Chapter 10 -Trigonometric Identities and Equations

10.1, 10.2 - Angles in four quadrants and exact values - Pg. 2 - 3

10.3 - Trigonometric identities - Pg. 4 - 6

10.4 - Simple trigonometric equations - Pg. 7 - 9

10.5 - Harder trigonometric equations - Pg. 10 - 12

10.6 - Equations and Identities - Pg. 13 - 14

Personal notes:

10.1, 10.2 - Angles in four quadrants and exact values

<u>Starter</u>

Use your calculator, fill in the table with the following options:
positive, **negative**, **1, 0, -1**, or **undefined**

Value of θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\theta = 0°$			
$0 < \theta < 90°$			
$\theta = 90°$			
$90 < \theta < 180°$			
$\theta = 180°$			
$180 < \theta < 270°$			
$\theta = 270°$			
$270 < \theta < 360°$			

We can elaborate the table in a diagram - _______________

<u>Exact values</u>

	0°	30°	45°	60°	90°
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

10.1, 10.2 - Angles in four quadrants and exact values

Notes

- Why using CAST diagram?
  It replaces the trig graphs to find out all the other solutions.

- We can rewrite angles in $2^{nd}, 3^{rd}$ and 4^{th} quadrants to 1^{st} quadrant using CAST diagram.

Steps of using CAST

1. Draw a pair of axes and label CAST in the correct quadrants and the angle given by the question
2. Work out the angle between the x-axis
3. Check the trig ratio against the quadrant (is it positive or negative?)
4. Reflect that angle in another quadrant that gives the same sign.
5. Count the two angles from 0^o

Example

Use CAST diagram to find the other angle(s) within the interval $0^o \leq x \leq 360^o$.

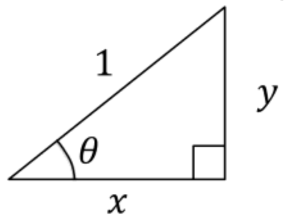
a) $\cos(315°)$
b) $\sin(120°)$
c) $\tan(-120°)$
d) $\sin(240°)$
e) $\cos(-225°)$

Practice Q1

a) $\sin(150°)$
b) $\cos(240°)$
c) $\tan(-45°)$
d) $\cos(-135°)$
e) $\tan(245°)$
f) $\sin(405°)$

10.3 - Trigonometric identities

Consider the following triangle:



Example
Simplify $5 - 5\sin^2\theta$

Example
Simplify $\sin^2 3\theta + \cos^2 3\theta$

Example
Prove that $1 - \tan\theta\sin\theta\cos\theta \equiv \cos^2\theta$

Example
Prove that $\frac{\tan x \cos x}{\sqrt{1-\cos^2 x}} \equiv 1$

BF MATHS

10.3 - Trigonometric identities

Practice Q1

a) Simplify $5\sin^2 3\theta + 5\cos^2 3\theta$

b) Simplify $(1+\sin x)^2 + (1-\sin x)^2 + 2\cos^2 x$

Practice Q2

Prove that $\tan^2\theta \equiv \frac{1}{\cos^2\theta} - 1$

Practice Q3

a) Prove that $\frac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

b) Prove that $2 - (\sin\theta - \cos\theta)^2 \equiv (\sin\theta + \cos\theta)^2$

Practice Q4 (tricky)

Prove that $\frac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} \equiv 1 - \tan^2\theta$

BF MATHS

10.3 - Trigonometric identities

Example

Given that $\cos\theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin\theta$ without using a calculator.

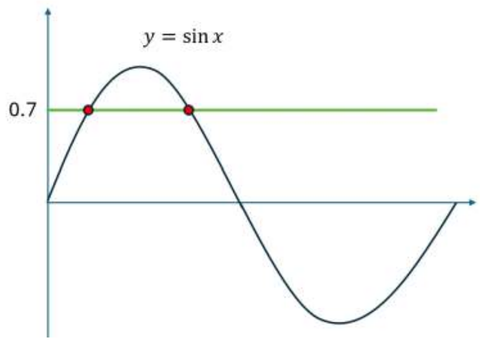
Practice Q5

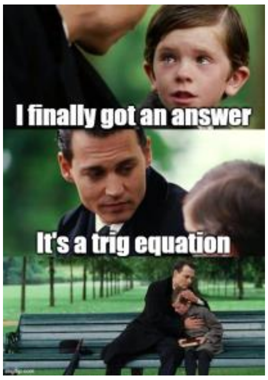
Given that $\sin\theta = \frac{2}{5}$ and that θ is obtuse, find the value of $\cos\theta$ without using a calculator.

10.4 - Simple trigonometric equations

Notes

- The equations $\sin\theta = k$ and $\cos\theta = k$ are only solvable if $-1 \leq k \leq 1$.
- We know there are more than 1 solutions to most trigonometric equations.





