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10.2 Forces as Vectors

1) a) $(2i - 4j) + (-i + 5j) = (i + j)N$

b) $\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}N$

c) $(i - 2j) + (4i - 2j) + (-2i + 6j) = (3i + 2j)N$

d) $\begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 \\ -8 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}N$

2) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = 0$

$$\begin{pmatrix} 5+a \\ 1+b \end{pmatrix} = 0$$

$$\begin{aligned} a &= -5 \\ b &= -1 \end{aligned}$$

3) $\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ -6 \end{pmatrix} + \begin{pmatrix} -4 \\ a \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = 0$

$$\begin{pmatrix} -3+a \\ 1+b \end{pmatrix} = 0$$

$$\begin{aligned} a &= 3 \\ b &= -1 \end{aligned}$$

4) $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 8 \\ 2b \end{pmatrix} + \begin{pmatrix} -3a \\ -8 \end{pmatrix} = 0$

$$\begin{pmatrix} 8-2a \\ -8+3b \end{pmatrix} = 0$$

$$8 - 2a = 0$$

$$2a = 8$$

$$a = 4$$

$$-8 + 3b = 0$$

$$3b = 8$$

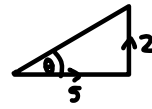
$$b = \frac{8}{3}$$

$$a = 4$$

$$b = \frac{8}{3}$$

5) a) i) $\sqrt{5^2 + 2^2} = \sqrt{29}N$

ii)

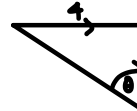


$$\tan \theta = \frac{2}{5}$$

$$\tan^{-1}\left(\frac{2}{5}\right) = 21.8^\circ$$

b) i) $\sqrt{(4)^2 + (-3)^2} = 5N$

ii)

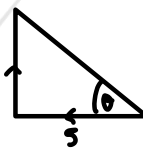


$$\tan \theta = \frac{3}{4}$$

$$\tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$$

c) i) $\sqrt{(-5)^2 + (4)^2} = \sqrt{41}N$

ii)



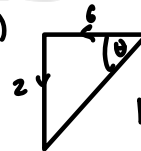
$$\tan \theta = \frac{4}{5}$$

$$\tan^{-1}\left(\frac{4}{5}\right) = 38.65^\circ$$

$$180 - 38.65 = 141.3^\circ$$

d) i) $\sqrt{(-6)^2 + (-2)^2} = 2\sqrt{10}N$

ii)



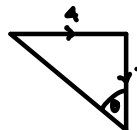
$$\tan \theta = \left(\frac{2}{6}\right) = 18.4$$

$$180 - 18.4 = 161.6^\circ$$

6) a) i) $(5i - 3j) + (2i + j) + (-3i - 5j) = (4i - 7j)N$

ii) $\sqrt{(4)^2 + (-7)^2} = \sqrt{65}N$

iii)



$$\tan \theta = \frac{7}{4}$$

$$\tan^{-1}\left(\frac{7}{4}\right) = 29.7^\circ$$

$$180 - 29.7 = 150.3^\circ$$

$$b) i) (3i + 2j) + (-4i - 5j) + (4i + 2j) = (3i - j)N \quad b) (16i + 4j) + (-4i - 16j) = (12i - 12j)$$

$$ii) \sqrt{(3)^2 + (-1)^2} = \sqrt{10}N$$

$$\sqrt{(12)^2 + (-12)^2} = 12\sqrt{2}N$$

$$iii) \begin{array}{l} \text{Diagram: A right-angled triangle with a horizontal base of length 3 and a vertical height of length 1. The angle \theta is at the bottom-left vertex.} \\ \tan\left(\frac{3}{1}\right) \\ \tan^{-1}(3) = 71.6^\circ \end{array}$$

$$c) \begin{array}{l} \text{Diagram: A right-angled triangle with a horizontal base of length 12 and a vertical height of length 12. The angle \theta is at the bottom-left vertex.} \\ \tan\theta = \frac{12}{12} \\ \tan^{-1}(1) = 45^\circ \end{array}$$

$$180 - 71.6 = 108.4^\circ$$

$$180 - 45 = 135^\circ$$

$$7) a) \begin{pmatrix} 2a \\ b \end{pmatrix} + \begin{pmatrix} b \\ -a \end{pmatrix} + \begin{pmatrix} -11 \\ 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2a + b - 11 \\ -a + b + 1 \end{pmatrix} = 0$$

$$\begin{array}{r} 2a + b = 11 \\ -a + b = -1 \end{array} \quad \begin{array}{r} -2a + b = 11 \\ -a + b = -1 \end{array}$$

$$\frac{3a = 12}{a = 4}$$

$$\begin{array}{r} 2(4) + b = 11 \\ 8 + b = 11 \\ b = 3 \end{array}$$

$$\begin{array}{l} a = 4 \\ b = 3 \end{array}$$

$$b) \begin{pmatrix} 8 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 11 \\ -1 \end{pmatrix}$$

$$\sqrt{(11)^2 + (-1)^2} = \sqrt{122}N$$

$$c) \begin{array}{l} \text{Diagram: A right-angled triangle with a horizontal base of length 11 and a vertical height of length 1. The angle \theta is at the bottom-left vertex.} \\ \tan\theta = \left(\frac{1}{11}\right) \end{array}$$

$$\tan^{-1}\left(\frac{1}{11}\right) = 5.19 \approx 5^\circ$$

$$9) R = (-4i + 7j) + (2ai - aj)$$

$$R = k(3i + j)$$

$$(-4i + 7j) + (2ai - aj) = k(3i + j)$$

$$(-4i + 7j) + (2ai - aj) = 3ki + kj$$

$$\begin{array}{r} -4 + 2a = 3k \\ 7 - a = k \times 2 \end{array}$$

$$\begin{array}{r} -4 + 2a = 3k \\ 14 - 2a = 2k \\ \hline 10 = 5k \\ k = 2 \end{array}$$

$$-4 + 2a = 3(2)$$

$$-4 + 2a = 6$$

$$2a = 10$$

$$a = 5$$

$$8) a) (4ai + aj) + (-2bi - 8bj) + (-12i + 12j) = 0$$

$$\begin{pmatrix} 4a - 2b - 12 \\ a - 8b + 12 \end{pmatrix} = 0 \quad \begin{array}{r} -16a - 8b = 48 \\ a - 8b = -12 \end{array} \quad \begin{array}{r} 2b = 4 \\ b = 2 \end{array}$$

$$\frac{15a = 60}{a = 4}$$

$$\begin{array}{r} 4a - 2b = 12 \times 4 \\ a - 8b = -12 \end{array}$$

$$\begin{array}{r} a = 4 \\ 4(4) - 2b = 12 \\ 16 - 2b = 12 \end{array}$$

$$\begin{array}{l} a = 4 \\ b = 2 \end{array}$$