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EXAM QUESTION BANK

SECTION A: STATISTICS

① a)

	Read music	Can't read music	Total
Female	10	8	18
Male	6	9	15
Total	16	17	33

b) i) $P(\text{Reads music}) = \frac{1}{16}$

ii) $P(\text{Can't read music} \mid \text{Female}) = \frac{8}{18} = \frac{4}{9}$

iii) $P(\text{Male} \mid \text{reads music}) = \frac{6}{16} = \frac{3}{8}$

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②

Mass of reactant, m (grams)	3	5	7	10	15	20	30
Mass of product, p (grams)	2.7	4.1	5.2	6.8	9.1	11.3	15.4

$\Rightarrow p = am^k$; $x = \log m$; $y = \log p$; $y = 0.081 + 0.749x$

a) $\log p = 0.081 + 0.749(\log m)$

$\Rightarrow \log p = (\log m)^{0.749} + 0.081$

$\Rightarrow \cancel{\log}\, p = \cancel{\log}\, m^{0.749} + 0.081$

10 10

$\Rightarrow p = m^{0.749} \times 10^{0.081}$

Let $\Rightarrow a = 10^{0.081} \approx 1.21$ (3sf)

$\Rightarrow p = am^{0.749}$ $\Rightarrow k = 0.749$

b) 50 grams is outside the range of the data $(3 \leq m \leq 30)$

This leads to extrapolation.

③ a) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

b) $\mu = 3.2$ Error of margin (E) = 1 ; z score for (95%) = 1.96

$E = z \times \frac{\sigma}{\sqrt{n}}$ $\Rightarrow 1 = 1.96 \times \frac{3.2}{\sqrt{n}}$ $\Rightarrow \sqrt{n} = 6.272$ $\Rightarrow n = (6.272)^2$

$\Rightarrow n \approx 39.337..$ $\Rightarrow n = 40$

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④

Sunshine, *s* (hours)	5.9	14.5	4.6	6.8	6.5	10.7
Visibility, *v* (Dm)	3200	3400	1900	2200	2500	3600

a) $r = 0.731$ (3SF)

Product Moment Coefficient					
Level					
0.10	0.05	0.025	0.01	0.005	Sample size, n
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10

b) $H_0: \rho = 0$ $H_1: \rho > 0$

SL: 5% (0.05)
Critical value:
$\Rightarrow 0.7293 < 0.731$
$\Rightarrow$ Reject H_0. There is evidence of positive correlation at the 5% level of significance, so claim is correct.

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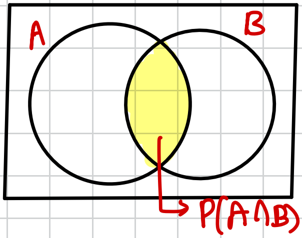
⑤ Given: $P(A|B) = P(A) = P(B) = P(B|A) = P(A' \cap B')$; $P(A) = x$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\Rightarrow P(A) = \frac{P(A \cap B)}{P(B)}$



$\Rightarrow x = \frac{P(A \cap B)}{x}$ $\Rightarrow x^2 = P(A \cap B)$

$\Rightarrow P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A' \cap B') = 1 - (x + x - x^2)$ $\Rightarrow$ $1 - (2x - x^2) = x$

$\Rightarrow 1 - 2x + x^2 - x = 0$ $\Rightarrow x^2 - 3x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{c}$

$x = \frac{3 \pm \sqrt{5}}{2}$; $x = \frac{3 - \sqrt{5}}{2}$ $(x \leq 1)$

6 a) ① n is large ② p is close to 0.5

b) 62.4 is the variance ($\sigma^2 = 62.4$) ; so the standard deviation is $\sqrt{62.4}$.

c) $X \sim B(250, 0.48) \Rightarrow Y \sim N\left(np, \left(\sqrt{np(p-1)}\right)^2\right) \Rightarrow Y \sim N\left(120, \left(\sqrt{62.4}\right)^2\right)$

$\Rightarrow P(X < 130) \approx P(Y < 129.5) = 0.8554$ (4SF)

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7 a) Beijing

Temperature, t (°C)	19.7	21.3	21.0	20.6	21.0	20.8	18.8
Windspeed, w (kn)	10.1	12.3	9.9	7.4	6.7	7.8	8.5

b) $r = 0.133$ (3SF)

c) The data shows a very weak positive correlation so linear model may not be best. There may be other variables affecting the relationship or a different model might be a better fit.

