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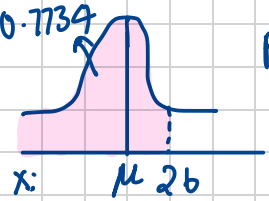
3.5 Finding μ and σ

① $X \sim N(\mu, 4^2)$; $P(X < 26) = 0.7734$; $Z \sim N(0, 1^2)$

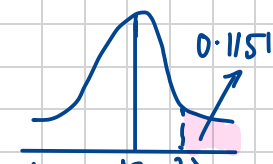


$P(Z < z) = 0.7734 \Rightarrow z = 0.75$

$z = \frac{X - \mu}{\sigma} \Rightarrow 0.75 = \frac{26 - \mu}{4} \Rightarrow 3 = 26 - \mu$

$\Rightarrow \mu = 23.0$ (3SF)

② $X \sim N(17, \sigma^2)$; $P(X > 20) = 0.1151$; $Z \sim N(0, 1^2)$

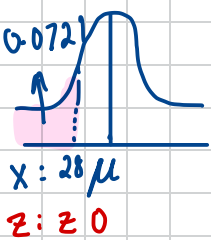


$P(Z > z) = 0.1151 \Rightarrow z = 1.1998$

$z = \frac{X - \mu}{\sigma} \Rightarrow 1.1998 = \frac{20 - 17}{\sigma} \Rightarrow \sigma = \frac{3}{1.1998}$

$\Rightarrow \sigma = 2.50$ (3SF)

③ $Y \sim N(\mu, 30)$; $P(Y < 28) = 0.0721$; $Z \sim N(0, 1^2)$

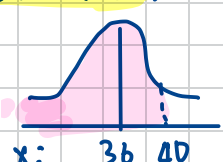


$P(Z < z) = 0.0721 \Rightarrow z = -1.46$

$z = \frac{X - \mu}{\sigma} \Rightarrow \frac{28 - \mu}{\sqrt{30}} = -1.46 \Rightarrow \mu = -1.46\sqrt{30} - 28$

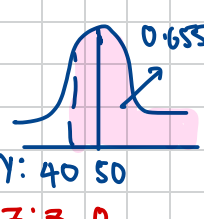
$\Rightarrow \mu = 36.0$ (3SF)

b) $P(Y < 40) =$



$\Rightarrow 0.7674$

④ $Y \sim N(50, \sigma^2)$; $P(Y > 40) = 0.6554$; $Z \sim N(0, 1^2)$

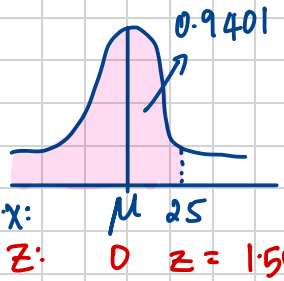


$P(Z > z) = 0.6554 \Rightarrow z = -0.3999$

$z = \frac{X - \mu}{\sigma} \Rightarrow \frac{40 - 50}{\sigma} = -0.3999 \Rightarrow \sigma = \frac{10}{0.399} = 25.0$ (3SF)

b) $P(45 < X < 50) = 0.0793$

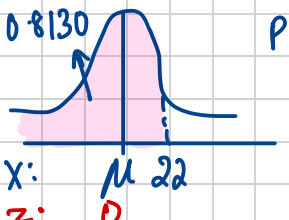
⑤ $X \sim N(\mu, \sigma^2)$; $Z \sim N(0, 1^2)$; $P(X<25) = 0.9401$; $P(X<22) = 0.8130$



$P(Z<z) = 0.9401 \Rightarrow z = 1.5556$

$Z = \frac{X-\mu}{\sigma} \Rightarrow 1.5556 = \frac{25-\mu}{\sigma}$

$\Rightarrow 1.5556\sigma + \mu = 25$ —①



$P(Z<z) = 0.8130 \Rightarrow z = 0.889$

$Z = \frac{X-\mu}{\sigma} \Rightarrow 0.889 = \frac{22-\mu}{\sigma}$

$\Rightarrow 0.889\sigma + \mu = 22$ —②

Solve simultaneously eq ① and ②

$1.5556\sigma + \mu = 25$

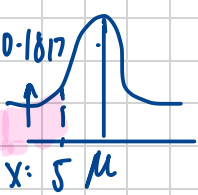
$0.889\sigma + \mu = 22$

$0.6666\sigma = 3$

$\sigma = 4.50$ (3sf)

