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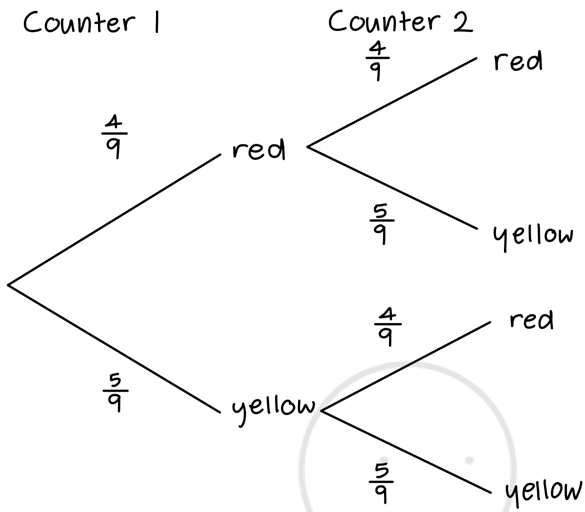
5.4 Tree Diagrams

1.

a)



b)

i) Probability that both counters are red:

P(both red) = P(red) × P(red)

$\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

ii) Probability that only one counter is red:

P(one red) = P(red) × P(yellow)

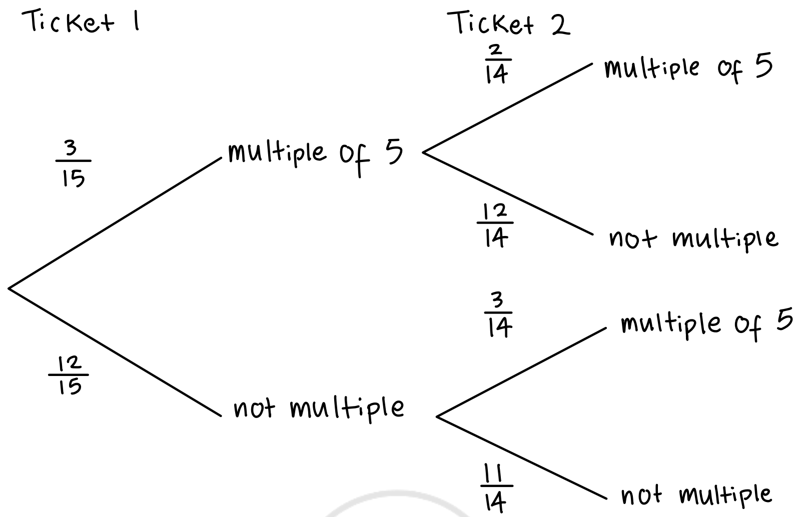
$\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$ (red × yellow)

$\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$ (yellow × red)

$\frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

2.

a)



b)

i) Probability of choosing two winning tickets:

P(2 wins) = P(mult. of 5) × P(mult. of 5)

$$\frac{3}{15} \times \frac{2}{14} = \frac{1}{35}$$

ii) Probability of choosing exactly one winning ticket:

P(1 win) = P(mult. of 5) × P(not mult.)

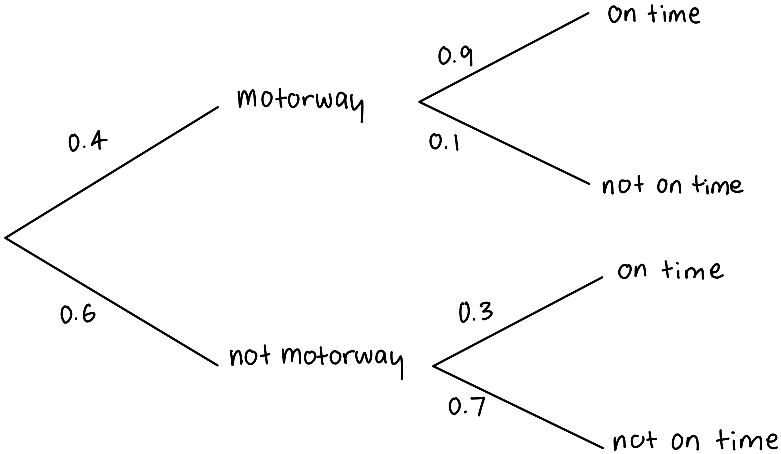
$$\frac{3}{15} \times \frac{12}{14} = \frac{36}{210} = \frac{6}{35}$$ (mult. of 5 × not mult.)