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8 a) Fair six-sided dice : $p = \frac{1}{6}$

	1	2	3	4	5	6
1		✓	✓		✓	
2	✓	✓	✓		✓	
3	✓	✓	✓		✓	
4					✓	
5	✓	✓	✓	✓	✓	
6					✓	

$= \frac{17}{36}$

b) $X \sim B\left(180, \frac{17}{36}\right)$

c) n is large ($50 < 80$)

p is close to 0.5 ($\frac{17}{36} = 0.4722$)

$Y \sim N\left(np, \left(\sqrt{np(1-p)}\right)^2\right)$

$\Rightarrow Y \sim N(85, 6.7^2)$

d) $P(X = 80) \approx P(79.5 \leq Y \leq 80.5) = 0.0451$ (3SF)

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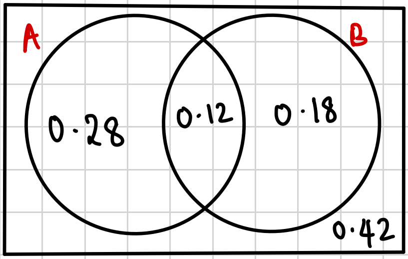


Given: $P(A) = 0.4$ $P(B) = 0.3$

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow 0.4 + 0.3 - 0.58$

$\Rightarrow 0.7 - 0.58 = 0.12$

b) $P(B' | A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{0.28}{0.4} = 0.7$

c) $P(A | B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.12}{0.3} = 0.4$

d) $P(A) = P(A|B)$, therefore it is independent

10

Water, W	24000	109000	19000	14000	67000	13000	3000	46000
Electricity, E	1090	3040	840	2190	10670	830	1600	1980

a) $r = 0.496$ (3sf)

Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10

b) $H_0: \rho = 0$ $H_1: \rho > 0$

Critical region: $0.5067 > 0.496$

Accept H_0. There is no evidence of linear correlation at 10% level of significance.

Water, W	24000	32000	14000	22000	37000	43000	27000	19000
Electricity, E	1090	1640	670	870	2380	2460	1950	940

c) $r = 0.937$ (3sf)

Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10

d) $H_0: \rho = 0$ $H_1: \rho > 0$

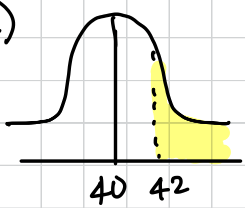
Critical region: $0.5067 < 0.937$

Reject H_0. There is evidence of linear correlation at 10% level of significance

e) Weaker positive correlation in part b due to different effeciencies of the different generators.

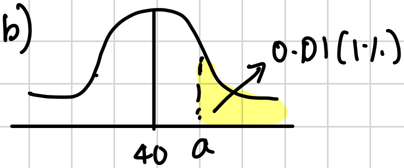
(11) $X \sim N(\mu, \sigma^2) = X \sim N(40, 2^2)$

a)



$P(X > 42) = 0.1587$ (4dp)

b)



$\Rightarrow P(X > a) = 0.01 \Rightarrow a = 44.652$ kg

$\Rightarrow X > 44.65$ (4sf)

c) probability weigh more than 41 kg $= P(X > 41) = 0.3085$

$X \sim B(3, 0.3085) \Rightarrow P(X = 2) = 0.1974$

If two of them weigh more than 41 kg, other one has to weigh less than 41 kg which share the same probability

