebraic equation (function)

a) A mapping

- A mapping is a function if every input has a distinct output (i.e. each input value only gives one output value).



<u>Graphically</u>

2.2 - Functions and Mappings

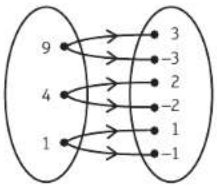
Practice (Pair and share)

For each of the following mappings:

i State whether the mapping is one-to-one, many-to-one or one-to-many.

ii State whether the mapping is a function.

a



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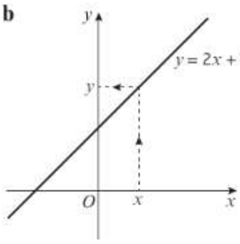

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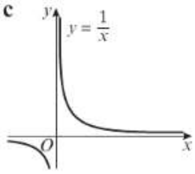

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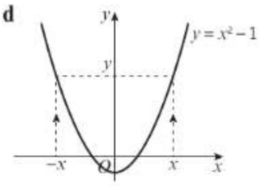

- Domain is the set of all possible input values (i.e. values of x)
- Range is the set of all possible output values (i.e. values of y)

Example

The function $f(x)$ is defined by

$$f: x \mapsto \begin{cases} 5 - 2x, & x < 1 \\ x^2 + 3, & x \geqslant 1 \end{cases}$$

a Sketch $y = f(x)$, and state the range of $f(x)$.

b Solve $f(x) = 19$.

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2.3, 2.4 - Composite and Inverse Function

Notes (Composite Function)

Starter

The functions f and g are defined by $f(x) = 3x + 2$ and $g(x) = x^2 + 4$. Find:

a the function $fg(x)$

b the function $gf(x)$

c the function $f^2(x)$

d the values of b such that $fg(b) = 62$.

Example

The functions f and g are defined by

$f: x \mapsto |2x - 8|$

$g: x \mapsto \frac{x+1}{2}$

a Find $fg(3)$. **b** Solve $fg(x) = x$.

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2.3, 2.4 - Composite and Inverse Function

Practice (Ex. 2C Q9)

E/P **9** The functions p and q are defined by

p: $x \mapsto \ln(x + 3)$, $x \in \mathbb{R}$, $x > -3$

q: $x \mapsto e^{3x} - 1$, $x \in \mathbb{R}$

Hint The range of p will be the set of possible inputs for q in the function qp.

a Find qp(x) and state its range. **(3 marks)**

b Find the value of qp(7). **(1 mark)**

c Solve qp(x) = 124. **(3 marks)**

Notes (Inverse Function)

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2.3, 2.4 - Composite and Inverse Function

Example

The function, $f(x) = \sqrt{x-2}$, $x \in \mathbb{R}$, $x \geqslant 2$.

a State the range of $f(x)$.

b Find the function $f^{-1}(x)$ and state its domain and range.

c Sketch $y = f(x)$ and $y = f^{-1}(x)$ and the line $y = x$.

Practice

The function $f(x)$ is defined by $f(x) = x^2 - 3$, $x \in \mathbb{R}$, $x \geqslant 0$.

a Find $f^{-1}(x)$.

b Sketch $y = f^{-1}(x)$ and state its domain.

c Solve the equation $f(x) = f^{-1}(x)$.

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2.5 - $\left|f(x)\right|$ and $f(|x|)$

Starter : Work out the values of y

	$y = \left|x^2 - 3x - 10\right|$	$y = \left|x^2\right| - 3|x| - 10$
$x = 2$		
$x = -2$		
$x = 0$		
$x = 5$		
$x = -5$		
Conclusion		

Graphical Differences

Sketching Graphs	$y = \left|f(x)\right|$	$y = f(|x|)$
Steps	• Sketch the graph of y = f(x) • Reflect the section below the x-axis in the x-axis	• Sketch the graph of $y = f(x)$ where $x \geq 0$ • Reflect the section in the y-axis
Example		

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2.5 - $|f(x)|$ and $f(|x|)$

<u>Example</u>

$f(x) = x^2 - 3x - 10$

a Sketch the graph of $y = f(x)$.

b Sketch the graph of $y = |f(x)|$.

c Sketch the graph of $y = f(|x|)$.

<u>Practice</u>

$g(x) = \sin x$, $-360° \leqslant x \leqslant 360°$

a Sketch the graph of $y = g(x)$.

b Sketch the graph of $y = |g(x)|$.

c Sketch the graph of $y = g(|x|)$.

