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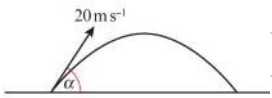
6·4 : Projectile motion formulae

① 

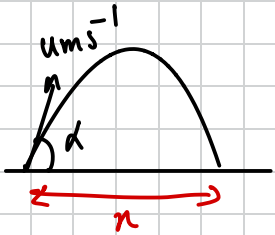
	x	y
s		h
u	$20\cos\alpha$	$20\sin\alpha$
v		0
a	0	$-g$
t		

y: $v^2 = u^2 + 2as$ — greatest height ($v=0$)

$\Rightarrow 0 = 20^2 \times \sin^2\alpha - 2gh$

$\Rightarrow 400\sin^2\alpha = 2gh$

$\Rightarrow \dfrac{200\sin^2\alpha}{g} = h$

② 

	x	y
s	R	0
u	$u\cos\alpha$	$u\sin\alpha$
v		
a	0	$-g$
t	t	t

y: $s = ut + \frac{1}{2}at^2$

$\Rightarrow 0 = u\sin\alpha t - \frac{1}{2}gt^2$

$\Rightarrow 0 \cdot 5gt^2 - u\sin\alpha t = 0$

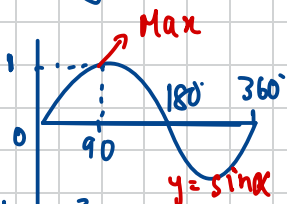
$\Rightarrow t(0 \cdot 5gt - u\sin\alpha) = 0$

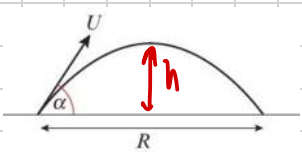
$\Rightarrow t = \dfrac{2u\sin\alpha}{g}$

$\Rightarrow$ x: $s = ut + \frac{1}{2}at^2 \Rightarrow s = u\cos\alpha\left(\dfrac{2u\sin\alpha}{g}\right) \Rightarrow \dfrac{2u^2\sin\alpha\cos\alpha}{g}$

$\rightarrow s = \dfrac{u^2\sin 2\alpha}{g}$

b) Maximum occurs at $2\alpha = 90°$; $\boxed{\alpha = 45°}$



③ 

	x	y
s	R	h
u	$u\cos\alpha$	$u\sin\alpha$
v		0
a	0	$-g$
t	t	t

a) x: $s = ut + \frac{1}{2}at^2$

$\Rightarrow R = u\cos\alpha t + \frac{1}{2}(0)t^2$

$\Rightarrow t = \dfrac{R}{u\cos\alpha}$; $t = \dfrac{R}{2u\cos\alpha}$ at max

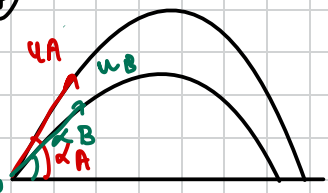
y: $s = ut + \frac{1}{2}at^2 \Rightarrow h = u\sin\alpha\left(\dfrac{R}{2u\cos\alpha}\right) - \frac{1}{2}g\left(\dfrac{R}{2u\cos\alpha}\right)^2$

$\Rightarrow h = \dfrac{R\tan\alpha}{2} - \frac{1}{2}g\left(\dfrac{R^2}{4u^2\cos^2\alpha}\right) \Rightarrow h = \dfrac{R\tan\alpha}{2} - \dfrac{gR^2}{8u^2\cos^2\alpha}$

b) $h = \dfrac{R\tan\alpha}{2} - \dfrac{gR^2}{8u^2} \times \dfrac{1}{\cos^2\alpha} \Rightarrow \dfrac{R\tan\alpha}{2} - \dfrac{gR^2}{8u^2}\sec^2\alpha$

$h = \dfrac{R\tan\alpha}{2} - \dfrac{gR^2}{8u^2}(1 + \tan^2\alpha)$

④



For A:

	↑	y
s	x_A	0
u	$u_A\cos\alpha_A$	$u_A\sin\alpha_A$
v		
a	0	$-g$
t	?	?

