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3.1 - Normal distribution

① a) Not suitable : number of children is discrete

b) Not suitable : the distribution is unlikely to be symmetrical

c) Suitable, e.g. likely to be similar times

d) Suitable, e.g. likely to be similar length.

②

③ The distribution shown does not have a bell-shaped curve.

④ a) $P(6.1 < X < 7.9) = 0.6826$ (68%) b) $P(5.2 < X < 8.8) = 0.9544$ (95%)

⑤ $M \sim N(5, \sigma^2) \Rightarrow P(2.6 < X < 7.4) = 0.95 \Rightarrow P(X < 2.6) = \frac{1 - 0.95}{2} = 0.025$

$P\left(Z < \frac{2.6-5}{\sigma}\right) = 0.025 \Rightarrow \frac{2.6-5}{\sigma} = -1.96 \Rightarrow \sigma = \frac{3.6-5}{-1.96} \Rightarrow \sigma = \frac{-2.4}{-1.96} \approx 1.2$ (2sf)

⑥ The area between the mean μ and one standard deviation (σ) is 68% (approx)

$\Rightarrow$ area to the left of 34.7kg = 100% - 68% - 32% (area to the right is 32% ÷ 2 = 16%)

$\mu + 2.2 = 34.7$ $\mu = 34.7 - 2.2 = 32.5$kg

⑦ $P(Y \geq 40.5) = 0.84 \Rightarrow 1 - \Phi\left(\frac{40.5-\mu}{\sigma}\right) = 0.84$

$\Rightarrow \Phi\left(\frac{40.5-\mu}{\sigma}\right) = 0.16 \Rightarrow \frac{40.5-\mu}{\sigma} \approx -1 \quad \mu - \sigma = 40.5$

$P(Y \geq 45) = 0.025 \Rightarrow 1 - \Phi\left(\frac{45-\mu}{\sigma}\right) = 0.025 \Rightarrow \mu + 1.96\sigma = 45$

$\Rightarrow \mu = 40.5 + \sigma$ —① $\Rightarrow 40.5 + \sigma + 1.96\sigma = 45$

$\Rightarrow 2.96\sigma = 4.5 \Rightarrow \sigma = 1.520\ldots = 1.5g\ (1dp)$

$\Rightarrow \mu = 40.5 + 1.52 = 42.02\ldots = 42g$

⑧ $X \sim N(2.5, 0.15)$ a) $P(T < 2.5) = 0.5$ b) $P(2.35 < T < 2.5) = 0.34$

c) $P(T > 2.8) = 0.02275\ldots = 0.023$

⑨ Mean: $\frac{\bar{x}}{n} = 11.66451613 \quad \sigma = 2.6$

a) i) $P(T < 11.6) = \frac{16}{31} = 0.516\ (3sf)$ ii) $P(9 < T < 14.2) = \frac{19}{31} = 0.613\ (3sf)$

iii) $P(6.5 < T < 16.8) = \frac{30}{31} = 0.968\ (3sf)$

b) Using the normal distribution the probabilities are 0.5, 0.68, 0.95. The probabilities are close, so the model is suitable