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1) a)

$$(27)^{\frac{5}{3}}$$
$$= \left(27^{\frac{1}{3}}\right)^5$$
$$= 3^5$$
$$= 243$$

b)

$$\frac{\left(4x^{\frac{1}{2}}\right)^3}{8x^2}$$
$$= \frac{64x^{\frac{3}{2}}}{8x^2}$$
$$= 8x^{\frac{3}{2}-2}$$
$$= 8x^{-\frac{1}{2}}$$

☆ Rationalise denominator

2)

$$\frac{(4\sqrt{3}-2)(7+\sqrt{3})}{(7-\sqrt{3})(7+\sqrt{3})}$$
$$= \frac{28\sqrt{3} - 4\sqrt{3}\sqrt{3} - 14 - 2\sqrt{3}}{7^2 - (\sqrt{3})^2}$$
$$= \frac{28\sqrt{3} - 12 - 14 - 2\sqrt{3}}{49-3}$$
$$= \frac{26\sqrt{3} - 2}{46}$$
$$= \frac{26}{46}\sqrt{3} - \frac{2}{46}$$
$$= \boxed{\frac{13}{23}}\sqrt{3} - \boxed{\frac{1}{23}}$$

$\therefore p = \frac{13}{23}$

$q = \frac{1}{23}$

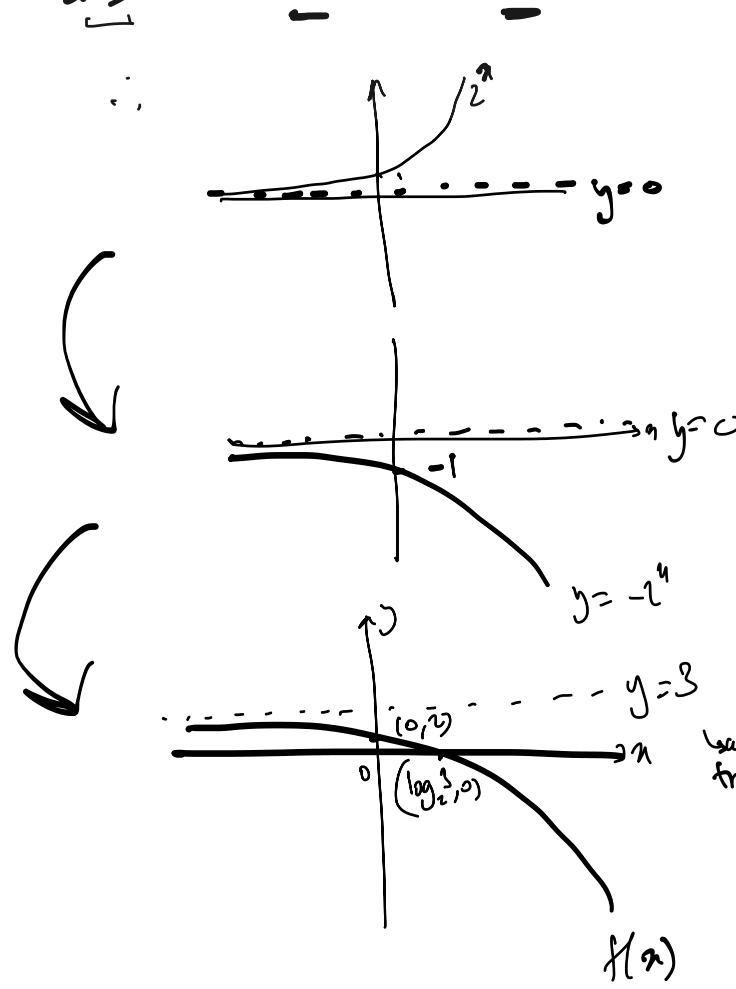
3) $f(x) = -2^x + 3$

let $g(x) = 2^x$

$\therefore -g(x) + 3$

use CBAD $\quad$ a bc d

$\therefore$



why is there an asymptote at $y=0$?

$g(x) = 2^x$

as $x \rightarrow -\infty$ (gets smaller & smaller)

as $y \rightarrow 0$ but never reaches it

"anything" to the power of something will never be 0.

has asymptote translated by vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

To find "x" intercept: when $y=0$

$\therefore -2^x + 3 = 0$

$-2^x = -3$

$2^x = 3$

$\log_2 2^x = \log_2 3$

$x = \log_2 3$

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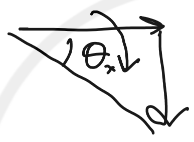
4 a) $\sqrt{98} - \sqrt{50}$
$= \sqrt{49}\sqrt{2} - \sqrt{25}\sqrt{2}$
$= 7\sqrt{2} - 5\sqrt{2}$
$= 2\sqrt{2}$

b) $\dfrac{16\sqrt{3}}{\sqrt{98}-\sqrt{50}} \longrightarrow$ replace with $2\sqrt{2}$ //

$= \dfrac{16\sqrt{3}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{16\sqrt{6}}{4} = 4\sqrt{6}$ //

5 a) $\underline{a} = 6i - 3j$
$\therefore |\underline{a}| = \sqrt{6^2 + (-3)^2}$
$= 3\sqrt{5}$
$= \dfrac{6}{3\sqrt{5}} i - \dfrac{3}{3\sqrt{5}} j$
$= \dfrac{2\sqrt{5}}{5} i - \dfrac{\sqrt{5}}{5} j$ //

b) $\cos\theta_x = \dfrac{2\sqrt{5}}{5} \quad \left(\dfrac{6}{|\underline{a}|}\right)$



$\theta_x = 26.6^\circ$ (3sf)
below x axis.