$$\frac{12}{15} \times \frac{3}{14} = \frac{36}{210} = \frac{6}{35}$$ (not mult. × mult. of 5)

$$\frac{6}{35} + \frac{6}{35} = \frac{12}{35}$$

3.

a)



b) P(on time) is different for each of the two initial choices so the events 'takes the motorway' and 'arrives on time' are not independent

c) Probability that the coach doesn't arrive on time:

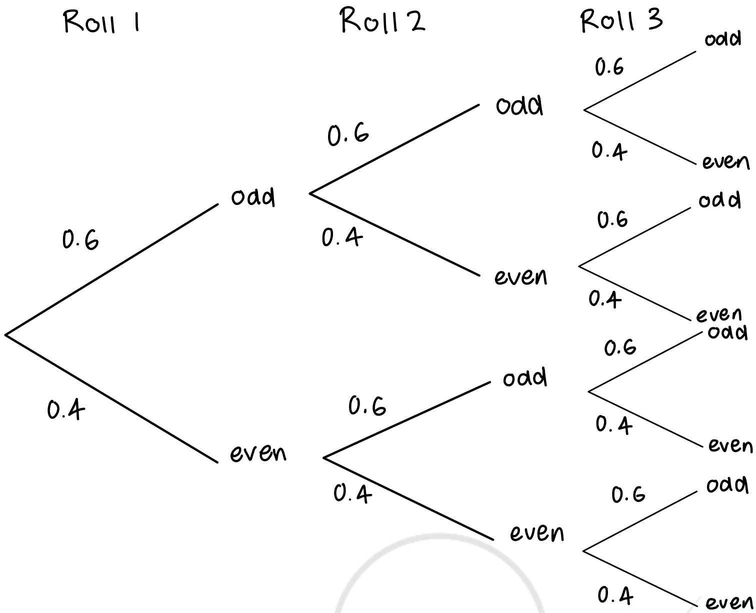
P(not on time) = P(motorway) × P(not on time) + P(not motorway) + P(not on time)

$0.4 \times 0.1 = 0.04$ (motorway × not on time)

$0.6 \times 0.7 = 0.42$ (not motorway × not on time)

$0.04 + 0.42 = 0.46$

4.



a) P(3 odd numbers) = P(odd on roll 1) × P(odd on roll 2) × P(odd on roll 3)
$= 0.6 \times 0.6 \times 0.6$
$= 0.216$

b) P(1 odd number) = P(odd) × P(even) × P(even)
$= 0.6 \times 0.4 \times 0.4$
$= 0.096$

$= 0.4 \times 0.6 \times 0.4$
$= 0.096$

$= 0.4 \times 0.4 \times 0.6$
$= 0.096$

$0.096 + 0.096 + 0.096 = 0.288$

c) P(3 odd numbers) $= 0.6 \times 0.6 \times 0.6 = 0.216$
P(3 even numbers) $= 0.4 \times 0.4 \times 0.4 = 0.064$

P(3 odd or 3 even) $= 0.216 + 0.064 = 0.28$

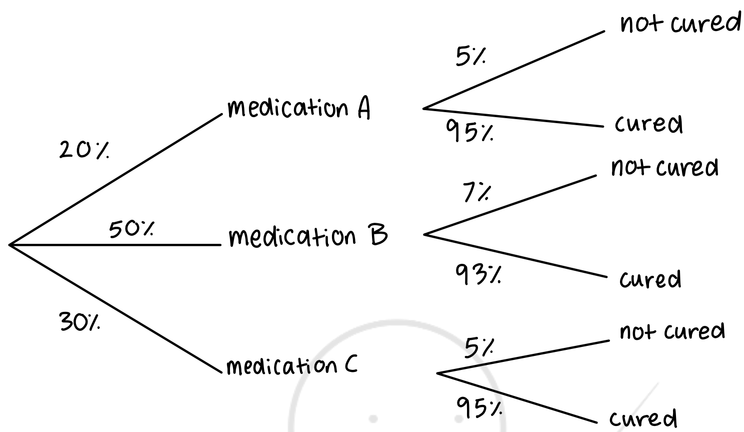
P(3 odd or 3 even in all 3 repetitions) $= 0.28 \times 0.28 \times 0.28$