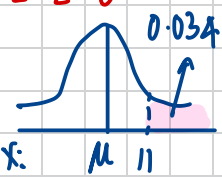
$\mu = 25 - 1.5556(\sigma) \Rightarrow \mu = 18.0$ (3sf)

⑥ $Y \sim N(\mu, \sigma^2)$; $P(Y<5) = 0.1817$; $P(Y>11) = 0.0345$; $Z \sim N(0, 1^2)$



$P(Z<z) = 0.1817 \Rightarrow z = -0.908$

$z = \frac{X-\mu}{\sigma} \Rightarrow -0.908 \Rightarrow \frac{5-\mu}{\sigma} \Rightarrow \mu - 0.908\sigma = 5$ —①



$P(Z>z) = 0.0345 \Rightarrow z = 1.8184$

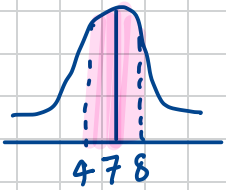
$z = \frac{X-\mu}{\sigma} \Rightarrow 1.8184 = \frac{11-\mu}{\sigma} \Rightarrow 1.8184\sigma + \mu = 11$ —②

Solve simultaneously eq ① and ②

$\mu - 0.908\sigma = 5$ —①

$1.8184\sigma + \mu = 11$ —②

$\mu = 7.00$ (3.sf) $\sigma = 2.20$ (3.sf)

b) $P(4 < Y < 8) \Rightarrow$ 

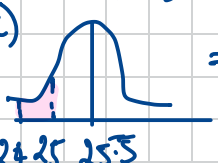
$\Rightarrow 0.5889$

⑦ a) The lengths tend to be symmetrical around a mean. The distribution of lengths will form a bell curve.

b) $\mu = \frac{Q_1 + Q_3}{2} \Rightarrow \mu = \frac{24.9 + 26.1}{2} \Rightarrow \mu = \frac{51}{2} = 25.5\,cm$

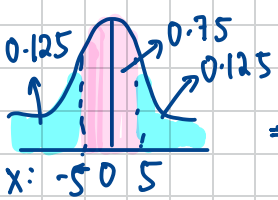
$IQR = Q_3 - Q_1 = 26.1 - 24.9 = 1.2 \; ; IQR \approx 1.35\sigma$

$\Rightarrow \sigma = \frac{1.2}{1.35} \Rightarrow \sigma = 0.890\,(3sf)$

c) 

$\Rightarrow P(24 < x < 25) = 0.2412$

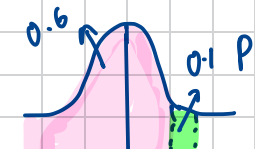
⑧ $X \sim N(0, \sigma^2) \; ; P(-5 < X < 5) = 0.75$



$P(Z < z) = 0.125 \Rightarrow z = -1.15 \Rightarrow -1.15 = \frac{-5}{\sigma}$

$\Rightarrow \sigma = \frac{5}{1.15} \Rightarrow \sigma = 4.35\,(3sf)$

⑨ $X \sim N(1.45, \sigma^2) \; ; P(X > 2a) = 0.3 \; ; \; P(X < a) = 0.6$



$P(Z < z) = 0.6 \Rightarrow z = 0.2533 \Rightarrow 0.2533 = \frac{a - 1.45}{\sigma}$

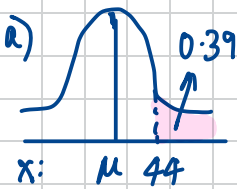
$\Rightarrow 0.2533\sigma - a = -1.45 \; -①$

$P(Z < z_1) = 0.7 \Rightarrow z = 0.5244 \Rightarrow 0.5244 = \frac{2a - 1.45}{\sigma}$

$\Rightarrow 0.5244\sigma - 2a = -1.45 \; -②$

Solve eq ① & ② simultaneously $\Rightarrow \sigma = 81.9\,(3sf) \quad a = 22.2\,(3sf)$

