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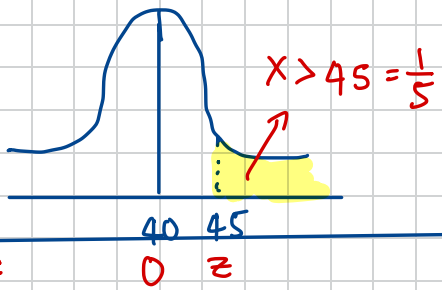
Set A

BRONZE:

Given: $X \sim N(40, \sigma^2)$

a)



b) $P(Z > z) = 0.2 \Rightarrow z = 0.8416$

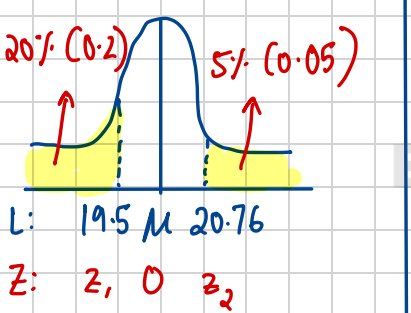
$z = \frac{X-\mu}{\sigma} \Rightarrow 0.8416 = \frac{45-40}{\sigma}$

$\Rightarrow \sigma = 5.94$ (3sf)

c) $X \sim N(40, 5.94^2\ldots) \Rightarrow P(X > 42) = 0.368$ (3sf)

SILVER:

Given: $L \sim N(\mu, \sigma^2)$



a) $P(Z < z_1) = 0.2 \Rightarrow z_1 = -0.841$

$z = \frac{X-\mu}{\sigma} \Rightarrow -0.841 = \frac{19.5-\mu}{\sigma}$

$\Rightarrow -0.841\sigma + \mu = 19.5 \quad -(1)$

$P(Z > z_2) = 0.05 \Rightarrow z_2 = 1.6448$

$z = \frac{X-\mu}{\sigma} \Rightarrow 1.6448 = \frac{20.76-\mu}{\sigma}$

$\Rightarrow 1.6448\sigma + \mu = 20.76 \quad -(2)$

Solve simultaneously eq (1) & (2)

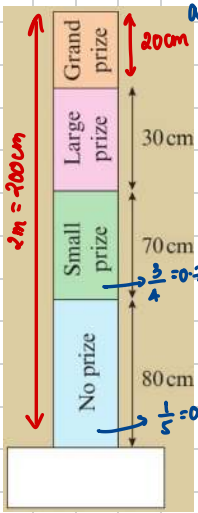
$\Rightarrow \sigma = 0.507$ (3sf) $\quad \mu = 19.9$ (3sf)

b) $L \sim N(19.9, 0.507^2) \Rightarrow P(19.75 < L < 20.5) = 0.5071$

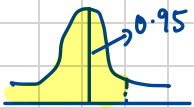
$\Rightarrow \frac{0.5071}{1} \times 100 \Rightarrow 50.7\%$ (3sf)

GOLD:

$X \sim N(\mu, \sigma^2)$



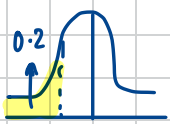
a) $P(X<150) = 0.2 + 0.75 \Rightarrow P(X<150) = 0.95$



$P(Z<z) = 0.95 \Rightarrow z = 1.6448 \Rightarrow Z = \frac{X-\mu}{\sigma}$

$\Rightarrow 1.6448 = \frac{150-\mu}{\sigma} \Rightarrow 1.6448\sigma + \mu = 150$ - (1)

$P(X<80) = \frac{1}{5} \Rightarrow P(X<80) = 0.2$



$P(Z<z) = 0.2 \Rightarrow z = -0.841 \Rightarrow z = \frac{X-\mu}{\sigma}$

$\Rightarrow -0.841 = \frac{80-\mu}{\sigma} \Rightarrow -0.841\sigma + \mu = 80$ - (2)

Solve simultaneously eq (1) & (2)

$\mu = 104$ cm (3SF) $\quad \sigma = 28.2$ cm (3SF)

b) $n = 300$

Probability of winning large prize

$\Rightarrow [P(X<180) - P(X<150)] = 0.9966 - 0.9499 = 0.0467$

$\Rightarrow 300 \times 0.0467 = 14$ prizes

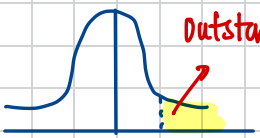
c) 200 cm is more than 3 standard deviations from the mean, so according to the model the probability of getting a value > 200 cm is very small so the model is still suitable.

Set B

BRONZE:

Given: $X \sim N(50, 10^2)$; $n = 8$; $Z \sim N(0, 1^2)$

a)



$Z = \frac{X - \mu}{\sigma} \Rightarrow Z = \frac{65 - 50}{10} = 1.5$

$\Rightarrow P(Z > 1.5) = 0.0668$ (3sf)

$Y \sim B(n, p) \Rightarrow Y \sim B(8, 0.0668)$

i) $P(Y \geq 1) = 0.4248$ (4sf)

ii) $P(Y = 2) = 0.0825$ (3sf)

iii) $P(Y < 3) = P(Y \leq 2) = 0.9871$ (4sf)

b) 100% is 5 standard deviations from the mean so virtually impossible to occur as an outcome, so the interviewer's claim is not valid

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SILVER:

Given: $X \sim N(38, 6.4^2)$

a) i) $P(X > 43) = 0.2713$

ii) $P(X > 45) = 0.1370$

b) If $X > 45$ then X must be greater than 43, so for be true we only need $X > 45$

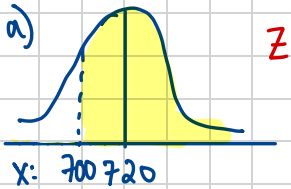
c) $P(X > 45 \mid X > 43) \Rightarrow \frac{P(X > 43) \cap P(X > 45)}{P(X > 43)} = \frac{P(X > 45)}{P(X > 43)}$

$\Rightarrow \frac{0.13763231}{0.21732773} = 0.6305$ (4sf)

GOLD:

Given: $X \sim N(720, 150^2)$; $Z \sim N(0, 1^2)$

a)



$Z = \frac{X - \mu}{\sigma} \Rightarrow Z = \frac{700 - 720}{150} \Rightarrow z = \frac{-2}{15}$

$P(Z > \frac{-2}{15}) = 0.553$ (3sf)

$\Rightarrow Y \sim B(6, 0.553) \Rightarrow P(Y \geq 5) = 0.1673$

b) $Y \sim B(6, 0.553) \Rightarrow P(Y = 6) = 0.9835$

$\Rightarrow Y \sim B(6, 0.9835) = 0.9049...$

$\Rightarrow P(X > 700) = 0.553 \Rightarrow (0.553)^6 = 0.02859...$

$\Rightarrow \frac{0.02859}{0.9049...} = 0.0316$ (3sf)

c) $P(X > 700)$ $Y \sim B(6, 0.553)$

$P(X > 300)$ $Y \sim B(6, 0.9974)$

$P(X > 400)$ $Y \sim B(6, 0.9835)$

SOLD: $Y \sim B(2, 0.9835) \Rightarrow P(Y = 2) = 0.9835$

$\frac{P(Y = 2 \mid X > 700)}{P(Y = 2 \mid X > 400)} = \frac{0.3058}{0.9672} = 0.3161$

4 NEW: $P(X > 300) = 0.9974$

$Y \sim B(6, 0.9974) \Rightarrow P(Y = 4) = 0.9896$

$\Rightarrow 0.3161 \times 0.9896 = 0.3128$