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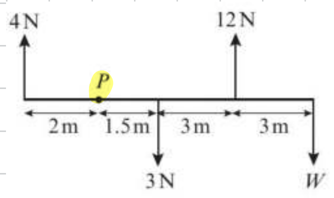
EXAM QUESTION BANK

SECTION B: MECHANICS

(21)



Take moments at P:

$4N: M = Fd \Rightarrow 4 \times 2m = 8\,Nm\ CW$

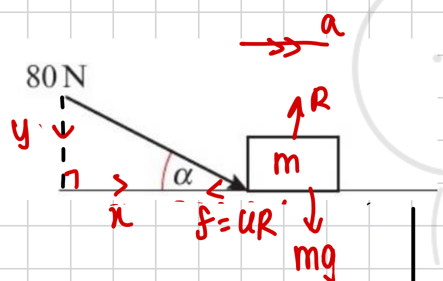
$3N: M = Fd \Rightarrow 3 \times 1.5m = 4.5\,Nm\ CW$

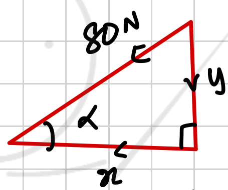
$WN: M = Fd \Rightarrow W \times 7.5m = 7.5W\,Nm$

$12N: M = Fd \Rightarrow 12 \times 4.5m = 54W\,Nm$

$\Rightarrow 8 + 4.5 + 7.5W - 54 = 12 \Rightarrow 7.5W = 53.5 \Rightarrow W = 7.13N\ (3sf)$

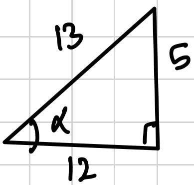
(22)





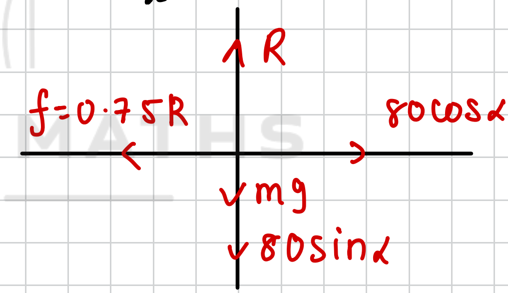
$y = 80\sin\alpha$

$x = 80\cos\alpha$



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$



$\Rightarrow \uparrow = \downarrow \Rightarrow R = mg + 80\sin\alpha$ — ①

$\Rightarrow \rightarrow = \leftarrow \Rightarrow 80\cos\alpha = 0.75R$ ② $\Rightarrow \frac{960}{13} = 0.75mg + \frac{300}{13}$

$\Rightarrow m = \frac{660}{13} \div (0.75g) \Rightarrow m = 6.91\,kg$ (3sf)

(23) At $t=0$; $r_0 = r + vt$ → constant velocity

Athena ⇒ $A(t) \Rightarrow \begin{pmatrix}12\\-6\end{pmatrix} + \begin{pmatrix}-1\\4\end{pmatrix}t \Rightarrow \begin{pmatrix}12-t\\4t-6\end{pmatrix}$

Jeff ⇒ $J(t) \Rightarrow \begin{pmatrix}2\\16\end{pmatrix} + \begin{pmatrix}2\\-2\end{pmatrix}t \Rightarrow \begin{pmatrix}2+2t\\16-2t\end{pmatrix}$

Equate i-direction position:

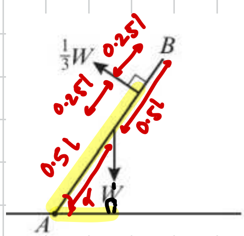
$12-t = 2+2t \Rightarrow 10 = 3t \Rightarrow t = \frac{10}{3}$

Equate j-direction position:

$4t-6 = 16-2t \Rightarrow 6t = 22 \quad t = \frac{11}{3}$

So, the hikers will not meet

(24)



Take moments at A:

$\frac{1}{3}W \Rightarrow M = Fd \Rightarrow M = \frac{1}{\cancel{3}} W \times \frac{\cancel{3}}{4} l = \frac{W}{4} l \text{ Nm}$

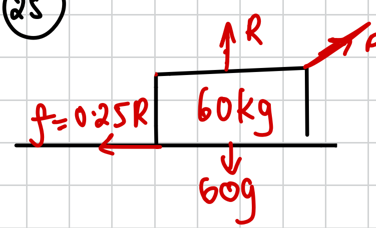
$W \Rightarrow M = Fd \Rightarrow M = W \times 0.5 l \cos\alpha = W \times \frac{1}{2} l \cos\alpha \text{ Nm}$

acw = cw

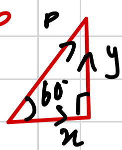
$\Rightarrow \frac{W}{4} l = W \times \frac{1}{2} l \cos\alpha \Rightarrow \cancel{Wl}\left(\frac{1}{4}\right) = \cancel{Wl}\left(\frac{1}{2}\cos\alpha\right)$

$\Rightarrow \cos\alpha = \frac{1}{2} \quad \Rightarrow \alpha = \arccos\left(\frac{1}{2}\right) \Rightarrow \alpha = 60^\circ$

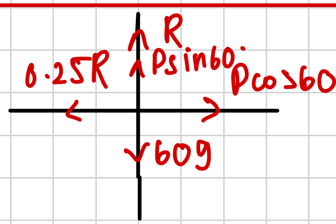
(25)



Resolve P:



$y = P\sin 60^\circ$
$x = P\cos 60^\circ$



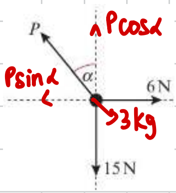
$\uparrow = \downarrow \Rightarrow R = 60g - \frac{\sqrt{3}}{2}P$ — (1) $\quad \rightarrow = \leftarrow \Rightarrow \frac{1}{2}P = 0.25R$ — (2)

equate (1) in (2) $\Rightarrow \frac{1}{2}P = 0.25\left(60g - \frac{\sqrt{3}}{2}P\right) \Rightarrow \frac{1}{2}P = 147 - \frac{\sqrt{3}}{8}P \quad P$

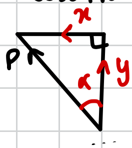
$\Rightarrow \frac{4+\sqrt{3}}{8}P : 147 \quad \Rightarrow 205.16\ldots \quad \Rightarrow P = 205 \text{ (3sf)}$

b) P would be 0 as there would be no frictional force acting on the particle.

