Author: Naga Karthik

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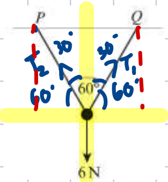
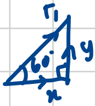
7.2 Modelling with statics

①



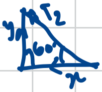
Resolve T_1:



$\sin 60° = \frac{y}{T_1}$ $\cos 60° = \frac{x}{T_1}$

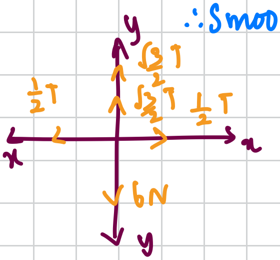
$y = \frac{\sqrt{3}}{2} T_1$ $x = \frac{1}{2} T_1$

Resolve T_2:



$\sin 60° = \frac{y}{T_2}$ $\cos 60° = \frac{x}{T_2}$

$y = \frac{\sqrt{3}}{2} T_2$ $x = \frac{1}{2} T_1$

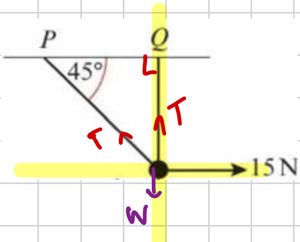


$\therefore$ Smooth $(T_1 = T_2) = T$

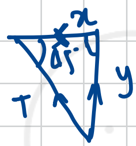
$\Rightarrow \frac{\cancel{2}\sqrt{3}}{\cancel{2}} T = 6$ $T = \frac{6}{\sqrt{3}} = 2\sqrt{3}\,N$

$T = 2\sqrt{3}\,N$

②

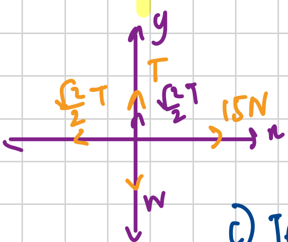


Resolve P:



$\cos 45° = \frac{x}{T}$ $\sin 45° = \frac{y}{T}$

$x = \frac{\sqrt{2}}{2} T$ $y = \frac{\sqrt{2}}{2} T$

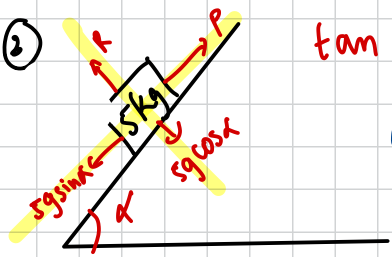


a) equilibrium $(\rightarrow = \leftarrow)$ $15 = \frac{\sqrt{2}}{2} T$ $T = 15\sqrt{2}\,N$

b) $\frac{\sqrt{2}}{2} T + T = W$ $\Rightarrow \frac{\sqrt{2}+2}{2}(15\sqrt{2}) \Rightarrow W = 15 + 15\sqrt{2}$
$= 15(1+\sqrt{2})\,N$

c) Tension is same on both sides of the bead.

③



$\tan \alpha = \frac{5}{12}$ $\alpha = \arctan \frac{5}{12}$ $\therefore$ equilibrium $(\uparrow = \downarrow;$
$\leftarrow = \rightarrow)$

a) $R = 5g\cos(\alpha) = g \times 5\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{60}{13} g\,N$

b) $P = 5g\sin\alpha = g \times 5\sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{25}{13} g\,N$

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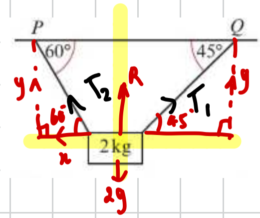
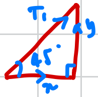


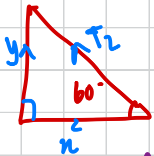
Resolve T_1:



$\Rightarrow \cos 45° = \frac{x}{T_1}$ $\qquad$ $\sin 45° = \frac{y}{T_1}$

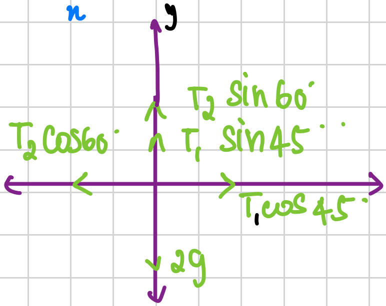
$x = \cos 45° T_1 = \frac{\sqrt{2}}{2} T_1 \Rightarrow y = \sin 45° T_1 = \frac{\sqrt{2}}{2} T_1$

Resolve T_2:



$\Rightarrow \cos 60° = \frac{x}{T_2}$ $\qquad$ $\Rightarrow \sin 60° = \frac{y}{T_2}$.