2.5 - $|f(x)|$ and $f(|x|)$

Practice (Ex. 2E Q10)

E/P **10** The function $f(x)$ is defined by

$$f(x) = \begin{cases} -2x - 6, & -5 \leqslant x < -1 \\ (x+1)^2, & -1 \leqslant x \leqslant 2 \end{cases}$$

a Sketch $f(x)$ stating its range. **(5 marks)**

b Sketch the graph of $y = |f(x)|$. **(3 marks)**

c Sketch the graph of $y = f(|x|)$. **(3 marks)**

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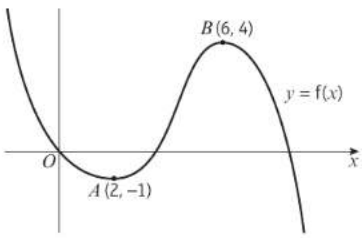
2.6 - Combining Transformation

Recall

Order of Transformations when more than 1

Example

The diagram shows a sketch of the graph of $y = f(x)$. The curve passes through the origin O, the point $A(2, -1)$ and the point $B(6, 4)$.



Sketch the graphs of:

a $y = 2f(x) - 1$

b $y = f(x + 2) + 2$

c $y = \frac{1}{4}f(2x)$

d $y = -f(x - 1)$

In each case, find the coordinates of the images of the points O, A and B.

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2.6 - Combining Transformation

Example

$f(x) = \ln x, x > 0$

Sketch the graphs of

a $y = 2f(x) - 3$ **b** $y = |f(-x)|$

Show, on each diagram, the point where the graph meets or crosses the x-axis.

In each case, state the equation of the asymptote.

2.6 - Combining Transformation

<u>Practice (Ex. 2F Q4,5)</u>

(E) **4** The function g is defined by

g: $x \mapsto (x-2)^2 - 9,\ x \in \mathbb{R}$.

a Draw a sketch of the graph of $y = \mathrm{g}(x)$, labelling the turning point and the x- and y-intercepts. **(3 marks)**

b Write down the coordinates of the turning point when the curve is transformed as follows:

i $2\mathrm{g}(x-4)$ **(2 marks)**

ii $\mathrm{g}(2x)$ **(2 marks)**

iii $|\mathrm{g}(x)|$ **(2 marks)**

c Sketch the curve with equation $y = \mathrm{g}(|x|)$. On your sketch show the coordinates of all turning points and all x- and y-intercepts. **(4 marks)**

5 $\mathrm{h}(x) = 2\sin x,\ -180° \leqslant x \leqslant 180°$.

a Sketch the graph of $y = \mathrm{h}(x)$.

b Write down the coordinates of the minimum, A, and the maximum, B.

c Sketch the graphs of:

i $\mathrm{h}(x - 90°) + 1$ **ii** $\frac{1}{4}\mathrm{h}\left(\frac{1}{2}x\right)$ **iii** $\frac{1}{2}|\mathrm{h}(-x)|$

In each case find the coordinates of the images of the points O, A and B.

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2.7 - Solving Modulus Problem

Example 1

Given the function $t(x) = 3|x - 1| - 2, x \in \mathbb{R}$,

a sketch the graph of the function

b state the range of the function

c solve the equation $t(x) = \frac{1}{2}x + 3$.

Practice (Ex. 2G Q1 cd)

Sketch the graph of each function and state the range.

c $f(x) = -2|x - 1| + 6, x \in \mathbb{R}$

d $f: x \mapsto -\frac{5}{2}|x| + 4, x \in \mathbb{R}$

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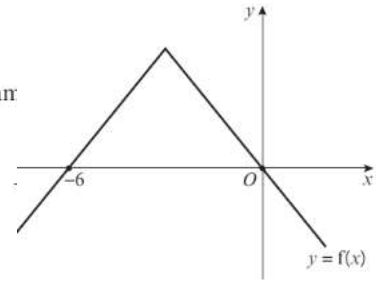
2.7 - Solving Modulus Problem

Example 2

The function f is defined by f: $x \mapsto 6 - 2|x + 3|$.

A sketch of the graph of the function is shown in the diagram.



a State the range of f.

b Give a reason why f^{-1} does not exist.

c Solve the inequality $f(x) > 5$.

Practice (Ex.2G Q6)

(E/P) **6** The functions m and n are defined as

$m(x) = -2x + k, x \in \mathbb{R}$

$n(x) = 3|x - 4| + 6, x \in \mathbb{R}$

where k is a constant.

The equation $m(x) = n(x)$ has no real roots.

Find the range of possible values for the constant k. **(4 marks)**

Problem-solving

$m(x) = n(x)$ has no real roots means that $y = m(x)$ and $y = n(x)$ do not intersect.

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