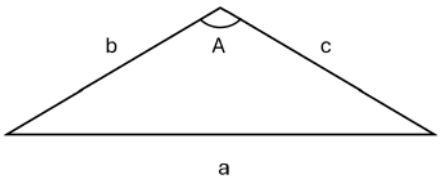
Chapter 9 - Trigonometric Ratios

Personal notes:

9.1, 9.2 - Cosine rule and Sine rule

Notes (Cosine rule)



$$a^2 = b^2 + c^2 - 2bc\cos A$$

- When to use it - *Triangles that involve all three lengths with an angle*
- The "a-value" is the length opposite to the angle *A*.

Example (missing length)
Calculate the length of the side *AB* of the triangle *ABC* in which *AC* = 6.5cm, *BC* = 8.7cm and $\angle \mathrm{ACB} = 100^{\mathrm{o}}$.

Example (missing angle)
Find the size of the smallest angle in a triangle whose sides have lengths 4cm, 7cm and 9cm.

Practice Q1
In triangle *ABC*, $AB = x\ cm$, $BC = (x+2)cm$, $AC = 5cm$ and $\angle \mathrm{ABC} = 60^{\mathrm{o}}$.
Find the value of x.

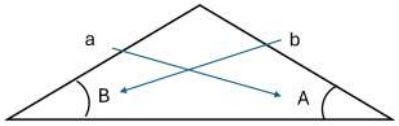
9.1, 9.2 - Cosine rule and Sine rule

Practice Q2

Coastguard station B is 8 km, on a bearing of $060°$, from coastguard station A.
A ship C is 4.8 km on a bearing of $018°$, away from A. Calculate how far C is from B.
(Tip: Bearing is always measured from the North line, clockwise)

Notes (Sine rule)

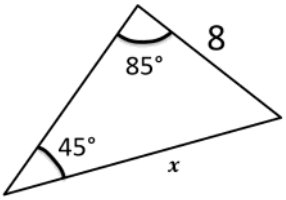
$$\frac{a}{\sin A} = \frac{b}{\sin B} \;/\; \frac{\sin A}{a} = \frac{\sin B}{b}$$



- When to use it? Triangles involved with *two lengths and two angles*.
- The "a - value" is the length opposite to the angle A.

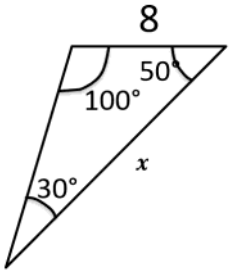
Example (missing length)

Find the value of x.



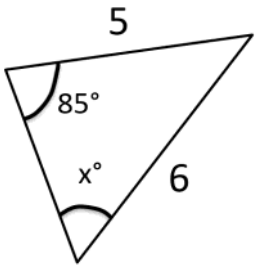
Practice Q3

Find the value of x.

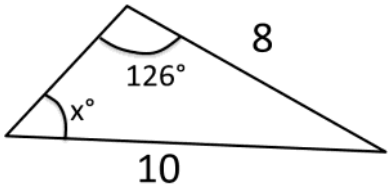


9.1, 9.2 - Cosine rule and Sine rule

Example (missing angle)
Find the value of x.

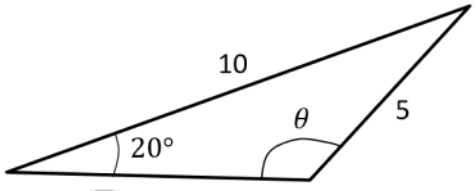


Practice Q4
Find the value of x.



Example (the Ambiguous case!)
In triangle $ABC, AB = 4cm, AC = 3cm$ and $\angle \mathrm{ABC} = 44^{\mathrm{o}}$. Work out the two possible values of $\angle \mathrm{ACB}$.

Practice
Work out the value of x and θ

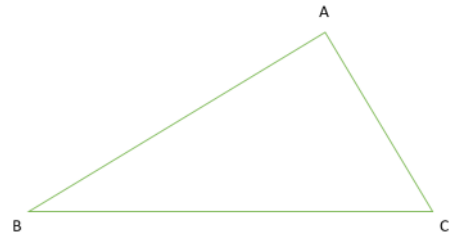


9.3 - Area of triangles

You would have learnt two formulae to work out area of triangle:

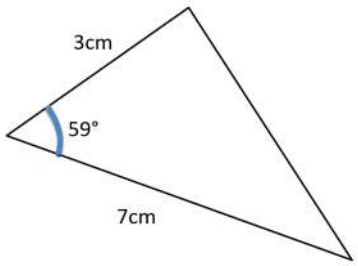
-
-

Proof of the formula



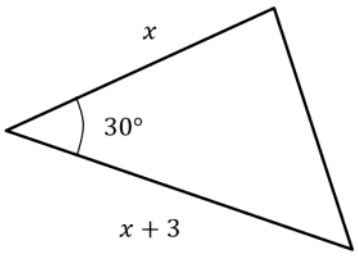
Example/Practice Q1

Work out the area of the triangle.



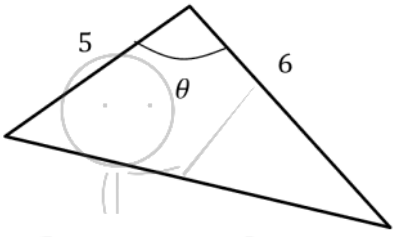
Example/Practice Q2 (harder)

Given the area is $10cm^2$. Work out value of x.



