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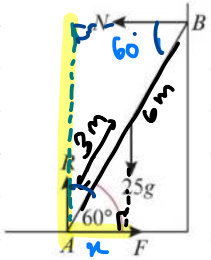
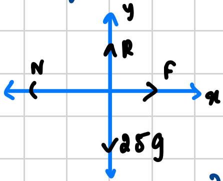
7.4 - Static rigid bodies

(1)



a) equilibrium ($\uparrow = \downarrow$; $\rightarrow = \leftarrow$)



$\Rightarrow \rightarrow = \leftarrow$; $\boxed{F = N}$

b) $\uparrow = \downarrow \Rightarrow R = 25g = 245N$

c) Take moments at A:

CW = ACW

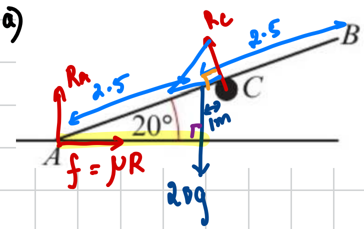
$25g : \rightarrow M = Fd \Rightarrow 25g \times 3\cos 60°$

$N : \rightarrow M = Fd \Rightarrow N \times 6\sin 60°$

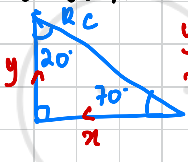
$\Rightarrow 25g \times 3\cos 60° = N \times 6\sin 60° \Rightarrow N = \dfrac{25g \times 3\cos 60°}{6\sin 60°}$

$\Rightarrow N = 70.7N$

(2) a)



b) Resolve R_C:



$y = R_C \cos 20°$

$x = R_C \sin 20°$

$\uparrow = \downarrow$

$R_C \cos 20° + R_A = 196$ — (1)

$\rightarrow = \leftarrow$

$R_C \sin 20° = \mu R_A$ — (2)

Take moments at A:

$20g : \rightarrow M = Fd = 20g \times 2.5\cos 20° \Rightarrow 20g \times 2.5\cos 20° = 3.5 R_C$

$R_C : \rightarrow M = Fd = R_C \times 3.5$

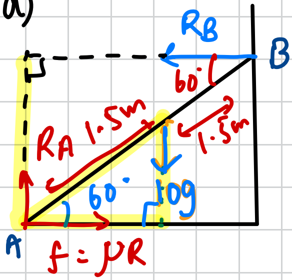
$\Rightarrow R_C = \dfrac{20g \times 2.5\cos 20°}{3.5} = 131.55... = 132N$ (3sf)

c) eq (1) $\Rightarrow R_A = 196 - R_C \cos 20° \Rightarrow 72.37....$

eq (2) $\Rightarrow \mu = \dfrac{R_C \sin 20°}{R_A} = 0.6216... = 0.622$ (3sf)

d) Reaction at the peg acts perpendicular to the rod.

3) a)



b) $\uparrow = \downarrow$; $\rightarrow = \leftarrow$

$R_A = 10g$ — ① $R_B = \mu R_A$ — ②

Take moments at A.

$10g$:→ $M = Fd$ = $10g \times 1.5\cos 60°$

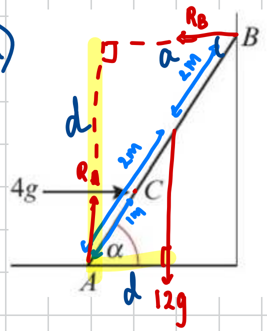
R_B :→ $M = Fd$ = $R_B \times 3\sin 60°$

$$R_B \times 3\sin 60° = 10g \times 1.5\cos 60°$$

$$\Rightarrow R_B = \frac{10 \times 1.5 \times \cos 60°}{3\sin 60°} \times g = \frac{5\sqrt{3}}{3}g$$

c) $R_B = \mu R_A$ $\Rightarrow \mu = \frac{5\sqrt{3}}{3}\cancel{g} \div 10\cancel{g} = \boxed{\frac{\sqrt{3}}{6} = \mu}$

4) a)



Smooth:→ No friction ; Equilibrium:→ $\uparrow = \downarrow$; $\rightarrow = \leftarrow$

$\uparrow = \downarrow \Rightarrow R_A = 12g$ — ① | $\rightarrow = \leftarrow$; $R_B = 4g$ — ②

Take moments at A:

$12g$:→ $M = Fd$ = $12g \times 2\cos\alpha$ acw

R_B :→ $M = Fd$ = $R_B \times 4\sin\alpha$ cw

$4g$:→ $M = Fd$ = $4g \times 1\sin\alpha$ acw

sum of acw = sum of cw

$$24g\cos\alpha + 4g\sin\alpha = R_B \times 4\sin\alpha$$

$$\Rightarrow 24g\cos\alpha = (4g) \times 4\sin\alpha - 4g\sin\alpha$$

$$\Rightarrow 24g\cos\alpha = 16g\sin\alpha - 4g\sin\alpha$$

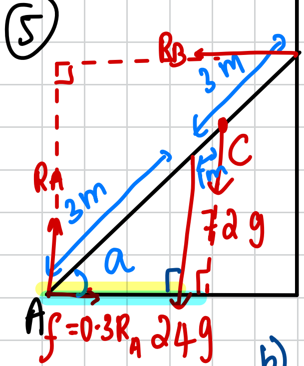
$$= 24\cancel{g}\cos\alpha = 12\cancel{g}\sin\alpha$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{24}{12} \Rightarrow \tan\alpha = 2 \Rightarrow \alpha = 63.4° \text{ (3SF)}$$

b) Assumption means no friction at points of contact with the ladder. Wall and floor are unlikely to be smooth in real life

c) α (a) would be smaller.

⑤

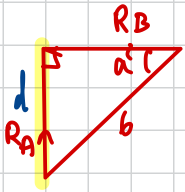


Equilibrium $\Rightarrow$ $\uparrow = \downarrow$; $\rightarrow = \leftarrow$

$\uparrow = \downarrow \Rightarrow R_A = 24g + 72g$ — ①

$\rightarrow = \leftarrow \Rightarrow R_B = f(0.3R_A)$ — ②

a) $R_A = 940.8\text{N} \Rightarrow f = 0.3 \times 940.8 = 282.24.$
$= 282\text{N (3sf)}$



b) Take moments at A:

24g: $\rightarrow M = Fd \Rightarrow 24g \times 3\cos\alpha$ CW

72g: $\rightarrow M = Fd \Rightarrow 72g \times 4\cos\alpha$ CW

R_B : $\rightarrow M = Fd \Rightarrow R_B \times 6\sin\alpha$ ACW

Sum of acw = Sum of cw

$R_B \times 6\sin\alpha = 24g \times 3\cos\alpha + 72g \times 4\cos\alpha$

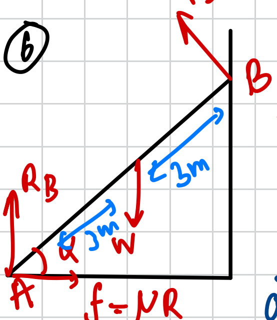
$(R_B = f)$

$f \times 6\sin\alpha = 705.6\cos\alpha + 2822.4\cos\alpha$

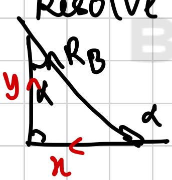
$1693.44\sin\alpha = 3528\cos\alpha$

$\Rightarrow \dfrac{\sin\alpha}{R_B \cdot \cos\alpha} = \dfrac{3528}{1693.44} \Rightarrow \tan\alpha = \dfrac{3528}{1693.44} \Rightarrow \alpha = 64.4°$ (3sf)

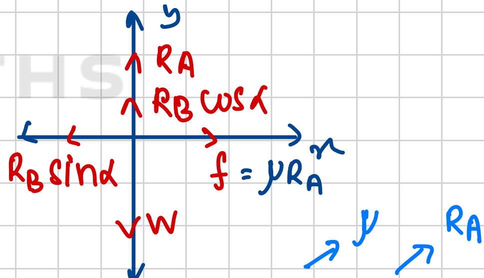
⑥



Resolve R_B:



$y = R_B\cos\alpha$
$x = R_B\sin\alpha$



a) $\leftarrow = \rightarrow$; $R_B\sin\alpha = \mu R_A \Rightarrow R_B = \dfrac{0.8 \times 80}{\sin\alpha}$

$\Rightarrow R_B = \dfrac{0.8 \times 80}{\sin(\tan^{-1}(0.75))} = 106.677\text{N} = 107\text{N (3sf)}$

b) $\uparrow = \downarrow \Rightarrow W = R_A + R_B\cos\alpha \Rightarrow W = 80 + (160.667 \times 0.8)$

$W = 165.33... = 165\text{N (3sf)}$

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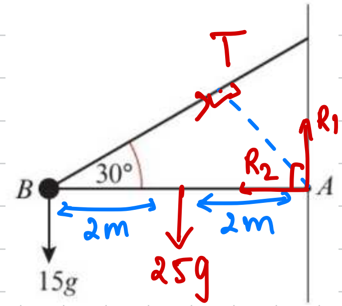
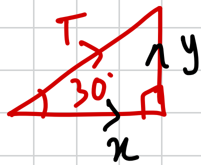
equilibrium = $\uparrow = \downarrow$; $\rightarrow = \leftarrow$