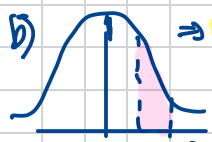
(10) $X \sim N(\mu, 7.2^2) \Rightarrow P(X>44) = 0.39$

a) 

$P(Z>z) = 0.39 \Rightarrow z = 0.2793$

$Z = \frac{X-\mu}{\sigma} \Rightarrow 0.2793 = \frac{44-\mu}{7.2} \Rightarrow \mu = 44 - 0.2793(7.2)$

$\Rightarrow \mu = 41.98904 \Rightarrow \mu = 42.0 \text{ (3sf)}$

b) 

$\Rightarrow P(48<X<53) = 0.1388$

c) $z = \frac{X-\mu}{\sigma} \Rightarrow \frac{50-42}{7.2} = \frac{8}{7.2} = \frac{10}{9}$

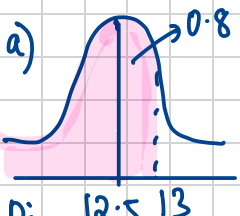
$\Rightarrow P(Z<\frac{10}{9}) = 0.8667$

$\Rightarrow P(X>50) = 1 - P(Z<\frac{10}{9}) \Rightarrow 1 - 0.8667 \doteq 0.1333$

Since the drivers are chosen independently, the probability that all three score more the 50 points is:

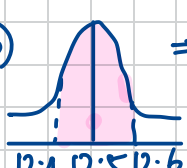
$P(X>50)^3 = (0.1333)^3 \Rightarrow 0.0023 \text{ (4dp)}$

(11) $D \sim N(12.5, \sigma)$; $P(D<13) = 0.8$

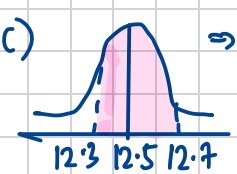
a) 

$P(Z<z) = 0.8 \Rightarrow z = 0.8416$

$Z = \frac{X-\mu}{\sigma} \Rightarrow \frac{13-12.5}{\sigma} = 0.8416 \Rightarrow \sigma = 0.5941 \text{ (4s.f)}$

b) 

$\Rightarrow P(12.4<D<12.6) = 0.1337$

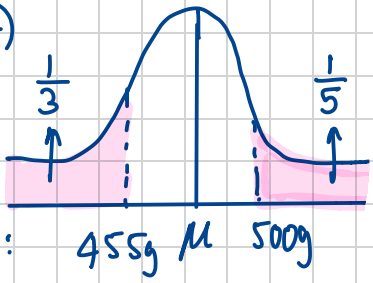
c) 

$\Rightarrow P(12.3<D<12.7) = 0.2636$

$D \sim B(10, 0.2636) \Rightarrow P(D \geq 5) = 0.0951 \text{ (3SF)}$

(12) $M \sim N(\mu, \sigma)$; $P(M<455) = \frac{1}{3}$; $P(M>500) = \frac{1}{5}$

a)



b) $z = \frac{x-\mu}{\sigma}$ for 455g: $P(z<z) = 1/3$

$\Rightarrow z = -0.43 \Rightarrow -0.43 = \frac{455-\mu}{\sigma}$

$\Rightarrow -0.43\sigma + \mu = 455$ — (1)

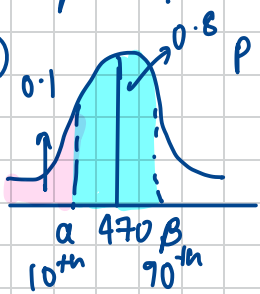
for 500g: $P(z>z) = 1/5 \Rightarrow z = 0.8416$

$\Rightarrow 0.8416 = \frac{500-\mu}{\sigma} \Rightarrow 0.8416\sigma + \mu = 500$ (2)

Solve eqn (1) & (2) simultaneously

$\Rightarrow \mu = 470$ (3sf) $\Rightarrow \sigma = 38.4$ (3sf)

c)



$P(a<M<\beta) = 0.8 \Rightarrow a = 424.63$, $\beta = 515.36$

$\Rightarrow \beta - a = 515.36 - 424.63 = 90.7$ (3sf)