Chapter 5 - Straight Line Graphs

Personal notes:

5.1, 5.2 - Equations of straight lines

Notes

- Equations of straight lines can be represented in two forms:

Example

Write $y = \frac{4}{5}x + 6$ into $ax + by + c = 0$ form.

Example

Find the gradient of a line with equation $2x - 3y + 7 = 0$

- A line meets $x - axis$:
- A line meets $y - axis$:

Example

The line $y = 4x - 8$ meets $x - axis$ at point P and $y - axis$ at point Q. Find the coordinates of P and Q.

Practice Q1

A line has equation $-2x + 3y - 9 = 0$.
Find a) the gradient, b) the coordinates which it crosses the x-axis and y-axis.

Practice Q2

Write the following equations in the form of $ax + by + c = 0$.

a) $y = \frac{3}{5}x + 4$ b) $x = 3y - \frac{1}{2}$

c) $2y = \frac{1}{5}x + \frac{3}{7}$

5.1, 5.2 - Equations of straight lines

Formula to find equation of straight lines

Example

Find the equation of the line that passes through (5, 7) and (3, -1).

Practice Q3

Find the equation of the line that passes through $(-1,6)$ and $(\frac{1}{2}, -3)$.

Example

The line $y = 3x - 9$ passes through A at $x - axis$. Find the equation of the line with gradient $\frac{2}{3}$ and passes through A.

5.1, 5.2 - Equations of straight lines

Example

Two lines $y = 4x - 7$ and $2x + 3y - 21 = 0$ intersect at point A.
Another point, B, has coordinates (-2, 8).
Find the equation of the line that passes through A and B.

Practice Q4

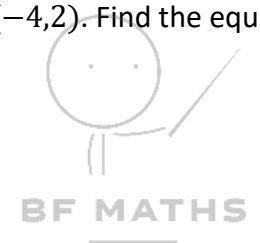
The line $y = \frac{2}{3}x - 4$ meets the x-axis at point P.
Find the equation of the line with gradient $-\frac{5}{2}$ that passes through the point P.
Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Practice Q5

The line $y = -2x + 6$ meets the x-axis at the point P. The line $y = \frac{3}{2}x - 4$ meets the y-axis at point Q. Find the equation of the line joining the points P and Q.

Practice Q6

The lines $y = x - 6$ and $y = 2x - 13$ intersect at the point A. The point B has coordinates $(-4,2)$. Find the equation of the line that pases through the points A and B.



5.3 - Parallel and perpendicular lines

Notes

- If two lines are…

  - Parallel :

  - Perpendicular :

Example

A line is parallel to $6x + 3y - 2 = 0$ and passes through (3, 0). Find the equation of the line.

Practice Q1

A line is parallel to the line $y = -\frac{1}{4}x + 3$ and passes through the point $(3, 1)$.

Work out the equation of the line in the form $ax + by + c = 0$, where a, b and c are integers.

Example

A line is perpendicular to another line with equation $y - 3x + 6 = 0$ and passes through the point (-2, 5). Find the equation of the line.

Practice Q2

A line l_1 has equation $3x - 8y + 3 = 0$ and crosses the x-axis at point A.

The line l_2 is perpendicular to l_1 and passes through A.

a) Find the coordinates of A.

b) Find the equation of the line l_2.

5.4 - Length and Area

Notes

- The distance between two coordinates could be found using the distance formula (which essentially derived from Pythagoras).

Practice Q1

Find the distance between these pairs of coordinates:

a) (3, 2) and (-2, 4) b) $(-1, 1+\sqrt{2})$ and $(-2\sqrt{2}, 3)$

Practice Q2

The distance between the points $(-1,13)$ and $(x, 9)$ is $\sqrt{65}$.

Find two possible values of x.

Example

A line l_1 has equation $4x - y = 0$. Another line l_2 has equation $2x + 3y - 21 = 0$. Given that l_1 and l_2 intersect at point A.

a) Find the coordinates of A.
b) Work out the area of triangle AOB, where l_2 meets B at $x - axis$.

5.4 - Length and Area

Practice Q3

Show the points A(-6, 2), B(4, 8), C(6, 1) and D(-9, -8) form a trapezium.
Hint: Trapezium has one pair of parallel sides, but different length.

Practice Q4

A straight line passes through the points $(4, -3)$ and $(-2,5)$.
a) Find the equation of this line, give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

This line crosses the x-axis at point *P* and y-axis at point *Q*.
b) Find the area of triangle *POQ*, where *O* is the origin.

5.5 - Modelling with straight lines

Notes

- Examples of linear graphs that represent real-life data:

- Modelling with straight lines means the axes are changed from x-y to different context.

Generic questions to help you get into the "modelling" mode

1.

2.

3.

5.5 - Modelling with straight lines

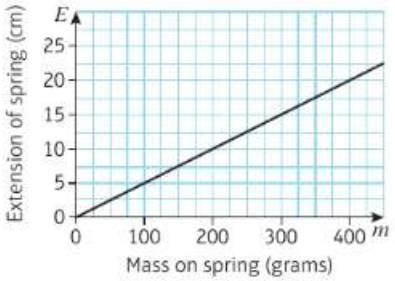
Example

The graph shows the extension, E, of a spring when different masses, m, are attached to the end of the spring.

a Calculate the gradient, k, of the line.

b Write an equation linking E and m.

c Explain what the value of k represents in this situation.



Practice Q1

A container was filled with water. A hole was made in the bottom of the container. The table below records the water remained at certain time intervals.

Time, t seconds	0	10	30	60	100	120
Depth of water, d cm	19.1	17.8	15.2	11.3	6.1	3.5

a) Given the data forms a straight line, deduce an equation in the form $d = at + b$

b) Interpret the meaning of the coefficients a and b.

c) Use the model to find the time when the container will be empty.

d) State one limitation of the model.

5.5 - Modelling with straight lines

Example

Joeliyn, an app designer, charges a fixed amount for the initial design work and then an additional daily rate for any revisions requested by clients.
Company X's app design takes 4 days and they are charged £1500.
Company Y's app design takes 8 days and they are charged £2400.

a) Write an equation linking the number of days, d, and the total cost, C, in the form $C = ad + b$, where a and b are constants to be determined.
b) Interpret the values of a and b.
c) Joeliyn charges a third company £17025. Calculate the number of days she has spent working on the app.

Exam Practice (June 2023 P1 Q7)

The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km.
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km.

Using a *linear model*, with V litres being the volume of fuel remaining in the car's fuel and d km being the distance the car had travelled.

a) find an equation linking V with d. **(4 marks)**

Given that, on a particular journey, the fuel tank was initially full and the car continued until it ran out of fuel, find

bi) the initial volume of fuel that was in the fuel tank of the car,
bii) the distance that the car travelled on this journey. **(3 marks)**

In fact, the car travelled 320 km on this journey.
c) Evaluate the model in light of this information. **(1 mark)**