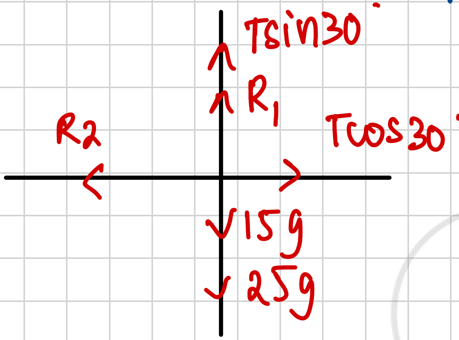


Resolve T:



$y = T\sin 30°$

$x = T\cos 30°$



$\uparrow = \downarrow \Rightarrow T\sin 30° + R_1 = 15g + 25g$

$\Rightarrow \frac{1}{2}T + R_1 = 392$ — 1

$\rightarrow = \leftarrow \Rightarrow R_2 = T\cos 30°$

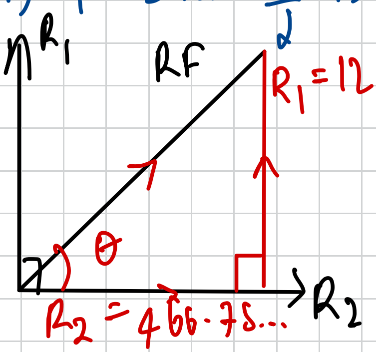
$R_2 = \frac{\sqrt{3}}{2}T$ — 2

a) Take moments at A:

$\sin 30 = \frac{d}{4} \Rightarrow d = 2$

$25g \times 2 + 15g \times 4 = T \times (2) \Rightarrow T = \frac{110g}{2} = 539N$

b) $R_1 = 392 - \frac{1}{2}(T) = 122.5N$ $\quad R_2 = \frac{\sqrt{3}}{2}(539) = 466.78\ldots$



$R_F = \sqrt{R_1^2 + R_2^2} = 49\sqrt{97} \approx 483$ (3sf)

$\tan\theta = \frac{R_1}{R_2} = \frac{122.5}{466.78} \Rightarrow \theta = 14.7°$

$\Rightarrow$ 483 N (3sf) at an angle of 14.7° (3sf) above the horizontal.

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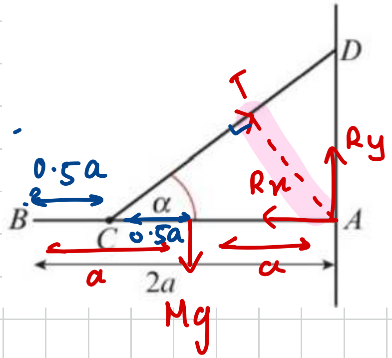
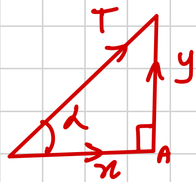


$f = 0.4R$

Resolve T:



$y = T\sin\alpha$
$x = T\cos\alpha$

$\Rightarrow \uparrow = \downarrow \Rightarrow Mg = R_y + T\sin\alpha$ — (1) $\rightarrow = \leftarrow \Rightarrow R_x = T\cos\alpha$ — (2)

Take moments at A:

a) $Mg: M = F \times d \Rightarrow Mga$
$\Rightarrow T: M = F \times d \Rightarrow T \times 1.5a\sin\alpha$
CW = ACW $\Rightarrow Mg\cancel{a} = T \times \frac{3}{2}\sin\alpha\,\cancel{a} \Rightarrow T = \frac{2Mg}{3\sin\alpha}$

b) $R(\rightarrow)$: $R_x = T\cos\alpha \Rightarrow \frac{2Mg}{3\sin\alpha} \times \cos\alpha \Rightarrow \frac{2Mg}{3\tan\alpha}$

$R(\uparrow)$: $R_y + T\sin\alpha = Mg \Rightarrow R_y + \frac{2Mg}{3\cancel{\sin\alpha}} \times \cancel{\sin\alpha} = Mg \Rightarrow Mg - \frac{2}{3}Mg$

$\Rightarrow R_y = \frac{1}{3}Mg$.

$\Rightarrow R_y = 0.4R_x \Rightarrow \frac{1}{3}\cancel{Mg} = 0.4\left(\frac{2\cancel{Mg}}{3\tan\alpha}\right)$

$\Rightarrow \frac{1}{3} = \frac{4}{15}\left(\frac{1}{\tan\alpha}\right) \Rightarrow \frac{\tan\alpha}{3} = \frac{4}{15} \Rightarrow \frac{4}{5} = \tan\alpha$

$\alpha = \arctan\left(\frac{4}{5}\right) \Rightarrow \alpha = 38.7°$ (3sf)

d) The angle would be larger because $\tan\alpha = 2\mu$.