For B:

	x	y
s	x_B	0
u	$u_B\cos\alpha_B$	$u_B\sin\alpha_B$
v		
a	0	$-g$
t	?	?

$\Rightarrow$ For A: y: $s=ut+\frac{1}{2}at^2 \Rightarrow 0=u_A\sin\alpha_A t-4.9t^2 \Rightarrow t(4.9t-u_A\sin\alpha_A)=0$

$\Rightarrow t=\frac{2u_A\sin\alpha_A}{g}$; For A: x: $s=ut+\frac{1}{2}at^2 \Rightarrow x_A=u_A\cos\alpha_A t$

$\Rightarrow x_A=u_A\cos\alpha_A\left[\frac{2u_A\sin\alpha_A}{g}\right]=\frac{u_A^2\sin 2\alpha_A}{g}$

$\Rightarrow$ For B: y: $s=ut+\frac{1}{2}at^2 \Rightarrow 0=u_B\sin\alpha_B t-4.9t^2 \Rightarrow t(4.9t-u_B\sin\alpha_B)=0$

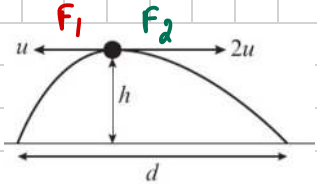
$\Rightarrow t=\frac{2u_B\sin\alpha_B}{g}$; for B: x: $s=ut+\frac{1}{2}at^2 \Rightarrow x_B=u_B\cos\alpha_B t$

$\Rightarrow x_B=u_B\cos\alpha_B\left[\frac{2u_B\sin\alpha_B}{g}\right]=\frac{u_B^2\sin 2\alpha_B}{g}$

$\Rightarrow$ Distance $=x_A-x_B=\frac{u_A^2\sin 2\alpha_A}{g}-\frac{u_B^2\sin 2\alpha_B}{g}$

$\Rightarrow d=\frac{1}{g}\left[u_A^2\sin 2\alpha_A-u_B^2\sin 2\alpha_B\right]$

⑤



For F_1:

	x	y
s	s_x	h
u	u	0
v		
a	0	g
t	t	t

For F_2:

	x	y
s	s_x	h
u	2u	0
v		
a	0	g
t	t	t

y: $s=ut+\frac{1}{2}at^2$

$\Rightarrow h=\frac{1}{2}gt^2$

$\sqrt{\frac{2h}{g}}=t$

For F_1: x: $s=ut+\frac{1}{2}at^2 \Rightarrow s_1=u\times\sqrt{\frac{2h}{g}}$

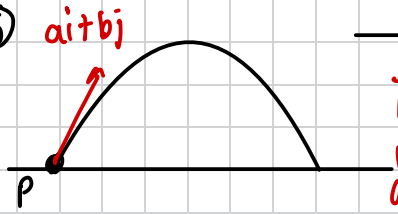
For F_2: x: $s=ut+\frac{1}{2}at^2 \Rightarrow s_2=2u\times\sqrt{\frac{2h}{g}}$

Total distance $=s_1+s_2$

$\Rightarrow u\sqrt{\frac{2h}{g}}+2u\sqrt{\frac{2h}{g}}$

$d=3u\sqrt{\frac{2h}{g}}$

⑥



	x	y
s	d	
u	a	b
v		0
a	0	-g
t	t	t/2

$y: v = u + at$

$\Rightarrow 0 = b - g\frac{t}{2}$

$x: s = ut + \frac{1}{2}at^2$

$d = at \Rightarrow \frac{d}{a} = t$

$\Rightarrow 0 = b - \frac{gd}{2a} \Rightarrow 2ab = gd \Rightarrow d = \frac{2ab}{g}$

b) $200 = \frac{2ab}{g} \Rightarrow 980 = ab$; $\begin{bmatrix} a \\ b \end{bmatrix} = k\begin{bmatrix} 12 \\ 5 \end{bmatrix}$ $a = 12k$, $b = 5k$