$x = T_2 \cos 60° = \frac{1}{2} T_2$ $\qquad$ $y = \sin 60° \times T_2 = \frac{\sqrt{3}}{2} T_2$



Equilibrium: $(\uparrow = \downarrow; \rightarrow = \leftarrow)$

$\Rightarrow (\uparrow = \downarrow)\ T_2 \sin 60° + T_1 \sin 45° = 2g$

$\Rightarrow \frac{\sqrt{3}}{2} T_2 + \frac{\sqrt{2}}{2} T_1 = 19.6$ — (1)

$\Rightarrow T_2 \cos 60° = T_1 \cos 45° \Rightarrow \frac{1}{\cancel{2}} T_2 = \frac{\sqrt{2}}{\cancel{2}} T_1$

$\Rightarrow T_2 = \sqrt{2} T_1$ (sub into (1))

$\Rightarrow \frac{\sqrt{3}}{2}(\sqrt{2} T_1) + \frac{\sqrt{2}}{2} T_1 = 19.6 \Rightarrow \frac{\sqrt{6}+\sqrt{2}}{2} T_1 = 19.6 \Rightarrow T_1 = \frac{49\sqrt{6} - 49\sqrt{2}}{5}$

$T_2 = \sqrt{2}\left(\frac{49\sqrt{6} - 49\sqrt{2}}{5}\right) = \frac{-98 + 98\sqrt{3}}{5}$

$T_1 = 10.1$ N (3sf) $\qquad$ $T_2 = 14.3$ N (3sf)

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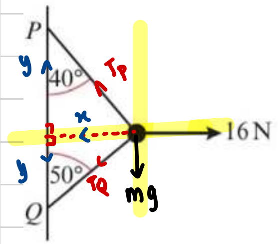
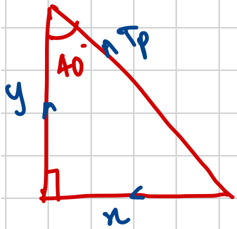


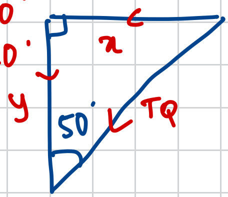
Resolve T_P:



$y = T_P \cos 40^\circ$

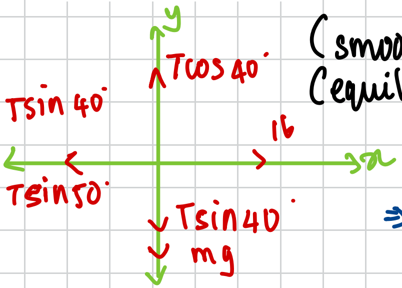
$x = T_P \sin 40^\circ$

Resolve T_Q:



$y = T_Q \cos 50^\circ$

$x = T_Q \sin 50^\circ$



(smooth : $T_P = T_Q$)

(equilibrium ⇒ ↑ = ↓ ; ← = →)

a) ← = → ⇒ $T\sin 40^\circ + T\sin 50^\circ = 16$

⇒ $T = \dfrac{16}{\sin 40^\circ + \sin 50^\circ} = 11.4\text{ N (3sf)}$

11.3569....

b) ↑ = ↓ ⇒ $T\cos 40^\circ = T\sin 40^\circ + mg$ ⇒ $m = \dfrac{T(\cos 40^\circ - \sin 40^\circ)}{g}$

⇒ $m = 0.143\text{ kg}$ (3sf)

c) The tension in the string would be greater at the top of the string as it would have to support the additional weight of the string below it.