6) $f(x) = 9x^{\frac{2}{3}} + 4x^{-2} - 3x^3 + 2$
$\therefore \int f(x)\,dx$
$= \int 9x^{\frac{2}{3}} + 4x^{-2} - 3x^3 + 2\,dx$
$= \dfrac{27x^{\frac{5}{3}}}{5} - 4x^{-1} - \dfrac{3}{4}x^4 + 2x + C$
$= \dfrac{27}{5}\sqrt[3]{x^5} - \dfrac{4}{x} - \dfrac{3}{4}x^4 + 2x + C$ //

7) LHS: $\dfrac{2\sin^2 x - 1}{\cos^2 x}$
$= \dfrac{2\sin^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$
$= 2\tan^2 x - \underbrace{\sec^2 x}$ $\quad 1 + \tan^2 x = \sec^2 x$
$\therefore 2\tan^2 x - 1 - \tan^2 x$
$= \tan^2 x - 1$ // $\rightarrow$ RHS

$\square$ //

Alternative working Q7:

$LHS = \dfrac{2\sin^2 x - (\sin^2 x + \cos^2 x)}{\cos^2 x}$

$= \dfrac{\sin^2 x - \cos^2 x}{\cos^2 x}$

$= \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\cos^2 x}$

$= \tan^2 x - 1 = RHS$

8) $\int_{\frac{1}{2}}^{1} (4-3x)^3\,dx$

Guess: $(4-3x)^4$
Check: $-12(4-3x)^3$
$\therefore$ answer: $-\frac{1}{12}(4-3x)^4$

$= \left[-\frac{1}{12}(4-3x)^4\right]_{\frac{1}{2}}^{1}$
$= -\frac{1}{12} - \left[-\frac{625}{192}\right]$
$= \dfrac{203}{64}$ //

9) $1 + \binom{n}{1}3x + \binom{n}{2}(3x)^2 \ldots$
$= \binom{n}{1}3 \times 6 = \binom{n}{2}9$
$= \dfrac{\cancel{n!}\,18}{1!(n-1)!} = \dfrac{\cancel{n!}\,9}{2!(n-2)!}$
$2 = \dfrac{(n-1)!}{(n-2)!\,2!}$
$2 = \dfrac{(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!}$
$2 \times 2 = n - 1$
$4 = n - 1$
$5 = n$ //

10) Points of Intersection:
a) $x^2 - 8x + 18 = 2x + 5$
$x^2 - 10x + 13 = 0$
$x = 5 \pm 2\sqrt{3}$

To find y coordinate sub "x" back into any equation

$\therefore$ A: $(5 - 2\sqrt{3}, 15 - 4\sqrt{3})$ B: $(5 + 2\sqrt{3}, 15 + 4\sqrt{3})$

b) $x^2 - 8x + 18 \leq y \leq 2x + 5$

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11) $f(x) = 8x^2 - \frac{80}{3}x^{\frac{3}{2}} + 5x$

$\therefore f(x)' = 16x - 40x^{\frac{1}{2}} + 5$

when $f(x)' = 4$

$16x - 40\sqrt{x} + 5 = -4$

$16x - 40\sqrt{x} + 9 = 0$

let $u = \sqrt{x}$

$16u^2 - 40u + 9 = 0$

$u_1 = \frac{9}{4}$ | $\therefore x_1 = \frac{81}{16}$

$u_2 = \frac{1}{4}$ | $x_2 = \frac{1}{16}$

12) a i) $(2x-3)^2(x+2)$

$\downarrow 9 \quad \downarrow 2$

$\therefore 9 \times 2 = 18$

$\therefore k = 18$

ii) Min: $(2x-3)^2$

$\therefore f(x+c) = (2(x+c)-3)^2$

$= (2x + \underbrace{2c - 3}_{\text{must be 0 for it to pass through origin}})^2$

