Chapter 6 - Circles

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Personal notes:

6.1 - Midpoints and perpendicular bisectors

Notes

- Midpoint between two points (x_1, y_1) and (x_2, y_2)
  $=$

Example

a) Find the midpoint between (-3, 2) and (0, 6)

b) Find the midpoint between $(\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3})$ and $(3\sqrt{2} + \sqrt{3},\ -\sqrt{2} + 2\sqrt{3})$

Example

Given the midpoint between AB is (-5, 3) and coordinates of A is (5, -2). Find the coordinates of B.

Practice Q1

a) Work out the midpoint between $(-3,6)$ and $(2,-5)$

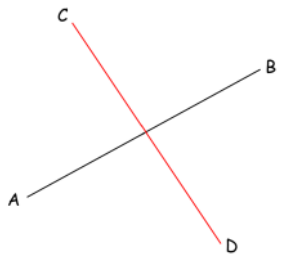
b) Work out the midpoint between $(a + b,\ 2a + b)$ and $(3a + b, -a + b)$

c) Given $M(-3,-2)$ is the midpoint between *AB*, and *A* has coordinates $(5,1)$, work out the coordinates of *B*.

6.1 - Midpoints and perpendicular bisectors

Notes

- Perpendicular bisector of a line segment AB is the straight line that *passes through the* ***midpoint*** *of AB and is* ***perpendicular*** *to AB.*



Example

A and B has coordinates (-1, 4) and (3, -6) respectively. Find the equation of the line l which is the perpendicular bisector of AB.

Practice Q2

Find the perpendicular bisector of *AB*, given coordinates of *A* and *B* are $(-5,8)$ and $(7,2)$ respectively.

Practice Q3

The line segment *ST* is a diameter of a circle, where *S* is $(-3,6)$ and *T* is $(5,-2)$. Find:

a) the coordinates of the centre of the circle.

b) the radius of the circle in the form $n\sqrt{2}$, where n is a constant to be found.

The perpendicular bisector of *ST* meets the circle at the points *A* and *B*.

c) Show that $SA = AT = TB = BS = 8$, draw a diagram to help visualise.

6.2 - Equation of a circle

Starter

a) Complete the square of $x^2 + 14x$
b) Complete the square of $x^2 + 6x + 4$
c) Complete the square of $y^2 - 36y + 15$

Notes

- Linear Equation :
- Quadratic :
- Cubic :
- Circle :

Example

A circle has equation $(x - 3)^2 + (y + 4)^2 = 20$

a) Write down the radius and centre of circle
b) Show (5, -8) is a point on the circle

Example

AB is a diameter of a circle. Given the coordinates of A and B are (4, 7) and (-8, 3) respectively. Find the equation of the circle.

Practice Q1

Write down the coordinates of the centre and the radius of each circle:

a) $(x - 3)^2 + (y + 1)^2 = 81$ b) $(x + 3)^2 + (y + 6)^2 = 11$

6.2 - Equation of a circle

Practice Q2

a) Write down the equation of the circle with centre $(1,4)$ and radius 5.

b) Write down the equation of the circle with centre $(-3,2)$ and radius $\sqrt{10}$.

Example

Find the centre and radius of the circle with the equation $x^2 + y^2 - 14x + 16y - 12 = 0$.

Practice Q3

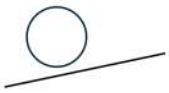
Find the centre and radius of the circle with the equation $x^2 + y^2 + 18x - 4y + 79 = 0$.

Practice Q4

The point $P(7, -14)$ lies on the circle with equation $x^2 + y^2 + 6x - 14y = 17$. Given *PQ* is a diameter of the circle, find the coordinates of *Q*.

6.3 - Intersections of straight lines and circles

Intersection of lines = _____!!

Diagram	Number of Intersection point(s)	Discriminant
	0	$b^2 - 4ac < 0$
	1	$b^2 - 4ac = 0$
	2	$b^2 - 4ac > 0$

Example

Find the coordinates of the points where the straight line $y = 2 - x$ meets the circle $(x + 2)^2 + (y - 1)^2 = 9$.

Example

Show $y = x - 7$ does not meet the circle with equation $(x + 2)^2 + y^2 = 33$.

6.3 - Intersections of straight lines and circles

Practice Q1

Find the point(s) of intersection between the line $x + y = 4$ and $(x + 2)^2 + \left(y - 3\right)^2 = 30$, give your answers in exact value.

Exam Practice (June 2019 AS Q10)

A circle *C* has equation

$x^2 + y^2 - 4x + 8y - 8 = 0$

ai) Find the coordinates of the centre of *C*.

aii) Find the exact radius of *C*. **(3 marks)**

The straight line with equation $x = k$, where k is a constant, is a tangent to *C*.

b) Find the possible values of k. **(2 marks)**

6.4 - Tangent and chord properties

Recap (from GCSE Circle Theorem)

- Radius and tangent will form a right angle.