$= 0.021952$

$= \dfrac{343}{15625}$

5.

a)



$P(A) = 0.20$ (20% received medication A)

$P(B) = 0.50$ (50% received medication B)

$P(C) = 0.30$ (30% received medication C)

$P(N) = 0.06$ (6% overall not cured)

$P(N \mid A) = 0.05$ (5% not cured by medication A)

$P(N \mid B) = 0.07$ (7% not cured by medication B)

Use law of total probability:

$P(N) = P(N|A) \times P(A) \quad + \quad P(N|B) \times P(B) \quad + \quad P(N|C) \times P(C)$

Substitute known values:

$0.06 = (0.05 \times 0.20) + (0.07 \times 0.50) + P(N|C) \times 0.30$

$0.06 = (0.01) + (0.035) + P(N|C) \times 0.30$

$0.06 = 0.045 + 0.30 \times P(N|C)$

$0.015 = 0.30 \times P(N|C)$

$P(N|C) = 0.05 \Rightarrow$ 5% patients not cured by medication C

b) $P(N|B) = 0.035 \neq P(B) \times P(N)$

$P(B) \times P(N) = 0.5 \times 0.06 = 0.03$

So, events 'the patient was given medication B' and 'the patient was not cured' are not statistically independent.

6.

a) First Draw (Bag A → Bag B)

- Bag A has 7 balls: 4 green (G) and 3 blue (B)
- The probability of drawing a green ball from Bag A is $P(G_1) = \frac{4}{7}$ and the probability of drawing a blue ball is $P(B_1) = \frac{3}{7}$

Second Draw (Bag A → Bag B)

- After the first ball is moved to Bag B, Bag A has 6 balls left
  - If the first ball was green, Bag A now has 3 green and 3 blue balls
  - If the first ball was blue, Bag A now has 4 green and 2 blue balls

- If the first ball was green:

  $P(G_2|G_1) = \frac{3}{6} = \frac{1}{2}$

  $P(B_2|G_1) = \frac{3}{6} = \frac{1}{2}$

- If the first ball was blue:

  $P(G_2|B_1) = \frac{4}{6} = \frac{2}{3}$

  $P(B_2|B_1) = \frac{2}{6} = \frac{1}{3}$

Third Draw (from Bag B)

- Bag B now contains 10 balls
- If the first ball was green and the second ball was green (Bag B: 5 green, 5 blue)

  Probability of drawing green or blue from Bag B is $P(G_3) = \frac{5}{10} = \frac{1}{2}$ and $P(B_3) = \frac{5}{10} = \frac{1}{2}$

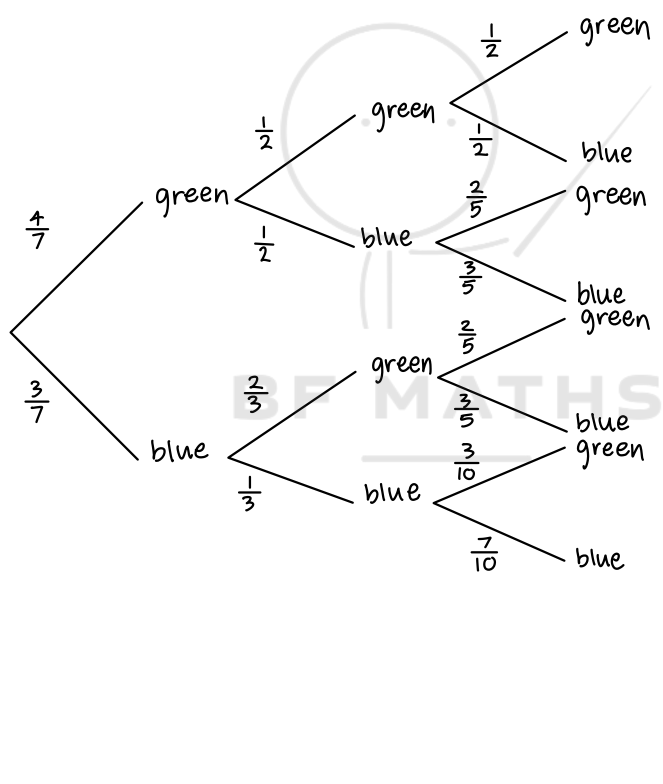
- If the first ball was green and the second ball was blue (Bag B: 4 green, 6 blue)

