# Chapter 3 - Equations and Inequalities



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Personal notes:

# 3.1,3.2 - Linear and quadratic simultaneous equations

**Notes**

There are two ways to solve a simultaneous equation:

- Elimination (useful in GCSE, not too useful in A-level)
- Substitution

Quadratic simultaneous equations

- Majority of them requires substitution to solve.
- Can have up to two pairs of solutions. Need to pair them up correctly.

Example (Elimination)

Solve the simultaneous equation

$2x + 3y = 8$
$3x - y = 23$

Example (Substitution)

Solve the simultaneous equation

$2x + 3y = 8$
$3x - y = 23$

Practice Q1

Solve the simultaneous equation by substitution

$2x - y = 1$
$4x + 2y = -30$

Practice Q2

Solve the simultaneous equation by substitution

$3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

# 3.1,3.2 - Linear and quadratic simultaneous equations

Example

Solve the simultaneous equations:

$x + 2y = 3$

$x^2 + 3xy = 10$

Practice Q3

Solve the simultaneous equations:

$3x^2 + y^2 = 21$

$y = x + 1$

Practice Q4

Solve the simultaneous equations:

$2x + 3y = 13$

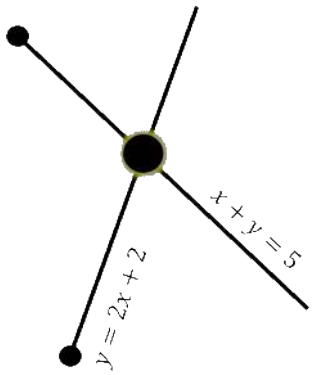
$x^2 + y^2 = 78$

# 3.3 - Simultaneous equations on graph

**Notes**

- The points of intersection of graphs = _____

$$\left.\begin{matrix} y = 2x + 2 \\ x + y = 5 \end{matrix}\right\} \; x = 1, y = 4$$




**Number of Solutions**

- Two *linear* equations can only intersect _____, i.e. Only _____.

- A *linear* equation and a *quadratic* equation can….

Example

Use discriminant to determine the number of points of intersection for the following pairs of simultaneous equations:

a) $2x + y = 3,$
$y = x^2 - 3x + 1$

b) $y = 2x - 2$
$y = x^2 + 4x + 1$

Practice Q1

Use discriminant to determine the number of points of intersection for these pairs of simultaneous equations:

$y = 6x^2 + 3x - 7$
$y = 2x + 8$

# 3.3 - Simultaneous equations on graph

Example

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that $k$ is a positive constant.

a) Find the value of $k$.
b) For this value of $k$, find the coordinates of the point of intersection.

Practice Q2

The values of $x$ and $y$ satisfy the simultaneous equations

$mx - y - 2 = 0$

$x^2 - 2x + y^2 - 4y = 4$

where $m$ is a real constant.

Given that these simultaneous equations have equal real roots, find the two possible values of $m$, giving your answers in exact value.

# 3.4,3.5 - Linear and quadratic inequalities

Starter

Find the set of values of $x$ for which:

a) $5x + 9 \leq x + 20$
b) $12 - 3x < 27$
c) $3(x - 5) > 5 - 2(x - 8)$

Example (and)

Find the set of values of $x$ for which:

$12 - 3x < 27$ **and** $3(x - 5) > 5 - 2(x - 8)$

Example (or)

Find the set of values of $x$ for which:

$5x + 9 \leq x + 20$ **or** $3(x - 5) > 5 - 2(x - 8)$

**Set notation**

- Sometimes you will be asked to write in set notation.
- Start with _____.