26)



Resolve P:



$y = P\cos\alpha$
$x = P\sin\alpha$

a) Equilibrium ($\uparrow = \downarrow$; $\rightarrow = \leftarrow$)

$\uparrow = \downarrow \Rightarrow P\cos\alpha = 15$

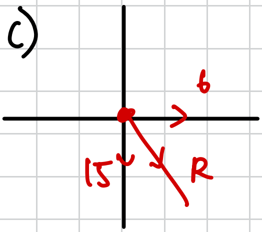
$\rightarrow = \leftarrow \Rightarrow P\sin\alpha = 6$

$\Rightarrow \dfrac{\cancel{P}\sin\alpha}{\cancel{P}\cos\alpha} = \dfrac{6}{15} \Rightarrow \tan\alpha = \dfrac{6}{15}$

$\alpha = 21.8^\circ$ (3SF)

b) $P\cos\alpha = 15 \Rightarrow P = 16.155\ldots \Rightarrow P = 16.2\text{ N}$ (3S.F)

c)



Resultant force $= \sqrt{15^2 + 6^2} = 3\sqrt{29}$

$\Rightarrow F = ma \Rightarrow 3\sqrt{29} = 3a \Rightarrow a = \sqrt{29}$

$s = 10$ $u = 0$ $a = \sqrt{29}$ $t = t$

$s = ut + \frac{1}{2}at^2 \Rightarrow 10 = \frac{1}{2}(\sqrt{29})t^2 \Rightarrow \sqrt{\dfrac{20}{\sqrt{29}}} = t$

$\Rightarrow t = 1.927\ldots \Rightarrow t = 1.93\text{ s}$ (3SF)

27) $r = 3t(kt^2+1)i + (5t^2-2)j,\ t \geq 0$

a) $v = \dfrac{dr}{dt} \Rightarrow v = \left[3t(2kt) + (kt^2+1)3\right]i + \left[10t\right]j$

$\Rightarrow v = \left[6kt^2 + 3(kt^2+1)\right]i + \left[10t\right]j \Rightarrow \left[6kt^2+3kt^2+3\right]i + \left[10t\right]j$

$\Rightarrow v = \left[9kt^2+3\right]i + \left[10t\right]j\ \text{ms}^{-1}$

b) $t = 2$; speed $= 5\sqrt{241}$

$v = \left[9k(2)^2+3\right]i + \left[10(2)\right]j\ \text{ms}^{-1} \Rightarrow v = \left[36k+3\right]i + \left[20\right]j$

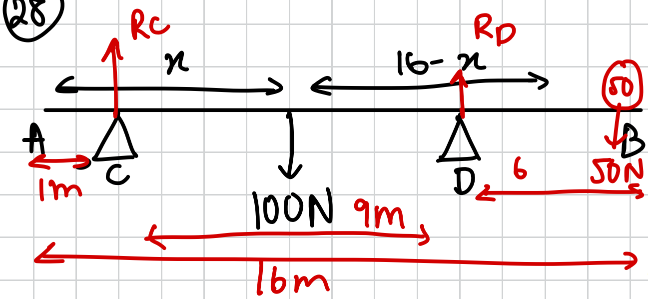
Speed $= \sqrt{v_i^2 + v_j^2} \Rightarrow 5\sqrt{241} = \sqrt{(36k+3)^2 + (20)^2}$

$\Rightarrow 6025 = (36k+3)^2 + 400 \Rightarrow 5625 = 1296k^2 + 9 + 216k$

$\Rightarrow 1296k^2 + 216k - 5616 = 0$

$\Rightarrow k = 2$ or $\cancel{k = -\frac{13}{6}}$ $\Rightarrow \boxed{k = 2}$

28



Take moment at A:

$R_C : \Rightarrow M = Fd \Rightarrow M = R_C \times 1 = R_C$

$R_D : \Rightarrow M = Fd \Rightarrow M = R_d \times 10 = 10R_d$

$50N : \Rightarrow M = Fd \Rightarrow M = 50 \times 16 = 800N$

$100N : \Rightarrow M = Fd \Rightarrow M = 100 \times x = 100x$

ACW = CW

$R_C + 10R_D = 800 + 100x$ — (1)

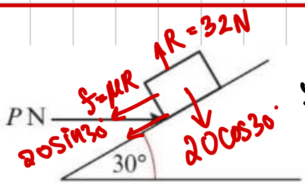
$\Rightarrow \uparrow = \downarrow \Rightarrow R_C + R_D = 150\,N \Rightarrow R_D = 150 - R_C$ — (2)

As B is the point of tipping ; $R_C = 0$

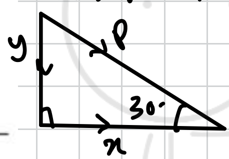
$\Rightarrow R_D = 150$ — (2) $\Rightarrow 10(150) = 800 + 100x \Rightarrow 700 = 100x$

$\Rightarrow \boxed{x = 7\,m}$

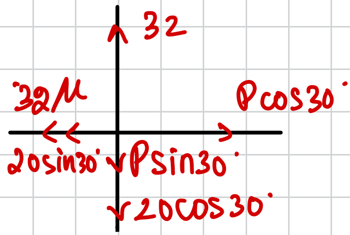
29



Resolve P:



$y = P\sin 30°$
$x = P\cos 30°$



a) Equilibrium $\Rightarrow (\uparrow = \downarrow ; \rightarrow = \leftarrow)$

$\uparrow = \downarrow \Rightarrow 32 = \frac{1}{2}P + 20\cos 30° \Rightarrow P = 2(32 - 20\cos 30°) \Rightarrow \boxed{P = 29.4N}$

b) $\rightarrow = \leftarrow ;\quad P\cos 30° = 32\mu + 20\sin 30° \Rightarrow \mu = \dfrac{P\cos 30° - 20\sin 30°}{32}$

$\Rightarrow \mu = 0.482$ (3sf)

30 $a = \frac{2}{3}e^{-\frac{2}{5}t}\ ms^{-2}$; $t = 0\ v = 1\,ms^{-1}$

a) $v = \int a\,dt \Rightarrow \int \frac{2}{3}e^{-\frac{2}{5}t}\,dt \Rightarrow \frac{2}{3}\int e^{-\frac{2}{5}t}\,dt \Rightarrow \frac{\cancel{2}}{3}\left[-\frac{5}{\cancel{2}}e^{-\frac{2}{5}t}\right] + c$

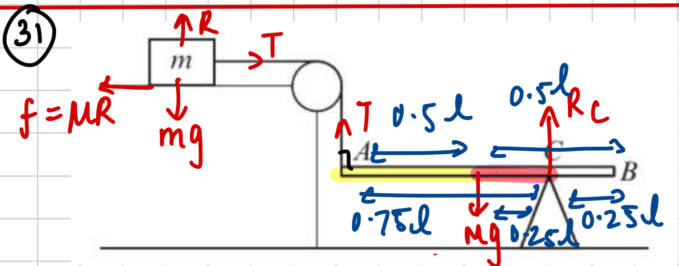
$\Rightarrow v = -\frac{5}{3}e^{-\frac{2}{5}t} + c \Rightarrow t = 0\quad v = 1 \Rightarrow 1 = -\frac{5}{3} + c \Rightarrow c = \frac{8}{3}$