$p = 0.197$ (3sf)

d) $P(X > 42 \mid R') = \dfrac{0.1587}{0.5} = 0.3174$

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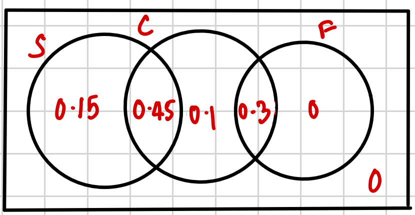
(12)

a)



b) i) $P(F \cup S) = 0.6 + 0.3 = 0.9$

ii) $P((C \cup S) \cup (C \cap F)) = 0.45 + 0.3 = 0.75$

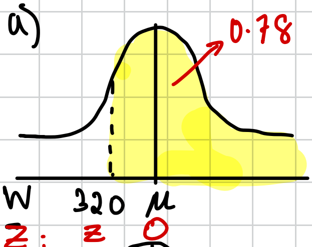
c) NO, $P(F' \cap S) = 0.6 \neq P(F') \times P(S)$

$0.6 \neq 0.7 \times 0.6$

$0.6 \neq 0.42$

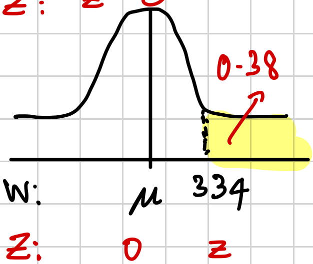
13) $W \sim N(\mu, \sigma^2)$ $\quad Z = \dfrac{X-\mu}{\sigma}$

a)



$\Rightarrow P(Z > z) = 0.78 \Rightarrow z = -0.772$

$\Rightarrow -0.772 = \dfrac{320-\mu}{\sigma} \Rightarrow -0.772\sigma + \mu = 320$ ①



$\Rightarrow P(Z > z) = 0.38 \Rightarrow z = 0.3054$

$\Rightarrow 0.3054 = \dfrac{334-\mu}{\sigma} \Rightarrow 0.3054\sigma + \mu = 334$ ②

Solve simultaneously eq ① & ②

$\sigma = 13.0$ (3sf) $\quad \mu = 330$ ml (3sf)

b) $P(W < 325) = 0.3493$; $Y \sim B(8, 0.3493)$

$\Rightarrow P(Y \leq 2) = 0.4294 \approx 0.430$ (3sf)

c) $X \sim B(600, 0.4294) \Rightarrow A \sim N(np, [\sqrt{np(1-p)}]^2) \Rightarrow A \sim N(257.64, 12.1^2)$

$\Rightarrow P(X < 250) = P(A < 249.5) = 0.251$ (3sf)

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14) $x = d \quad y = \log v$

$y = 0.0074x - 0.412$

a) $\log v = 0.0074d - 0.412 \Rightarrow \log_{10} v = 0.0074d - 0.412$

$\Rightarrow v = 10^{0.0074d} \times 10^{-0.412}$; $k = 10^{-0.412} = 0.387$; $b = 1.02$ (3sf)

Diameter of particle, d (mm)	0.2	1.3	5	11	20	45	80	160
Speed of current, v (m s^{-1})	0.1	0.25	0.5	0.75	1	1.5	2.5	3.5
	$\log 0.1$	$\log\frac{1}{4}$	$\log\frac{1}{2}$	$\log\frac{3}{4}$	$\log 1$	$\log\frac{3}{2}$	$\log\frac{5}{2}$	$\log 3.5$

b) $r = 0.799$ (3sf)