$\Rightarrow 60k^2 = 980 \quad k = \frac{7\sqrt{3}}{3} \Rightarrow a = 12\left[\frac{7\sqrt{3}}{3}\right] = 28\sqrt{3}$

$b = 5\left[\frac{7\sqrt{3}}{3}\right] = \frac{35\sqrt{3}}{3}$

Speed: $\sqrt{a^2+b^2} = \sqrt{(28\sqrt{3})^2 + \left(\frac{35\sqrt{3}}{3}\right)^2} = 52.5\ ms^{-1}$ (3sf)

⑦

	x	y
s	s_x	0
u	$u\cos\alpha$	$u\sin\alpha$
v		
a	0	-g
t	t	t

$y: s = ut + \frac{1}{2}at^2$

$0 = u\sin\alpha\, t - 4.9t^2$

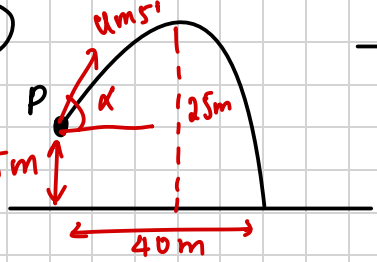
$\Rightarrow t(4.9t - u\sin\alpha) = 0$

$\Rightarrow t = \frac{2u\sin\alpha}{g}$

$\Rightarrow x: s_x = ut + \frac{1}{2}at^2 \Rightarrow S_x = u\cos\alpha\left[\frac{2u\sin\alpha}{g}\right] = \frac{u^2 2\sin\alpha\cos\alpha}{g}$

$\Rightarrow S_x = \frac{u^2\sin 2\alpha}{g}$

(8)



	x	y
s	40	10 (25-15)
u	$u\cos\alpha$	$u\sin\alpha$
v		
a	0	$-g$
t	t	t

x: $s = ut + \frac{1}{2}at^2$

$\Rightarrow 40 = u\cos\alpha\, t + \frac{1}{2}(0)t^2$

$\Rightarrow t = \frac{40}{u\cos\alpha}$ — (1)

$\Rightarrow$ y: $s = ut + \frac{1}{2}at^2 \Rightarrow 10 = u\sin\alpha\left[\frac{40}{u\cos\alpha}\right] + \frac{1}{2}(-g)\left(\frac{40}{u\cos\alpha}\right)^2$

$\Rightarrow 10 = 40\tan\alpha - \frac{1}{2}g\left(\frac{1600}{u^2\cos^2\alpha}\right) \Rightarrow 10 = 40\tan\alpha - \frac{800g}{u^2\cos^2\alpha}$

$\Rightarrow \frac{800g}{u^2} \times \frac{1}{\cos^2\alpha} - 40\tan\alpha - 10 = 0 \Rightarrow \frac{800g}{u^2}\sec^2\alpha - 40\tan\alpha - 10 = 0$

$\Rightarrow \frac{800g}{u^2}(1+\tan^2\alpha) - 40\tan\alpha + 10 = 0$

b) $u = 40\ ms^{-1} \Rightarrow \frac{800g}{40^2}(1+\tan^2\alpha) - 40\tan\alpha + 10 = 0$

$\Rightarrow 4.9(1+\tan^2\alpha) - 40\tan\alpha - 10 = 0 \Rightarrow 4.9 + 4.9\tan^2\alpha - 40\tan\alpha + 10 = 0$

$\Rightarrow 4.9\tan^2\alpha - 40\tan\alpha + 10 = 0$

$\tan\alpha = 7.772\ldots$ $\quad \tan\alpha = 0.39125\ldots$

$\alpha = \arctan(7.772\ldots)$ $\quad \alpha = \arctan(0.39125\ldots)$

$\alpha = 82.7^\circ$ (3sf) $\quad \alpha = 21.4^\circ$ (3sf)