$\Rightarrow v = \left(\dfrac{8 - 5e^{-\frac{2}{5}t}}{3}\right) ms^{-1}$

b) $v = \dfrac{8-5e^{-\frac{2}{5}t}}{3}\ ms^{-1}$; $v = ?$ $t = 3$

$\Rightarrow v = \dfrac{8-5e^{-\frac{2}{5}(3)}}{3}$ $\Rightarrow v = 2.164\cdots$ $\Rightarrow v = 2.16\ ms^{-1}$ (3sf)

31



$M \le k\mu m$

For the box: →

$\uparrow = \downarrow \Rightarrow R = mg$ — (1)

$\rightarrow = \leftarrow \Rightarrow T = f\ (\mu R)$

$\Rightarrow T = \mu mg$ — (2)

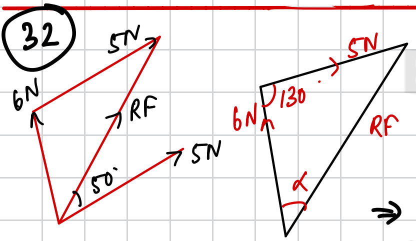
Take moments at C: →

* $Mg: \rightarrow M = Fd \Rightarrow M = Mg \times \frac{1}{4}l \Rightarrow M = \frac{Mg}{4}l$
* T at (A): $\rightarrow M = Fd \Rightarrow M = T \times \frac{3}{4}l \Rightarrow \frac{3}{4}Tl$

acw moments = cw moments $\Rightarrow \dfrac{Mg\cancel{l}}{\cancel{4}} = \dfrac{3}{\cancel{4}}T\cancel{l} \Rightarrow T = \frac{1}{3}Mg$

$\Rightarrow \mu m\cancel{g} \ge \frac{1}{3}M\cancel{g}$ $\Rightarrow 3\mu m \ge M$ $\Rightarrow \boxed{k = 3}$

32



a) $Rf^2 = A^2 + B^2 - 2AB\cos(180° - 50°)$

$R^2 = 6^2 + 5^2 - 2(6)(5)\cos(130°)$

$\Rightarrow R^2 = 61 - 60\cos(130°)$

$\Rightarrow R = \sqrt{61 - 60\cos 130°}$ $\Rightarrow R = 9.98N$ (3sf)

b) $\dfrac{RF}{\sin(180° - 50°)} = \dfrac{B}{\sin\theta}$ $\Rightarrow \sin\theta = \dfrac{5\sin(130°)}{9.97834..}$ $\Rightarrow \theta = 22.6°$ (3sf)

33. $F = (6i + 2j)$ $\quad m = 4 \quad a = a$

$F = ma \Rightarrow a = \dfrac{F}{m} \Rightarrow \dfrac{6i + 2j}{4} \Rightarrow a = \left(\dfrac{3}{2} i + \dfrac{1}{2} j\right) ms^{-2}$

$r(t) = r_0 + v_0 t + \frac{1}{2} a t^2$

$\Rightarrow r(t) = (4i - 2j) + (ui + vj)t + \frac{1}{2}\left(\frac{3}{2} i + \frac{1}{2} j\right) t^2$

$\Rightarrow (-3i + 8j) = (4i - 2j) + (ui + vj)6 + \frac{1}{2}\left(\frac{3}{2} i + \frac{1}{2} j\right) \times 36$

$\Rightarrow (-3i + 8j) = (4i - 2j) + 6ui + 6vj + 27i + 9j$

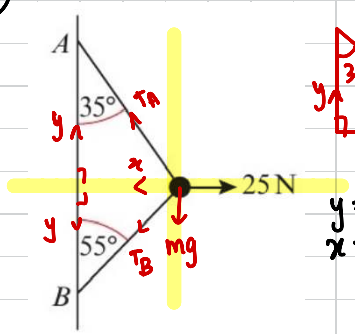
$\Rightarrow (-3i + 8j) = (4 + 6u + 27)i + (-2 + 6v + 9)j$

Equate the i & j

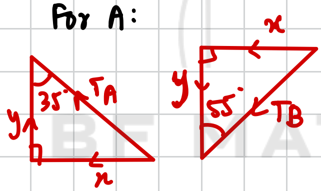
$\Rightarrow -3 = 4 + 6u + 27$ $\quad | \quad 8 = -2 + 6v + 9$ $\quad | \quad u = \left(-\dfrac{17}{3} i + \dfrac{1}{6} j\right) ms^{-1}$

$\Rightarrow u = -\dfrac{17}{3}$ $\quad | \quad \Rightarrow v = \dfrac{1}{6}$

34.

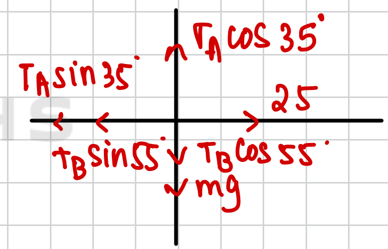


For A:



$y = T_A \cos 35°$ $\quad y = T_B \cos 55°$

$x = T_A \sin 35°$ $\quad x = T_B \sin 55°$



a) Equilibrium: $\uparrow = \downarrow$; $\rightarrow = \leftarrow$ $\Rightarrow T_A \cos 35° = T_B \cos 55° + mg$ — ①

$T_A \sin 35° + T_B \sin 55° = 25$ — ② $\quad (T_A = T_B) \rightarrow$ smooth

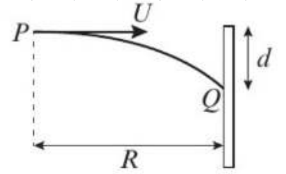
$\Rightarrow$ Eq ②: $\rightarrow T(\sin 35° + \sin 55°) = 25 \Rightarrow T = 17.95 \Rightarrow T = 18\text{ N}$

b) Eq ①: $\rightarrow m = \dfrac{T\cos 35° - T\cos 55°}{g} \Rightarrow m = 0.450\text{ kg (3sf)}$

c) The tension in the string is the same on both sides of the bead.

d) The tension in the string would be greater as the top has to support the mass of string below it

35



From P to Q:

	x	y
s	R	$-d$
u	U	0
v		
a	0	$-g$
t	t	t

a) y: $s = ut + \frac{1}{2}at^2$

$\Rightarrow -d = 0t + \frac{1}{2}(-g)t^2$

$\Rightarrow \cancel{-}2d = \cancel{-}gt^2$

$\Rightarrow \frac{2d}{g} = t \Rightarrow t = \sqrt{\frac{2d}{g}}$

x: $s = ut + \frac{1}{2}at^2 \Rightarrow R = Ut + \frac{1}{2}(0)t^2 \Rightarrow R = U\left(\sqrt{\frac{2d}{g}}\right)$