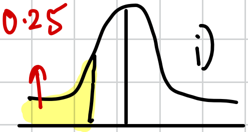
Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10

c) At 0.01 (1%) $\Rightarrow$ Critical region is 0.7887

$0.7887 < 0.799$

(15) $Q_1 = 12.7$ $IQR = 6.2$ $Q_3 = IQR + Q_1 = 18.9$; $X \sim N(\mu, \sigma^2)$

a) $Z \sim N(0, 1^2)$

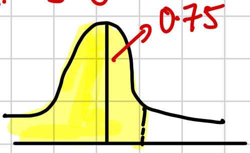


X: 12.7 μ

Z: z 0

i) $\Rightarrow P(Z < z) = 0.25 \Rightarrow z = -0.674$

$\Rightarrow z = \dfrac{x - \mu}{\sigma} \Rightarrow -0.674 = \dfrac{12.7 - \mu}{\sigma} \Rightarrow -0.674\sigma + \mu = 12.7$ (1)



x: μ 18.9

Z: 0 z

$\Rightarrow P(Z < z) = 0.75 \Rightarrow z = 0.674$

$\Rightarrow z = \dfrac{x - \mu}{\sigma} \Rightarrow 0.674 = \dfrac{18.9 - \mu}{\sigma} \Rightarrow 0.674\sigma + \mu = 18.9$ (2)

Solve simultaneously eq (1) & (2)

$\Rightarrow \mu = 15.8$ $\sigma = 4.60$ (3SF)

ii) $P(x > 16) = 0.483$ (3s.f)

b) 12min + 8 min = 20min $\Rightarrow \dfrac{P(x > 20)}{P(x > 12)} = \dfrac{0.1806}{0.7956} = 0.227$ (3SF)

c) 45 mins is more than 6 standard deviations away from the mean, so virtually impossible according to the normal distribution model. The model is unlikely to be suitable.

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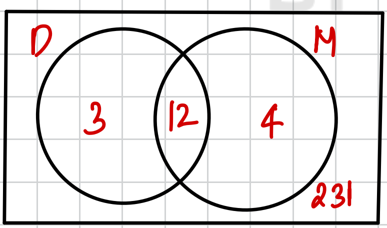
(16) a)



b) $P(M|D) = \dfrac{P(M \cap D)}{P(D)} = \dfrac{12}{15} = 0.8$

c) $P(D'|M) = \dfrac{P(M \cap D')}{P(M)} = \dfrac{4}{16} = 0.25$

d) 20% of the defective components are not identified by the machine and 25% of the components identified as defective are not so test is not very effective.

17

	7	6	5	3	4	1	9	10	2	8
Engine size, *E* (litres)	3.2	3.2	2.8	2.5	2.8	1.8	5.0	6.0	2.3	3.5
Fuel economy, *M* (mpg)	24	26	26	27	26	29	21	19	30	23

a) Arrange in ascending order:
$\Rightarrow$ 1·8, 2·3, 2·5, 2·8, 2·8, 3·2, 3·2, 3·5, 5·0, 6·0

$Q_1 = 2.5 \quad IQR = Q_3 - Q_1 = 1 \quad \Rightarrow Q_3 + 1.5(Q_3 - Q_1) = 3.5 + 1.5 = 5 < 6$

$Q_3 = 3.5$

So it is an outlier

b) It follows the trend of the data, or it is a reasonable engine capacity for a large car.

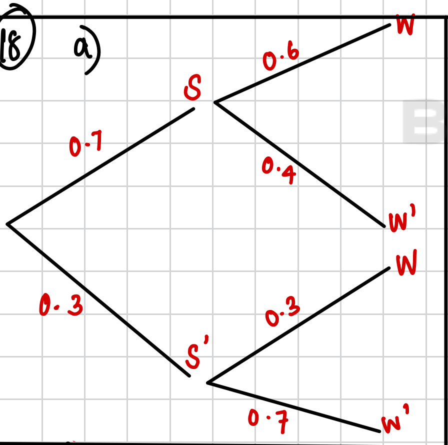
c) $r = -0.945$ (3sf)

d) Cars with larger engines are larger and therefore less fuel efficient.

e) Strong negative correlation, therefore a linear model is likely to be suitable

f) Negative correlation, so expect gradient to be negative.