- The perpendicular from the centre of a circle to a chord bisects the chord.

Example

Given the point P(10, 0) is on the circle $(x-2)^2+\left(y-6\right)^2=100$. Find the equation of the tangent passing through P.

Example

A circle C has equation $(x-5)^2+\left(y+3\right)^2=10$. The line l is a tangent to circle and has gradient -3. Find two possible equations of l.

6.4 - Tangent and chord properties

Practice Q1

The point $P(-1,2)$ lies on the circle centre $(3,5)$.

a) Find the equation of the circle.

b) Find the equation of the tangent to the circle at *P.*

Practice Q2

The points *R* and *S* lie on a circle with centre $C\ (a,-2)$, as shown in the diagram.

The point *R* has coordinates $(2,3)$ and the point *S* has coordinates $(10,1)$.

M is the midpoint of the line segment *RS*.

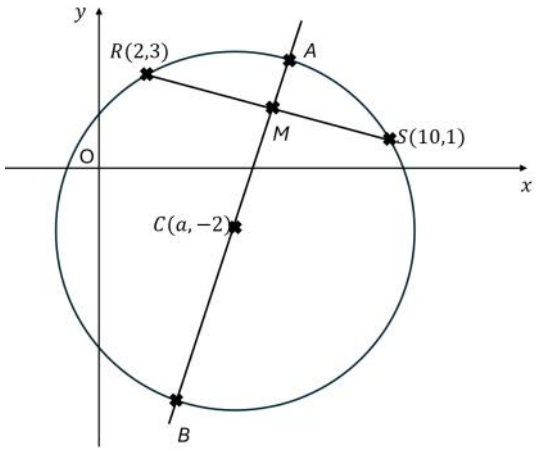
The line l passes through *M* and *C*.

a) Find an equation for l. **(4 marks)**

b) Find the value of a. **(2 marks)**

c) Find the equation of the circle. **(3 marks)**

d) Find the points of intersection, *A* and *B*, of the line l and the circle. **(5 marks)**



6.5 - Circles and triangles

Recap

- Angles in a semi-circle is $90°$.
- Distance between (x_1, y_1) and $(x_2, y_2) =$
- Midpoint between (x_1, y_1) and $(x_2, y_2) =$
- Gradient between (x_1, y_1) and $(x_2, y_2) =$

Example

Given the points P(-2, 6), Q(6, 0) and R(5, 7) all lie on a circle.

a) Show that PR is perpendicular to QR
b) Prove that PQ is the diameter of the circle and hence work out the length of the diameter.
c) Hence, find the equation of the circle.

Example

Given the points P(3, 16), Q(11, 12) and R(-7, 6) lie on a circle. The equation of the perpendicular bisector of PQ is $y = 2x$.

a) Find the equation of the perpendicular bisector of PR.
b) Find the centre of circle
c) Find the equation of circle.

6.5 - Circles and triangles

Practice Q1

Consider the points $P(3,15)$, $Q(-13,3)$, $R(-7,-5)$ and $S(8,0)$.

a) Show that *PQR* is a right-angled triangle.

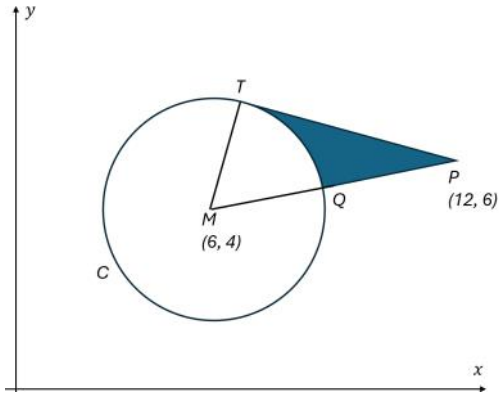
b) Given all the four points lie on a circle, find the equation of the circle.

Exam Practice

The diagram shows the circle *C*, with centre $M(6,4)$ and radius 3.

The point *T* lies on the circle and the tangent at *T* passes through the point $P(12,6)$.

The line *MP* cuts the circle at *Q*.



a) Write down the equation of the circle. **(1 marks)**

b) Show that the angle *TMQ* is 61.68^{o}, correct to 2 decimal places. **(3 marks)**

The shaded region *TPQ* is bounded by the straight lines *TP, QP* and the arc *TQ*, as shown in the diagram.

c) Find the area of the shaded region *TPQ*. Give your answer to 3 decimal places. **(4 marks)**