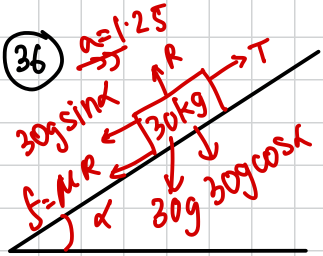
$\Rightarrow R = U\sqrt{\frac{2d}{g}}$

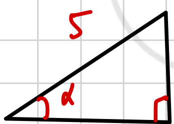
b) $d = d$; $U = 11{\cdot}2\,ms^{-1}$ $R = 2{\cdot}25m = U\sqrt{\frac{2d}{g}}$

$\Rightarrow 2{\cdot}25 = U \times \sqrt{\frac{2}{g}} \times \sqrt{d} \Rightarrow U\sqrt{d} = \frac{63\sqrt{10}}{40} \Rightarrow \sqrt{d} = \frac{63\sqrt{10}}{40} \times \frac{1}{11{\cdot}2}$

$\Rightarrow \sqrt{d} = \frac{9\sqrt{10}}{64} \Rightarrow d = \frac{405}{2048}\,m \approx 0{\cdot}1977m \Rightarrow d = 19{\cdot}8m\,(3sf)$

36





$\sin\alpha = \frac{3}{5}$

$\cos\alpha = 4/5$

$\uparrow = \downarrow \Rightarrow R = 30g\cos\alpha \Rightarrow R = \frac{1176}{5}\,N$

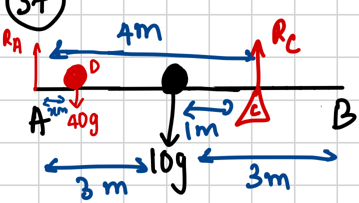
$R(\nearrow) \Rightarrow F = ma$

$\Rightarrow T - 30g\sin\alpha - 0{\cdot}2R = 30(1{\cdot}25)$

$\Rightarrow T = 30(1{\cdot}25) + 0{\cdot}2\left(\frac{1176}{5}\right) + 30g\left(\frac{3}{5}\right)$

$\Rightarrow T = 260{\cdot}94N \Rightarrow T = 261N\,(3sf)$

37



a) Take moments at A:

$\Rightarrow 10g: M = Fd \Rightarrow M = 10g \times 3 = 30g\,N$

$\Rightarrow R_C: M = Fd \Rightarrow M = R_C \times 4 = 4R_C\,N$

$\therefore$ acw = cw

$\Rightarrow 30g = 4R_C \Rightarrow R_C = 7{\cdot}5g\,N$

b) $\uparrow = \downarrow \Rightarrow R_A + R_C = 50g$

$\Rightarrow R_C + R_C = 50g$

$\Rightarrow 2R_C = 50g$

$\Rightarrow R_C = 25g$

b) Take moments at A:

$\Rightarrow 10g: M = Fd \Rightarrow M = 30gN$ cw

$\Rightarrow R_C: M = Fd \Rightarrow M = 100gN$ acw

$\Rightarrow 40g: M = Fd \Rightarrow M = 40gxN$ cw

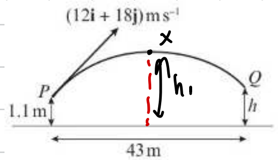
$\therefore$ acw = cw

$\Rightarrow 30g + (40g \times x) = 100g$

$\Rightarrow 40g \times x = 70g$

$\Rightarrow x = \frac{7}{4} \Rightarrow x = 1{\cdot}75 \Rightarrow AD = 1{\cdot}75m$

38



From P to X

	x	y
s	21.5	h_1
u	12	18
v		0
a	0	$-g$
t	t	t

a) $y: v^2 = u^2 + 2as$

$\Rightarrow (0)^2 = (18)^2 - 2gs$

$\Rightarrow \dfrac{-18^2}{-2g} = s$

$\Rightarrow s = 16.53$ (4sf)

$\Rightarrow h_1 = 1.1 + 16.5 = 17.6\text{ m}$ (3sf)

b) From P to Q

	x	y
s	43	h_0
u	12	18
v		
a	0	$-g$
t	t	t

$x: s = ut + \frac{1}{2}at^2$

$\Rightarrow 43 = 12t$

$\Rightarrow t = 3.583\ldots$ sec

$y: s = ut + \frac{1}{2}at^2$

$\Rightarrow h = 18(3.58\ldots) - \frac{1}{2}g(3.588)^2$

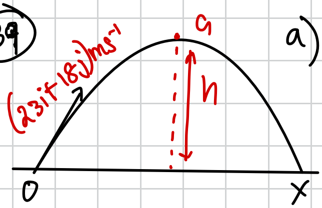
$\Rightarrow h_0 = 1.58263\ldots$

$\Rightarrow h = 1.1 + 1.582\ldots$

$h = 2.68\text{ m}$ (3sf)

c) Fielder catches the ball nearer the ground, or the ball might not reach the fielder before hitting the ground.

39



a) From O to G:

	x	y
s	s_x	h
u	23	18
v		0
a	0	$-g$
t	t	t

$y: v^2 = u^2 + 2as$

$\Rightarrow 0^2 = 18^2 - 2g(h)$

$\Rightarrow \dfrac{-18^2}{-2g} = h$

$\Rightarrow h = \dfrac{810}{49}\text{ m}$

b) From O to X:

	x	y
s	s_x	0
u	23	18
v		
a	0	$-g$
t	t	t

$y: s = ut + \frac{1}{2}at^2$

$\Rightarrow 0 = 18t - 4.9t^2$

$\Rightarrow t(4.9t - 18) = 0$

$\Rightarrow t = \dfrac{18}{4.9}$

$x: s = ut$

$\Rightarrow s_x = 23\left(\dfrac{18}{4.9}\right)$

$\Rightarrow s_x = \dfrac{4140}{49}\text{ m}$

c) $v(t) = (u_x, u_y - gt)$

$\Rightarrow (23,\ 18 - 9.8t)$

$\dfrac{v_y}{v_x} = -\dfrac{1}{2}$

$\Rightarrow \dfrac{18 - 9.8t}{23} = -\dfrac{1}{2}$

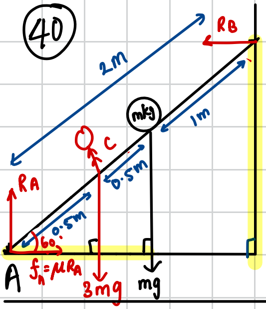
$\Rightarrow 2(18 - 9.8t) = -23$

$\Rightarrow 36 - 19.6t = -23$

$\Rightarrow \cancel{-}19.6t = \cancel{-}59$

$\Rightarrow t = 3.01$ (3sf)