Example

Use the set notation to describe the set of values of $x$ for which:

a) $3x - 5 < x + 8$ and $5x > x - 8$
b) $x - 5 > 1 - x$ or $15 - 3x > 5 + 2x$

Practice Q1

Use set notation to describe the set of values of $x$ for which:

a) $2(1 + x) < 4 - x$ and $x - 3 < 4x + 6$
b) $\frac{1}{2}(4x + 3) < 2.5$ or $\frac{x-3}{5} > 7$

# 3.4,3.5 - Linear and quadratic inequalities

**Notes**

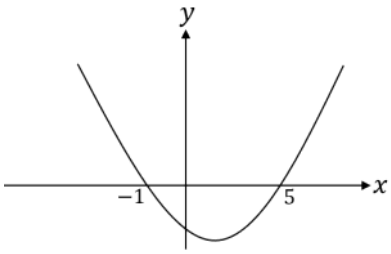
To solve a *quadratic inequality*:

i) Rearrange so that one side of the inequality is __.
ii) Make the inequality like a _____ and solve it to find the roots
iii) Sketch the graph
iv) If the inequality is > 0: choose the part of graph that is _____.

If the inequality is < 0: choose the part of graph that is _____.

Example

Here is a sketch of the graph $f(x) = x^2 - 4x - 5$, roots are $x = -1, x = 5$




Example

Solve $x^2 + 2x - 15 \leq 0$

Example

Solve $3 - 5x < 2x^2$ and write the set of values of $x$ in set notation.

# 3.4,3.5 - Linear and quadratic inequalities

Exam Practice (Edexcel C1 May 2010 Q3)

Find the set of values of $x$ for which

(*a*) $3(x-2) < 8-2x$, **(2)**

(*b*) $(2x-7)(1+x) < 0$, **(3)**

(*c*) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$. **(1)**

Dealing with $x$ in the denominator

- When we multiply/divide negative numbers in inequalities, the sign flips.
- But since we don't know value of $x$, it could be positive *or negative*, we need to _____.

Example

Find the set of values for which $\frac{6}{x} > 2 \; where \; x \neq 0$.

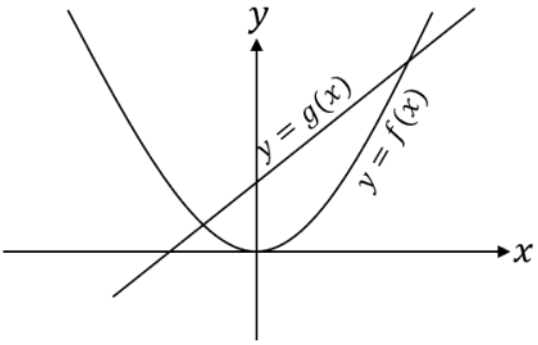
Practice Q2

Given that $x \neq 0$, find the set of values of $x$ for which
$6 + \frac{5}{x} > \frac{8}{x}$, give your answer in set notations.

# 3.6,3.7 - Inequalities on graph

**Notes**

- Below is a sketch of two functions $y = f(x)$ and $y = g(x)$




Example

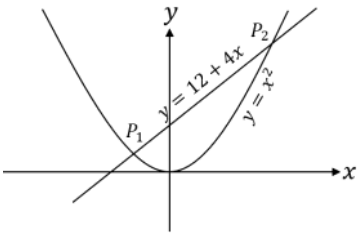
$L_1$ has equation $y = 12 + 4x$.

$L_2$ has equation $y = x^2$.

The diagram shows a sketch of $L_1$ and $L_2$ on the same axes.

a) Find the coordinates of $P_1$ and $P_2$, the points of intersection.

b) Hence write down the solution to the inequality $12 + 4x > x^2$.




Practice Q1

a) Sketch $f(x) = x^2 - 5$ and $g(x) = 7x + 13$ on the same pair of axes.

b) Find the coordinates of the point where $f(x)$ and $g(x)$ meet.

c) Write down the solutions to the inequality $f(x) \leq g(x)$.

# 3.6,3.7 - Inequalities on graph

**Shading regions**

You will be asked to shade regions that satisfy linear and quadratic inequalities.

1) Make $y$ the subject, draw the graph

2) If $y > f(x)$, shade the region ***above*** the graph

3) If $y < f(x)$, shade the region ***below*** the grpah

P.S: Dotted Line: > or <
Bold Line: $\geq$ or $\leq$

Example

Shade the region that satisfies the inequalities:

$2y + x < 14$

$y \geq x^2 - 3x - 4$

Practice Q2

Shade the region that satisfies the inequalities:

$y > (x - 3)^2$

$y + x \leq 5$