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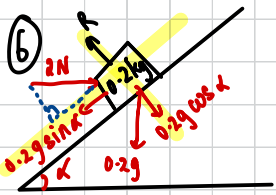
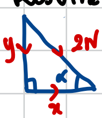
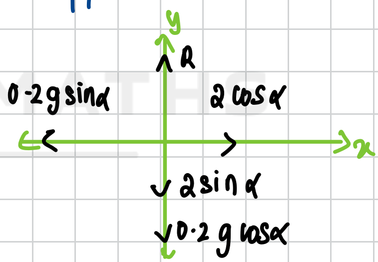


Resolve 2N:



$y = 2\sin\alpha$

$x = 2\cos\alpha$



a) equilibrium (→ = ←) ⇒ $0.2g\sin\alpha = 2\cos\alpha$ ⇒ $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{2}{0.2g}$

$\tan\alpha = \dfrac{2}{0.2g}$ ⇒ $\alpha = \arctan\left(\dfrac{2}{0.2g}\right)$ ⇒ $\alpha = 45.57\ldots = 45.6^\circ$ (3sf)

b) If the particle was heavier; the m > 0.2g → the angle would be smaller

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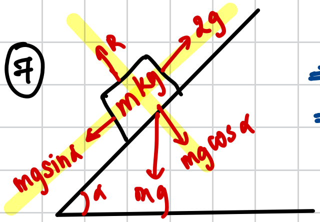

equilibrium: $(\uparrow = \downarrow \; ; \; \rightarrow = \leftarrow)$

$\Rightarrow \uparrow = \downarrow \Rightarrow R = mg\cos\alpha$ — (1)

$\Rightarrow \rightarrow = \leftarrow \Rightarrow 2\cancel{g} = m\cancel{g}\sin\alpha$ — (2)

$\Rightarrow \sin\alpha = \dfrac{2}{m} \quad \Rightarrow \quad \alpha = \arcsin\left(\dfrac{2}{m}\right)$

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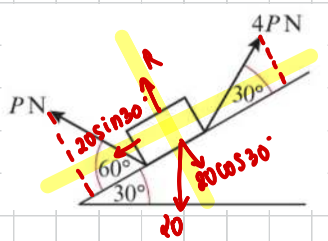
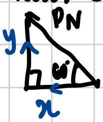


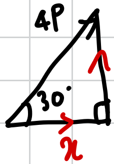
Resolve PN:



$y = P\sin 60° = \dfrac{\sqrt{3}}{2}P$

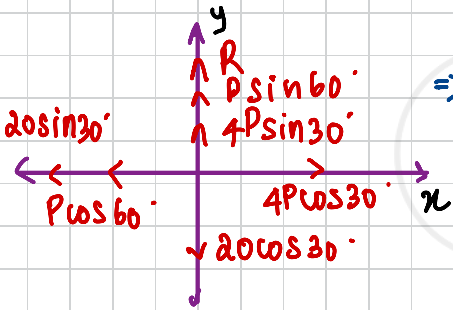
$x = P\cos 60° = \dfrac{1}{2}P$

Resolve 4PN:



$y = 4P\sin 30° = 2P$

$x = 4P\cos 30° = \dfrac{4\sqrt{3}P}{2}$



equilibrium $(\uparrow = \downarrow \; ; \; \rightarrow = \leftarrow)$

$\Rightarrow \leftarrow = \rightarrow$

$\Rightarrow 20\sin 30° + P\cos 60° = 4P\cos 30°$

$\therefore 10 + \dfrac{1}{2}P = \dfrac{4\sqrt{3}}{2}P$

$\Rightarrow 10 = \dfrac{-1+4\sqrt{3}}{2}P \Rightarrow P = \dfrac{20+80\sqrt{3}}{47}$

$\Rightarrow \uparrow = \downarrow \Rightarrow R + P\sin 60° + 4P\sin 30 = 20\cos 30°$

$\Rightarrow R = 10\sqrt{3} - \dfrac{\sqrt{3}}{2}P - 2P \Rightarrow R = 10\sqrt{3} - \dfrac{4+\sqrt{3}}{2}\left(\dfrac{20+80\sqrt{3}}{47}\right)$

$R = 7.651.. = 7.65 \text{ (3sf)}$