(40)



Equilibrium ($\uparrow : \downarrow$; $\rightarrow : \leftarrow$)

$(\uparrow = \downarrow) \Rightarrow R_A = 3mg + mg$

$\Rightarrow R_A = 4mg$ — ①

$(\rightarrow = \leftarrow) \Rightarrow R_B = \mu R_A$

$\Rightarrow R_B = \mu(4mg)$

$\Rightarrow R_B = 4mg\mu$ — ②

Take moments at A:

* $3mg \Rightarrow M = Fd \Rightarrow 3mg \times 0.5\cos 60°$

$\Rightarrow 0.75mg$ CW

$mg \Rightarrow M = Fd \Rightarrow mg \times 1\cos 60°$

$\Rightarrow 0.5mg$ CW

$R_B \Rightarrow M = Fd \Rightarrow R_B \times 2\sin 60°$

$\Rightarrow \sqrt{3}R_B$ ACW

CW moments = ACW moments

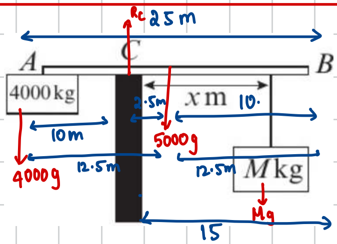
$\Rightarrow 0.75mg + 0.5mg = \sqrt{3}R_B \quad \Rightarrow R_B = \dfrac{1.25mg}{\sqrt{3}}$ — ③

$\Rightarrow$ Equate ③ in ② $\Rightarrow \dfrac{1.25mg}{\sqrt{3}} = 4mg\mu \Rightarrow \cancel{mg}\left(\dfrac{1.25}{\sqrt{3}}\right) = \cancel{mg}(4\mu)$

$\Rightarrow \mu = \dfrac{1.25}{4\sqrt{3}} \Rightarrow \mu = 0.180$ (3sf)

(41)

a)



$(\uparrow = \downarrow) \Rightarrow R_C = 4000g + 5000g + Mg \Rightarrow R_C = 9000g + Mg$ — ①

Take moments at A:

$R_C : M = Fd \Rightarrow M = R_C \times 10 = 10R_C$ acw

$5000g : M = Fd \Rightarrow M = 5000g \times 12.5 = 612500$ cw

$Mkg : M = Fd \Rightarrow M = Mg \times (10 + x)$ cw

acw moments = cw moments $\Rightarrow 10R_C = 612500 + Mg(10 + x)$

When $x = 6$, the M would be the greatest value

$x = 6 \Rightarrow 10(9000g + Mg) = 612500 + 16Mg$

$\Rightarrow 90{,}000g + 10Mg = 612500 + 16Mg$

$\Rightarrow \dfrac{90000(g) - 612500}{6g} = M \Rightarrow M = 4580$ kg (3sf)

When $x = 15$, the M would be the least value

$x = 15 \Rightarrow 10(9000g + Mg) = 612500 + 25Mg$

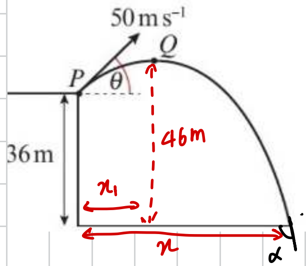
$\Rightarrow 90{,}000g + 10Mg = 612500 + 25Mg$

$\Rightarrow M = \dfrac{90{,}000g - 612500}{15g} \Rightarrow M = 1830$ kg (3sf)

$\Rightarrow 1830 \le M \le 4580$kg (3sf) [when $x: 6 \le x \le 15$]

b) The beam is unlikely to be uniform, or the beam does not rest freely on the tower.

42



From P to Q:

	x	y
S	x_1	$+10$
u	$50\cos\theta$	$50\sin\theta$
v		0
a	0	$-g$
t	t	t

From P to X:

	x	y
S	x	-36
u	$50\cos\theta$	$50\sin\theta$
v		
a	0	$-g$
t	t	t

a) From P to Q:

$y: v^2 = u^2 + 2as$

$\Rightarrow 0 = (50\sin\theta)^2 + 2(-g)(10)$

$\Rightarrow (50\sin\theta)^2 = 196$

$\Rightarrow \sin^2\theta = \frac{196}{2500}$

$\Rightarrow \sin\theta = \pm\frac{7}{25}$

$\Rightarrow \sin\theta = \frac{7}{25}$ (angle cannot be obtuse or reflex)

$\Rightarrow \theta = 16.3^\circ$ (3SF)

b) From P to Q:

$y: v = u + at$

$\Rightarrow 0 = 50\sin\theta - gt$

$\Rightarrow t = \frac{50\sin\theta}{g}$

$\Rightarrow t = 1.42857\ldots$

From P to X

$\Rightarrow 2t = 2(1.428\ldots)$

$\Rightarrow t = 2.857\ldots$

$\Rightarrow t = 2.86$ (3SF)

$\therefore$ It takes twice as the halfway is Q

c) From P to X:

$y: s = ut + \frac{1}{2}at^2$

$\Rightarrow -36 = 50\sin\theta t - 4.9t^2$

$\Rightarrow 4.9t^2 - 14 - 36 = 0$

$\Rightarrow t = \frac{10 + 2\sqrt{115}}{7}$

$v_y = u_y + at = 14 - g(t) = -\frac{14\sqrt{115}}{5}$

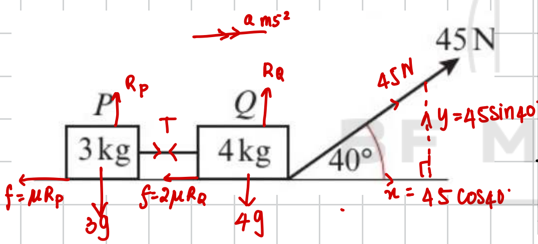
$v_x = u_x = 50\cos\theta = 48$

$\Rightarrow \tan\alpha = -\frac{30.026\ldots}{48}$

$\Rightarrow \alpha = 32.02\ldots$

$\Rightarrow \alpha = 32.0^\circ$ (3SF)

43



a) For Q: $\rightarrow$ $u = 0$ $t = 3$ $s = 5$ $a = a$

$\Rightarrow s = ut + \frac{1}{2}at^2$ $\Rightarrow 5 = 0(3) + \frac{1}{2}(a)(3^2)$

$\Rightarrow 10 = 9a$ $\Rightarrow a = \frac{10}{9}\text{ ms}^{-2}$

b) For Q: $\rightarrow$ $\uparrow = \downarrow$ $\Rightarrow R_Q + 45\sin 40^\circ = 4g$

$\Rightarrow R_Q = 4g - 45\sin 40^\circ$ — (1)