Example/Practice Q3

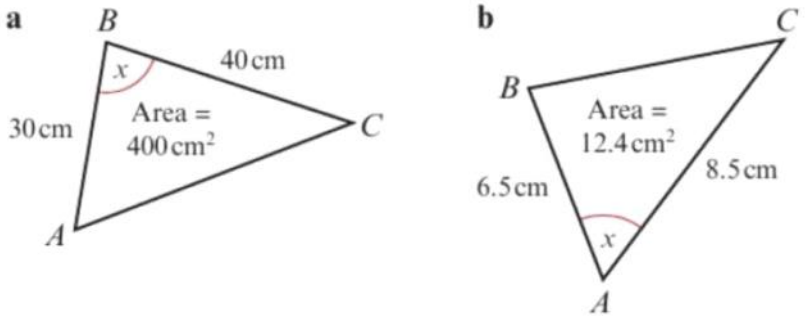
Given the area is 10 and θ is obtuse. Work out the value of θ.



9.3 - Area of triangles

Practice Q4

Work out the possible sizes of x in the following triangles:



Practice Q5

In ΔPQR, $PQ = (x + 2)$cm, $PR = (5 - x)cm$ and $\angle \mathrm{QPR} = 30^{\mathrm{o}}$.

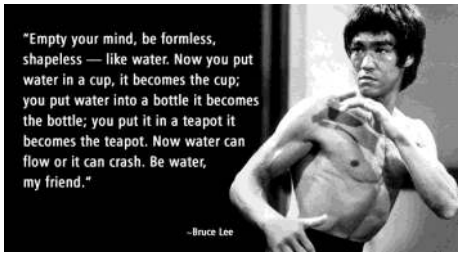
The area of the triangle is $A\ cm^2$.

a) Show that $A = \frac{1}{4}(10 + 3x - x^2)$

b) By completing the square, find the maximum value of A, and give the corresponding value of x.

9.4 - Solving triangle problems

About problem-solving, there is no "one rule fits all" - *Be water, my friend (Bruce Lee, 1971)*

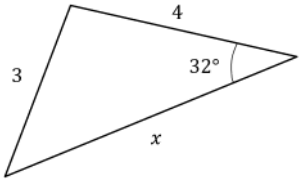


Notes

To solve problems involving triangles, consider the following:

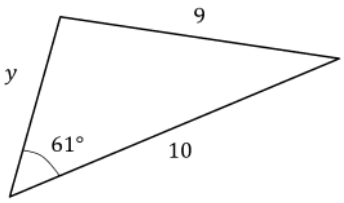
Example

Work out value of x.



Practice Q1

Find y.



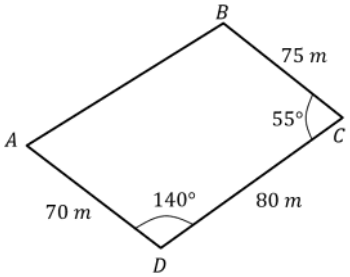
Example/Practice Q2

The diagram shows the locations of four mobile phone masts in a field, $BC = 75\ m,\ CD = 80m$, $\angle BCD = 55°$ and $\angle ADC = 140°$.

In order that the masts do not interfere with each other, they must be at least 70m apart.

Given that A is the minimum distance from D, find:

a) The distance *A* is from *B*
b) The angle *BAD*
c) The area enclosed by the four masts



9.4 - Solving triangle problems

Practice Q3

A helicopter flies on a bearing of 200^o from A to B, where $AB = 70km$. It then flies on a bearing of 150^o from B to C, where C is due south of A. Work out the distance of C from A.

9.5 - Graphs of sine, cosine and tangent

Graphs of $\sin x$, $\cos x$, $\tan x$

Example

Sketch the graph of $y = \sin\theta$ in the interval $-90^{\circ} \leq \theta \leq 270^{\circ}$.

9.5 - Graphs of sine, cosine and tangent

Practice Q1
Sketch the graph of $y = \cos\theta$ in the interval $-180^0 \leq \theta \leq 180^o$

Practice Q2
Sketch the graph of $y = \tan\theta$ in the interval $-270^o \leq \theta \leq 90^o$

Practice Q3
Sketch the graph of $f(x) = \sin x + 3$ in the interval $0^o \leq x \leq 360^o$
(hint: think about transformation)

9.6 - Transforming trigonometric graphs

Recall

Write down the geometrical transformation that is represented by each function:

- $y = f(x + a)$
- $y = f(x) + a$
- $y = f(ax)$
- $y = af(x)$
- $y = f(-x)$
- $y = -f(x)$

Think and Discuss

What transformation does each of the following equation represent?

a) $y = 4\sin x$

b) $y = \cos(x + 45^o)$

c) $y = -\tan x$

d) $y = \sin(\frac{x}{2})$

Example

Sketch the graph of $y = 4\sin x, 0 \leq x \leq 360^o$

9.6 - Transforming trigonometric graphs

Example

Sketch the graph of $y = \cos(x + 45^o)$, $0 \leq x \leq 360^o$

Practice Q1

Sketch the graph of $y = -\tan x$, $-180^o \leq x \leq 180^o$

Practice Q2

Sketch $y = \sin(\frac{x}{2})$, $-360^o \leq x \leq 360^o$