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3.7: Hypothesis testing with the normal distribution

① a) The mean of the 50 distances in the sample.

b) $H_0: \mu = 200$ $H_1: \mu > 200$ (new golf club has increased the distance)

c) $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \Rightarrow \bar{X} \sim N\left[200, \frac{15^2}{50}\right]$

d) Critical region: $\rightarrow P(\bar{X} > a) = 0.05 \Rightarrow a = 203.48$
$\Rightarrow \bar{X} > 203.5$ (3sf)

② $n = 30$; $X \sim N(\mu, 5.7^2)$; $\bar{x} = 16.6$
Given: $H_0: \mu = 18$ $H_1: \mu < 18$; SL : 10% (0.1)

a) Rejected if $P(\bar{X} \le \bar{x}) < 0.1$

b) $\bar{X} \sim N\left(18, \frac{5.7^2}{30}\right) \Rightarrow P(\bar{X} \le 16.6) = 0.0893$ (4sf).

$\Rightarrow 0.0893 < 0.1 \Rightarrow$ Reject H_0 as it falls in the critical region and accept H_1.

③ $X \sim N(45, 6^2)$; $n = 50$; $\bar{X} \sim N\left(45, \frac{6^2}{50}\right)$

a) $H_0: \mu = 45$ $H_1: \mu > 45$

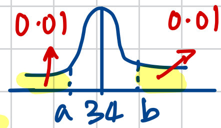
Critical region: $\rightarrow P(\bar{X} > a) = 0.05 \Rightarrow a = 46.395 \Rightarrow \bar{X} > 46.4$ hours

b) $\bar{x} = 50$ hours $\Rightarrow \bar{x} = 50 > 46.4$.
$\therefore$ There is evidence that the new battery is an improvement

④ a) A one-tailed test is used to test when it is claimed that the mean has increased or decreased. Whereas, a two-tailed test is used when it is thought that the mean has changed in either direction.

b) $n = 25$; $X \sim N(\mu, 1.1^2)$; $\bar{X} \sim N\left(34, \frac{1.1^2}{25}\right)$; SL: 1% (0.01)

Given: $H_0: \mu = 34$; $H_1: \mu \neq 34$



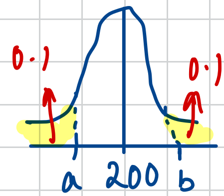
$P(\bar{X} < a) = 0.01 \Rightarrow a = 33.488 \Rightarrow \bar{X} < 33.5$

$P(\bar{X} > b) = 0.01 \Rightarrow b = 34.511 \Rightarrow \bar{X} > 34.5$ (3sf)

c) 33.7 is not in the critical region, so accept H_0.

⑤ $X \sim N(200, 6^2)$; $n = 20$; $\bar{X} \sim N\left[200, \frac{6^2}{20}\right]$; $\bar{x} = 201.5$

$H_0: \mu = 200$; $H_1: \mu \neq 200$



Critical region: $\longrightarrow$

$P(\bar{X} < a) = 0.1 \Rightarrow a = 198.28 \Rightarrow \bar{X} < 198.3$

$P(\bar{X} > b) = 0.1 \Rightarrow b = 201.71 \quad \bar{X} > 201.7$

$\bar{x} = 201.5$, which does not fall in critical region, so accept H_0. There is insufficient evidence of a change in the mean diameter of bowls produced by the automated pottery wheel.

⑥ $X \sim N(5.9, 0.84^2)$; $n = 16$; $\bar{x} = 6.6$; $\bar{X} \sim N\left[5.9, \frac{0.84^2}{16}\right]$; SL = 1%

$H_0: \mu = 5.9$ $\quad H_1: \mu > 5.9$

$P(\bar{X} > 6.6) = 4.2 \times 10^{-4} \Rightarrow 0.00042 < 0.01$

$\therefore$ Therefore, Reject H_0. There is sufficient evidence to support the zoologist's claim

(4) $X \sim N(12, 3.6^2)$; $n = 25$; $\bar{X} \sim N\left[12, \frac{3.6^2}{25}\right]$; $SL = 0.01$ (1%)

a) $H_0 : \mu = 12$ $H_1 : \mu < 12$

b) Critical region: $\rightarrow$ $P(\bar{X} \leq a) = 0.01 \Rightarrow a = 10.325$

$\Rightarrow$ 10.3 minutes (3SF)

Given: SL : 10% (0.1) ; $\bar{x} = 12.3$

c) $12.3 > 12$ so $P(\bar{X} < \bar{x})$ will be greater than 50%. There is insufficient evidence to reject H_0, so conclude the waiting times have not reduced.