$R(\rightarrow) \Rightarrow F = ma \Rightarrow 45\cos 40^\circ - 2\mu R_Q - T = 4a \Rightarrow 45\cos 40^\circ - 2\mu R_Q - T = \frac{40}{9}$

$\Rightarrow 45\cos 40^\circ - 8\mu g + 90\sin 40^\circ \mu - T = 40/9$ — (1)

$\Rightarrow$ For P: $\rightarrow$ $\uparrow = \downarrow$ $\Rightarrow R_P = 3g$

$R(\rightarrow) \Rightarrow F = ma \Rightarrow T - \mu R_P = 3a \Rightarrow T - \mu 3g = \frac{30}{9}$ — (2)

Solve simultaneously eq (1) & (2)

$\Rightarrow T - \mu 3g = 30/9$ — (2)

$-T - \mu(8g - 90\sin 40^\circ) = \frac{40}{9} - 45\cos 40^\circ$ (1)

$\mu = 0.534$ (3sf)

c) $T = \frac{30}{9} + (0.534)3g = 19.0$ (3sf)

d) Both particles have the same acceleration

(44) $a = 3(t+2)i + 2(t-1)j \quad ; \quad t=0 \quad v = -2i + 3j$

a) $\Rightarrow v = \int a\,dt \Rightarrow v = \int 3(t+2)i + 2(t-1)j \Rightarrow v = \left[\frac{3}{2}(t+2)^2\right]i + (t-1)^2 j + c$

$t=0 \quad v = -2i+3j \;;\; -2i+3j = \left[\frac{3}{2}(2)^2\right]i + (-1)^2 j + c$

$\Rightarrow -2i + 3j = 6i + j + c \Rightarrow c = -8i + 2j$

$v = \left[\frac{3}{2}(t+2)^2 - 8\right]i + \left[(t-1)^2 + 2\right]j \quad \Rightarrow \text{speed} = |v| = \sqrt{v_i^2 + v_j^2}$

when $t = 1.2 \;;\; v = 7.36i + 2.04j \quad \Rightarrow \text{speed} = \sqrt{7.36^2 + 2.04^2} = 7.64\,ms^{-1}$

b) v is parallel to $i+j$

$\Rightarrow \frac{1.5(t+2)^2 - 8}{(t-1)^2 + 2} = 1 \quad \Rightarrow 1.5(t+2)^2 - 8 = (t-1)^2 + 2$

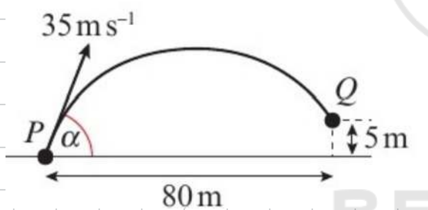
$\Rightarrow 1.5(t^2 + 4t + 4) - 8 = t^2 - 2t + 1 + 2$

$\Rightarrow 1.5t^2 + 6t + 6 - 8 = t^2 - 2t + 3$

$\Rightarrow t = 0.6023$ or $\cancel{t = -16.6}$

$\Rightarrow 0.5t^2 + 8t - 5 = 0 \quad \Rightarrow t = 0.602\ (3sf)$

(45)



From P to Q:

	x	y
s	80	5
u	$35\cos\alpha$	$35\sin\alpha$
v		
a	0	$-g$
t	t	t

$x: s = ut + \frac{1}{2}at^2$

$80 = 35\cos\alpha\, t + \frac{1}{2}(0)t^2$

$\Rightarrow t = \frac{80}{35\cos\alpha} = \frac{16}{7\cos\alpha}$

$\Rightarrow t = \frac{16}{7\cos\alpha} \quad — (1)$

$y: s = ut + \frac{1}{2}at^2 \Rightarrow 5 = 35\sin\alpha\left[\frac{16}{7\cos\alpha}\right] + \frac{1}{2}(-g)\left[\frac{16}{7\cos\alpha}\right]^2$

$\Rightarrow 5 = 80\tan\alpha - \frac{256}{98}g(1 + \tan^2\alpha) \quad \Rightarrow 5 = 80\tan\alpha - \frac{256}{98}(g) - \frac{256g}{98}\tan^2\alpha$

$\Rightarrow \frac{256g}{98}\tan^2\alpha - 80\tan\alpha + \left[\frac{256}{98}g + 5\right]$ } Quadratic form

$\Rightarrow \tan\alpha = 2.6787$ or $\tan\alpha = 0.4462$

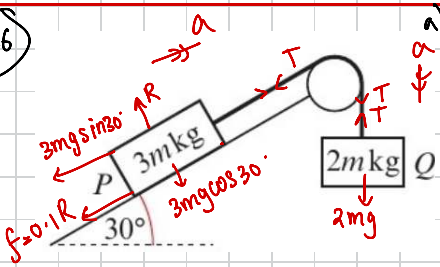
$\Rightarrow \alpha = \arctan(2.6787)$ or $\alpha = \arctan(0.4462)$

$\Rightarrow \alpha = 69.5°\ (3sf)$ or $\cancel{\alpha = 24.0\ (3sf)}$

↳ largest angle α

b) The ball is unlikely to land in the hole exactly. Valid reasons include the model does not consider air resistance and/or rotational motion of the ball, the golfer is unlikely to hit the ball at the exact angle or speed, and the measurements may not be exact.

46



a) For Q: $R(\downarrow) \Rightarrow F = ma \Rightarrow 2mg - T = 2ma$

$\Rightarrow T = 2mg - 2ma$ — ①

For P: $\uparrow = \downarrow \Rightarrow R = 3mg\cos 30°$ — ②

$R(\nearrow) \Rightarrow F = ma \Rightarrow T - 3mg\sin 30° - 0.1R = 3ma$

$\Rightarrow 2mg - 2ma - 3mg\sin 30° - 0.3mg\cos 30° = 3ma$

$\Rightarrow 2mg - 3mg\sin 30° - 0.3mg\cos 30° = 5ma$

$\Rightarrow \cancel{m}(2g - 3g\sin 30° - 0.3g\cos 30°) = 5\cancel{m}a$

$\Rightarrow a = \dfrac{2g - 3g\sin 30° - 0.3g\cos 30°}{5} \Rightarrow a = 0.47077...$

$u = 0 \quad v = 10 \quad a = 0.4707... \quad t = t$

$\Rightarrow v = u + at \Rightarrow 10 = 0 + 0.4707...t \Rightarrow t = \dfrac{10}{0.47...} \Rightarrow t = 21.2$ (3sf)

b) P does not reach the pulley. Q does not reach the floor.

c) the deceleration of P after the (string snaps) There is no tension in the string

$\Rightarrow -3mg\sin 30° - 0.1R = 3ma \Rightarrow -3mg\sin 30° - 0.3mg\cos 30° = 3ma$