  Probability of drawing green or blue from Bag B is $P(G_3) = \frac{4}{10} = \frac{2}{5}$ and $P(B_3) = \frac{6}{10} = \frac{3}{5}$

- If the first ball was blue and the second ball was green (Bag B: 4 green, 6 blue)
  Probability of drawing green or blue from Bag B is $P(G_3) = \frac{4}{10} = \frac{2}{5}$ and $P(B_3) = \frac{6}{10} = \frac{3}{5}$

- If the first ball was blue and the second ball was blue (Bag B: 3 green, 7 blue)
  Probability of drawing green or blue from Bag B is $P(G_3) = \frac{3}{10}$ and $P(B_3) = \frac{7}{10}$

Final tree diagram:



b) Bag A contains 7 balls: 4 green and 3 blue

Event Q consists of: both balls drawn are green

both balls drawn are blue

<u>Case 1: Both balls are green</u>

Probability of drawing a green ball on the first draw: $P(G_1) = \frac{4}{7}$

Bag A now has 6 balls left (3 green, 3 blue)

Probability of drawing a green ball on the second draw: $P(G_2|G_1) = \frac{3}{6} = \frac{1}{2}$

So probability of drawing two green balls is:

$P(\text{Both green}) = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14} = \frac{2}{7}$

<u>Case 2: Both balls drawn are blue</u>

Probability of drawing a blue ball on the first draw: $P(B_1) = \frac{3}{7}$

Bag A now has 6 balls left (4 green, 2 blue)

Probability of drawing a blue ball on the second draw: $P(B_2|B_1) = \frac{2}{6} = \frac{1}{3}$

So probability of drawing two blue balls is:

$P(\text{Both blue}) = \frac{3}{7} \times \frac{1}{3} = \frac{3}{21} = \frac{1}{7}$

Total Probability of Q:

$$P(Q) = P(\text{Both green}) + P(\text{Both blue})$$

$$= \frac{2}{7} + \frac{1}{7}$$

$$= \frac{3}{7}$$

c) $P(R) = P(G_1) \times P(G_2) \times P(B_3)$
$+$
$P(G_1) \times P(B_2) \times P(B_3)$
$+$
$P(B_1) \times P(G_2) \times P(B_3)$
$+$
$P(B_1) \times P(B_2) \times P(B_3)$

$$= \left(\frac{4}{7} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{4}{7} \times \frac{1}{2} \times \frac{3}{5}\right) + \left(\frac{3}{7} \times \frac{2}{3} \times \frac{3}{5}\right) + \left(\frac{3}{7} \times \frac{1}{3} \times \frac{7}{10}\right)$$

$$= \frac{4}{28} + \frac{12}{70} + \frac{18}{105} + \frac{21}{210}$$

$$= \frac{41}{70}$$

d)
$P(G_1) \times P(G_2) \times P(B_3) = \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{7}$
$P(G_1) \times P(B_2) \times P(G_3) = \frac{4}{7} \times \frac{1}{2} \times \frac{2}{5} = \frac{4}{35}$
$P(B_1) \times P(G_2) \times P(G_3) = \frac{3}{7} \times \frac{2}{3} \times \frac{2}{5} = \frac{4}{35}$
$P(B_1) \times P(B_2) \times P(G_3) = \frac{3}{7} \times \frac{1}{3} \times \frac{3}{10} = \frac{3}{70}$
$P(B_1) \times P(G_2) \times P(B_3) = \frac{3}{7} \times \frac{2}{3} \times \frac{3}{5} = \frac{6}{35}$
$P(G_1) \times P(B_2) \times P(B_3) = \frac{4}{7} \times \frac{1}{2} \times \frac{3}{5} = \frac{6}{35}$

$$\frac{1}{7} + \frac{4}{35} + \frac{4}{35} + \frac{3}{70} + \frac{6}{35} + \frac{6}{35} = \frac{53}{70}$$

$$1 - \frac{53}{70} = \frac{17}{70}$$

so $P(Q \text{ and } R) = \frac{17}{70}$

e) $P(Q) \times P(R) \neq P(Q \text{ and } R)$ so events Q and R are not independent