18 a)



b) $P(W) = (0.7 \times 0.6) + (0.3 \times 0.3) = 0.42 + 0.09$

$P(W) = 0.51$

c) $P(S|W) = \dfrac{P(S \cap W)}{P(W)} = \dfrac{0.6 \times 0.7}{0.51} = \dfrac{14}{17}$

d) $P(S'|W') = \dfrac{P(S' \cap W')}{1 - P(W)} = \dfrac{0.3 \times 0.7}{0.49} = \dfrac{3}{7}$

e) Assuming that each serve is independent and the probability of success is constant.

$S \sim B(100, 0.7) \Rightarrow H_0: p = 0.7 \quad H_1: p \neq 0.7$; SL = 0.025 (two-tailed test)

$P(S \leq 62) = 0.053 \Rightarrow 0.053 > 0.025$

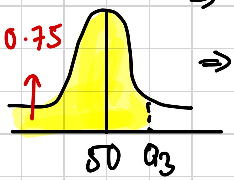
There is insufficient evidence to reject H_0. There is no evidence that proportion of successful first serves has change.

(19) $X \sim N(50, 8^2)$

a) $P(X > 55) = 0.2659$ (4sf)

b) Q_3 of X:

$\Rightarrow P(X < Q_3) = 0.75$

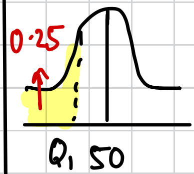


$\Rightarrow Q_3 = 55.395$

$\Rightarrow Q_3 = 55.4$ (3sf)

c) Q_1 of X: $P(X < Q_1) = 0.25$

$\Rightarrow Q_1 = 44.6$ (3sf)



d) $h = Q_1 - 1.5(Q_3 - Q_1) = 44.6 - 1.5(10.8)$

$h = 28.4$ (3sf)

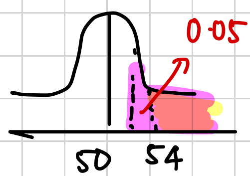
$k = Q_1 + 1.5(Q_3 - Q_1) = 55.4 + 1.5(10.8)$

$k = 71.6$ (3sf)

e) $P(X < 28.4) + P(X > 71.6) = 0.003467 + 0.003467 = 0.006934$

$= 0.007$

f) $X \sim N(50, 8^2) \Rightarrow \bar{X} \sim N\left[50, \left(\sqrt{\frac{8^2}{12}}\right)^2\right]$; $\bar{x} = 54$

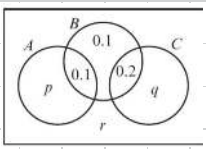


$H_0: \mu = 50$ $H_1: \mu > 50$

$P(\bar{X} > 54) = 0.0416 < 0.05$. Reject H_0. There is evidence to suggest the reading have improved.

g) Increase could have been due to other factors, e.g. second time taking test increases familiarity.

(20)



a) $P(A \cap B) = 0.1$

$P(A) \times P(B) = 0.1$

$(p + 0.1)(0.4) = 0.1$

$\Rightarrow 0.4p = 0.06$

$\Rightarrow p = 0.15$

b) $P(B|C) = \frac{4}{11}$

$\Rightarrow \frac{P(B \cap C)}{P(C)} = \frac{4}{11}$

$\Rightarrow \frac{0.2}{q + 0.2} = \frac{4}{11}$

$\Rightarrow 2.2 = 4q + 0.8$

$\Rightarrow q = 0.35$

Probability $= 1$

$p + 0.4 + q + r = 1$

$0.15 + 0.4 + 0.35 + r = 1$

$r = 0.1$

c) $P(A \cup C | B) = \frac{P((A \cup C) \cap B)}{P(B)}$

$= \frac{0.1 + 0.2}{0.4} = \frac{0.3}{0.4} = 0.75$