$\Rightarrow \cancel{m}(-3g\sin 30° - 0.3g\cos 30°) = 3\cancel{m}a \Rightarrow a = \dfrac{-3g\sin 30° - 0.3g\cos 30°}{3}$

$\Rightarrow a = -5.7487...\ ms^{-2}$

$v = 0 \quad u = 10 \quad a = -5.74... \quad s = s$

$\Rightarrow v^2 = u^2 + 2as \Rightarrow 0 = 10^2 - 2(5.74...)s \Rightarrow s = \dfrac{\cancel{-}10^2}{\cancel{-}2(5.74...)}$

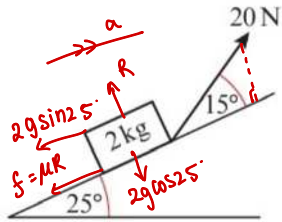
$\Rightarrow s = 8.6976... \Rightarrow s = 8.70\text{ m}$ (3sf)

d) $R(\swarrow)$: $F = ma \Rightarrow 3mg\sin 30° = 1.5mg$

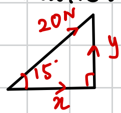
$\Rightarrow F_{max} = \mu R \Rightarrow 0.1 \times 3mg\cos 30° = \dfrac{3\sqrt{3}}{20}mg$

$\Rightarrow 1.5mg > \dfrac{3\sqrt{3}}{20}mg$; So P slides back down the slope.

(47)

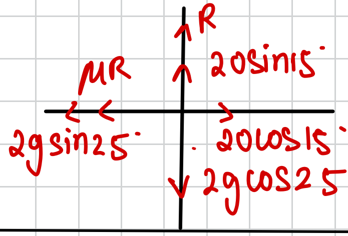


Resolve 20N:



$y = 20\sin 15°$

$x = 20\cos 15°$



a) $\uparrow = \downarrow \Rightarrow R + 20\sin 15° = 2g\cos 25° \Rightarrow R = 2g\cos 25° - 20\sin 15°$ —(1)

$R(\nearrow)$: $F = ma \Rightarrow 20\cos 15° - 2g\sin 25° - \mu R = 2a$

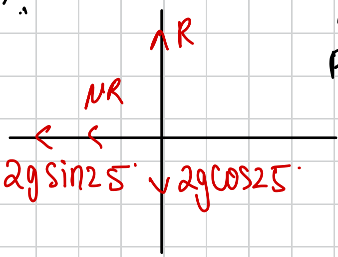
$\Rightarrow 20\cos 15° - 2g\sin 25° - \mu(2g\cos 25° - 20\sin 15°) = 2a$ $\quad \boxed{a=0}$

$\Rightarrow 20\cos 15° - 2g\sin 25° = \mu(2g\cos 25° - 20\sin 15°)$

$\Rightarrow \mu = \dfrac{20\cos 15° - 2g\sin 25°}{2g\cos 25° - 20\sin 15°}$ $\quad \Rightarrow \mu = 0.87669...$

$\Rightarrow \mu = 0.877$ (3sf)

b) When P is removed: the deceleration of the block



$\uparrow = \downarrow \Rightarrow R = 2g\cos 25°$ —(1)

$R(\swarrow) \Rightarrow -2g\sin 25° - \mu R = 2a$

$\Rightarrow a = \dfrac{-2g\sin 25° - 0.87669...(2g\cos 25°)}{2}$

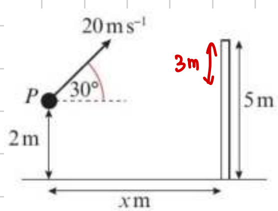
$\Rightarrow a = -11.9283..\ ms^{-2}$

$v = 0 \quad u = 8 \quad a = -11.9283... \quad s = s$

$\Rightarrow v^2 = u^2 + 2as \quad \Rightarrow 0 = 8^2 - 2(11.9283...)s \quad \Rightarrow s = \dfrac{-8^2}{-2(11.9283...)}$

$\Rightarrow s = 2.6826...\ m \quad \Rightarrow s = 2.68\ m$ (3sf)

(48)



	x	y
s	x	3
u	$20\cos 30°$	$20\sin 30°$
v		
a	0	$-g$
t	t	t

a) $y: s = ut + \frac{1}{2}at^2$

$3 = 20\sin 30°\, t - 4.9t^2$

$\Rightarrow 4.9t^2 - 10t + 3 = 0$

$\Rightarrow t_1 = 1.6753$ or $t_2 = 0.3654$

$x: s = ut \Rightarrow x = 20\cos 30°\, t$

$\Rightarrow x = 20\cos 30°(1.6753)$

$\Rightarrow x = 29\,m$ (3sf)

$\Rightarrow x = 20\cos 30°(0.3654...)$

$\Rightarrow x = 6.33$ (3sf)

$\Rightarrow 6.33 < x < 29.0$ (3sf)

b)

	x	y
s	x	-2
u	$20\cos 30°$	$20\sin 30°$
v		
a	0	$-g$
t	t	t

$y: s = ut + \frac{1}{2}at^2$

$\Rightarrow -2 = 10t - 4.9t^2$

$\Rightarrow 4.9t^2 - 10t - 2 = 0$

$\Rightarrow t = 2.22...s$

$v_y = u_y + at$

$\Rightarrow v_y = 10 - 9.8(2.21...)$

$\Rightarrow -11.73\ ms^{-1}$

$\Rightarrow v_x = u_x + at$

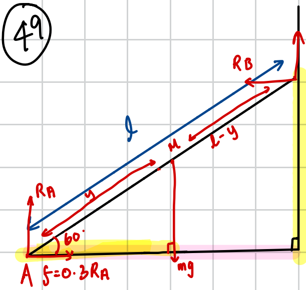
$\Rightarrow v_x = 10\sqrt{3}\ ms^{-1}$

Speed $= \sqrt{v_x^2 + v_y^2}$

$\Rightarrow \sqrt{(10\sqrt{3})^2 + (-11.73)^2}$

$\Rightarrow$ Speed $= 21.0\ ms^{-1}$ (3sf)

(49)



Equilibrium ($\uparrow=\downarrow$; $\rightarrow=\leftarrow$)

$\uparrow=\downarrow$; $0.7R_B+R_A=mg$ — (1)

$\rightarrow=\leftarrow$; $0.3R_A=R_B$ — (2)

Take moment at A:

$\Rightarrow mg: M=Fd \Rightarrow mg\times y\cos 60°$ CW

$\Rightarrow R_B: M=Fd \Rightarrow R_B\times l\sin 60°$ ACW

$\Rightarrow 0.7R_B: M=Fd \Rightarrow 0.7R_B\times l\cos 60°$ ACW

ACW = CW

$\Rightarrow 0.7R_B\times l\cos 60° + R_B\times l\sin 60° = mg\times y\cos 60°$

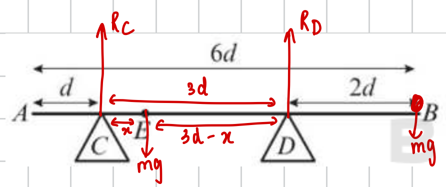
$\Rightarrow 0.7(0.3R_A)\times l\cos 60° + 0.3R_A\times l\sin 60° = (0.7(0.3R_A)+R_A)\times y\cos 60°$

$\Rightarrow R_A(0.105l) + R_A\left(\frac{3\sqrt{3}}{20}l\right) = R_A(0.605y)$

$\Rightarrow \cancel{R_A}\left(0.105l + \frac{3\sqrt{3}}{20}l\right) = \cancel{R_A}(0.605y)$

$\Rightarrow l\left[0.105+\frac{3\sqrt{3}}{20}\right]\div 0.605 = y \quad \Rightarrow y = 0.603\,l$ (3sf)

(50)



a) Let CE be x

Take moments at D:

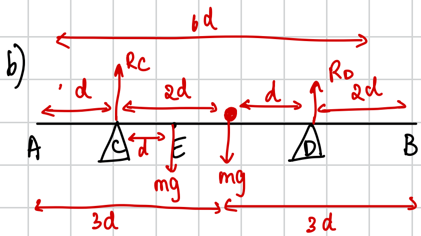
$mg: M=F\times d \Rightarrow mg(3d-x)$

$mg: M=F\times d \Rightarrow mg\times 2d$

ACW = CW

$\Rightarrow (3d-x)\cancel{mg} = \cancel{mg}\times 2d \Rightarrow 3d-x=2d \Rightarrow \boxed{x=d} \Rightarrow AE=d+x=2d$

b)



Equilibrium ($\uparrow=\downarrow$)

$R_C+R_D=2mg$

Take moments at C:

$mg: M=F\times d \Rightarrow mg\times d$

$mg: M=F\times d \Rightarrow mg\times 2d$

$R_D: M=F\times d \Rightarrow R_D\times 3d$

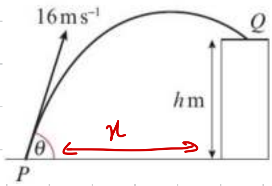
$\Rightarrow$ ACW = CW

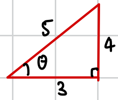
$\Rightarrow dmg+2dmg = 3dR_D$

$\Rightarrow 3\cancel{d}mg = 3\cancel{d}R_D$

$\Rightarrow R_D = mg$ N

51





$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

	x	y
s	x	h
u	$16\cos\theta$	$16\sin\theta$
v	v_x	v_y
a	0	$-g$
t	t	t

a) $v_x = u_x + at$

$\Rightarrow v_x = 16\cos\theta + 0(t)$

$\Rightarrow v_x = 16 \times \frac{3}{5} = 9.6\,ms^{-1}$

$\Rightarrow v_y = u_y + at$

Speed $= \sqrt{v_x^2 + v_y^2}$ $\Rightarrow 12^2 = 9.6^2 + v_y^2$ $\Rightarrow v_y = 7.2\,ms^{-1}$

$\Rightarrow$ y: $v^2 = u^2 + 2as$ $\Rightarrow (7.2)^2 = (12.8)^2 + 2(-g)h \Rightarrow h = \frac{(7.2)^2 - (12.8)^2}{-2g}$

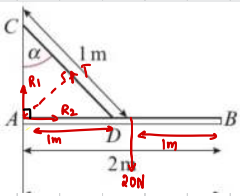
$\Rightarrow h = 5.71\,m$ (3sf)

b) $s = h$ $a = -g$ $u = 16\sin\theta$ $t = t$

y: $s = ut + \frac{1}{2}at^2$ $\Rightarrow h = 12.8t - 4.9t^2 \Rightarrow 4.9t^2 - 12.8t + h = 0$

$t = \frac{100}{49}$ or $t = \frac{4}{7}$; $\Rightarrow x: s = ut$ $\Rightarrow s = 16\left[\frac{3}{5}\right]\left[\frac{100}{9}\right] = 19.6\,m$ (3sf)

52



a) $AD = \sin\alpha$

Moments about A:

T: $M = Fd \Rightarrow T \times \sin\alpha \times \cos\alpha$

20N: $M = Fd \Rightarrow 20 \times 1$

acw = cw

$\Rightarrow T\sin\alpha\cos\alpha = 20$

$\Rightarrow T = \frac{20}{\frac{1}{2}\sin 2\alpha}$

$\Rightarrow T = \frac{40}{\sin 2\alpha}$

b) $T = 80\,N$

$80 = \frac{40}{\sin 2\alpha}$

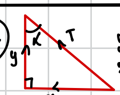
$\sin 2\alpha = \frac{1}{2}$

b) $2\alpha = 30^\circ, (180^\circ - 30^\circ)$

$2\alpha = 30^\circ, 150^\circ$

$\alpha = 15^\circ, 75^\circ$

c)



$y = T\cos 15^\circ$

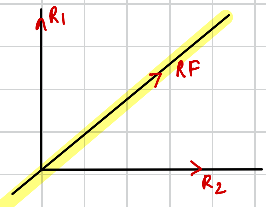
$x = T\sin 15^\circ$

$\uparrow = \downarrow \Rightarrow R_1 + T\cos 15^\circ = 20$ — ①

$\rightarrow = \leftarrow \Rightarrow R_2 = T\sin 15^\circ$ — ②

Take moments at A

CW = ACW $\Rightarrow T = \frac{40}{\sin 30^\circ}$ $\Rightarrow T = 80\,N$



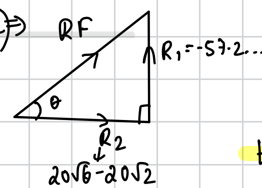
$R_1 = 20 - T\cos 15^\circ$ — ①

$R_1 = -57.2740661$

$R_2 = T\sin 15^\circ$

$R_2 = 20\sqrt{6} - 20\sqrt{2}$

c) $\Rightarrow$



$RF = \sqrt{R_1^2 + R_2^2}$

$RF = 60.901\ldots N$

$RF = 60.9\,N$ (3sf)

$\tan\theta = \frac{R_1}{R_2}$ $\Rightarrow \theta = \arctan\left(\frac{R_1}{R_2}\right)$

$\Rightarrow \theta = -70.124^\circ\ldots \approx 70.1^\circ$ (3SF)

$\Rightarrow$ 60.9 N at an angle of 70.1° to the horizontal below the beam.

d) The angle would be closer to the horizontal