- _______________could help us work out all solutions within the given range of θ.
- Use CAST diagram as soon as you $\sin^{-1} ⬚$, $\cos^{-1} ⬚$, $\tan^{-1} ⬚$

Example

Find the solutions of the equation $\sin\theta = \frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

Example

Find the solutions of the equation $\tan\theta = -\frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

Practice Q1

Find the solutions of the equation $\cos\theta = \frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 360°$.

Practice Q2

Find the solutions of the equation $\sin\theta = -\frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 360°$.

BF MATHS

10.4 - Simple trigonometric equations

<u>Example</u>
Solve, in the interval $0 \leq x \leq 360°$, $5\cos x = -2$.

<u>Example</u>
Solve $\sin\theta = \sqrt{3}\cos\theta$ in the interval $0 \leq \theta \leq 360°$.

<u>Practice Q3</u>
Solve the following equations for θ, in the interval $0 \leq \theta \leq 360^{o}$:

a) $7\sin\theta = 5$
b) $3\cos\theta = -2$
c) $3\tan\theta = -11$
d) $2\sin\theta - 3\cos\theta = 0$

BF MATHS

10.4 - Simple trigonometric equations

Example

Solve $\sin\theta = -\frac{5}{7}$ in the interval $-180° \leq \theta \leq 540°$.

Practice Q4

a) Solve $\tan x = \frac{-\sqrt{3}}{3}, -180^o \leq x \leq 360^o$
b) Solve $\cos x = 0.84, -100^o \leq x \leq 440^o$

Practice Q5

Solve $\sqrt{3}\sin\theta = \cos\theta$ in the interval $-180° \leq \theta \leq 720°$.

10.5 - Harder trigonometric equations

Notes

- Harder trigonometric equations involve expressions like $\cos 3\theta$, $\sin 4\theta$ and $\tan\frac{1}{2}\theta$...etc
- Remember to *find new range* for 3θ because 3θ means the graph is stretched horizontally by $\frac{1}{3}$ (aka compressed 3 times horizontally). Hence, there will be 3 pairs of solutions.

Example

Solve $\cos 3\theta = \frac{\sqrt{3}}{2}$, in the interval $0 \leq \theta \leq 360°$.

Example

Solve the equation -2 $\sin 2\theta = \cos 2\theta$ in the interval $0 \leq \theta \leq 360°$

Practice Q1

Solve the equation $\sin 3x = -\frac{1}{2}$ in the interval $-180° \leq x \leq 180°$.

10.5 - Harder trigonometric equations

Practice Q2

Solve the equation $\cos\frac{1}{2}x = -\frac{\sqrt{2}}{2}$ in the interval $0^o \leq x \leq 900°$.

Example

Solve $\sin(2x + 30°) = \frac{1}{\sqrt{2}}$ in the interval $0 \leq x \leq 360°$.

Practice Q3

Solve the equation $\sin(2x + 60°) = 0.3$ in the interval $0 \leq x \leq 180°$.

BF MATHS

10.5 - Harder trigonometric equations

Exam Practice Q4

a) Given that $4\sin x = 3\cos x$, write down the value of $\tan x$. **(1 mark)**

b) Hence, solve, for $0^o \le \theta \le 360^o$, $4\sin 2\theta = 3\cos 2\theta$. Give your answers to 1 d.p. **(4 marks)**

Exam Practice Q5

Solve, for $0 \le x < 180^o$,

$$\cos(3x - 10^o) = -0.4,$$

Giving your answers to 1 decimal place. You should show each step in your working. **(7 marks)**

BF MATHS

10.6 - Equations and Identities

Notes

- You will need to be able to solve quadratics which involved trigonometry and trigonometric identities.
- Steps to solve trigonometric quadratics:
  1) Use identities when situation arises to convert into only one trig ratio
  2) Solve quadratics normally
  3) Use CAST diagram to work out all solutions

Example

Solve $5\sin^2 x + 3\sin x - 2 = 0$ in the interval $0 \leq x \leq 360°$.

Example

Solve $\sin^2(\theta - 30°) = \frac{1}{2}$ in the interval $0 \leq \theta \leq 360°$.

Practice Q1

Solve $10\cos^2\theta - 3\cos\theta - 1 = 0$ for the interval $0 \leq \theta \leq 360^o$

BF MATHS

10.6 - Equations and Identities

Example

Solve $2\cos^2 x + 9\sin x = 3\sin^2 x$ in the interval $-180° \leq x \leq 180°$.

Practice Q2

Solve for θ, in the interval $0^o \leq \theta \leq 360^o$, the following equations:

a) $2\cos^2\theta - 5\cos\theta + 2 = 0$
b) $8\sin^2\theta + 6\cos\theta - 9 = 0$

Exam Practice

a) Show that the equation
$5\sin x = 1 + 2\cos^2 x$
can be written in the form
$2\sin^2 x + 5\sin x - 3 = 0$

(2 marks)

b) Solve, for $0 \leq x < 360^o$,
$2\sin^2 x + 5\sin x - 3 = 0$

(4 marks)