$\therefore 2c - 3 = 0$

$2c = 3$

$c = \frac{3}{2}$ //

13 a) $a = 1$

$b = k - 4$

$c = \frac{1}{2}k + 3$

$\therefore b^2 - 4ac$

$= (k-4)^2 - 4(\frac{1}{2}k + 3)$

$= k^2 - 8k + 16 - 2k - 12$

$= k^2 - 10k + 4$

b) two roots $\Rightarrow b^2 - 4ac > 0$

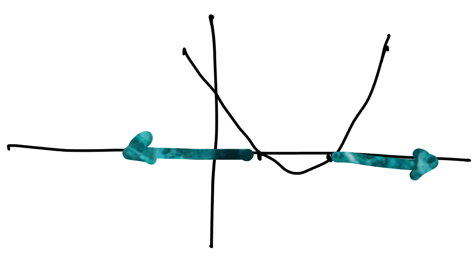
$\therefore k^2 - 10k + 4 > 0$

critical values

$k^2 - 10k + 4 = 0$

$k_1 = 5 + \sqrt{21}$

$k_2 = 5 - \sqrt{21}$



$\therefore k < 5 - \sqrt{21}$ or $k > 5 + \sqrt{21}$

c) when $k = 8$

$f(x) = x^2 + 4x + 7$

$(x+2)^2 - 4 + 7$

$(x+2)^2 + 3$

$(x+2)^2 \geq 0$

$(x+2)^2 + 3 > 0$

$\therefore f(x) > 0$ for all real values of x

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14) a) $\sum F = 6\underline{i} + 3p\underline{j} + 4p\underline{i} - 5\underline{j}$

$= (6+4p)\underline{i} + (3p-5)\underline{j}$

$\begin{pmatrix} 6+4p \\ 3p-5 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$6 + 4p = \lambda$

$3p - 5 = 0$

$p = \frac{5}{3}$ //

b) $\therefore \begin{pmatrix} 6 + 4(\frac{5}{3}) \\ 3(\frac{5}{3}) - 5 \end{pmatrix}$

$F_R = \begin{pmatrix} 6 + \frac{20}{3} \\ 0 \end{pmatrix}$

$F_R = \begin{pmatrix} \frac{38}{3} \\ 0 \end{pmatrix}$

$\therefore |F_R| = \sqrt{(\frac{38}{3})^2 + (0)^2}$

$= \frac{38}{3}$ //

15) a) $f(x) = \frac{1}{3} \cdot x^{\frac{1}{2}} + 3 \cdot x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{6}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

$= \frac{1}{6\sqrt{x}} - \frac{3}{2(\sqrt{x})^3}$

b) $\therefore$ let $f'(x) = 0$

$\frac{1}{6\sqrt{x}} - \frac{3}{2\sqrt{x^3}} = 0$

$\frac{1}{6\sqrt{x}} = \frac{3}{2\sqrt{x^3}}$

$2\sqrt{x^3} = 18\sqrt{x}$

$x = 9$

$\therefore$ when $x = 9$, $y = 2$

$(9, 2)$

c) $f''(x) = -\frac{1}{12}x^{-\frac{3}{2}} + \frac{9}{4}x^{-\frac{5}{2}}$

when $x = 9$

$f''(9) = \frac{1}{162}$

$\therefore$ since $f''(x) > 0$

as $\frac{1}{162} > 0$

when $x = 9$, $\Rightarrow$ local MINIMUM.

16 a) Ship : $\begin{pmatrix} -1 \\ 3 \end{pmatrix} \rightarrow 1$ hour

boat : $\begin{pmatrix} 6 \\ -2 \end{pmatrix} \rightarrow 1$ hour

$\therefore$ 2 hours $\Rightarrow \times 2$

$\therefore$ ship : $\begin{pmatrix} -2 \\ 6 \end{pmatrix} = -2\underline{i} + 6\underline{j}$

boat : $\begin{pmatrix} 12 \\ -4 \end{pmatrix} = 12\underline{i} - 4\underline{j}$

b) distance $= \sqrt{(12 - -2)^2 + (-4-6)^2}$

$= 2\sqrt{74}$

distance between boat + ship $= \begin{pmatrix} 12 \\ -4 \end{pmatrix} - \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 14 \\ -10 \end{pmatrix}$

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17 a) $y = \tan(x + k)$

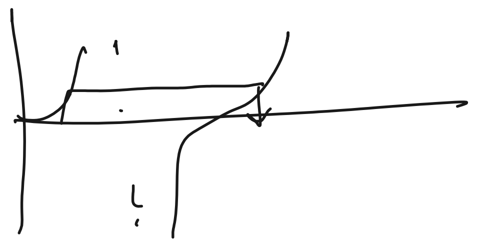
when $x = 60$

$y = 1$

$\therefore\ 1 = \tan(60 + k)$

$k = \underline{-15°}$ as $\tan 45 = 1$

$225 = 1$



or $60 + k = 225$

$k = \underline{165°}$

b) $90 + 15 = 105°$

$90 - 165 = -75°$

$\therefore\ x = 105°$ or $x = -75°$

18) $\log_{16}(1 - 3x) = \frac{3}{4}$

$1 - 3x = 16^{\frac{3}{4}}$

$1 - 3x = 8$

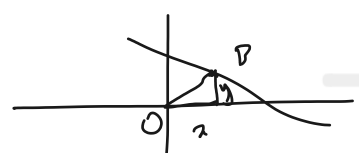
$-7 = 3x$

$-\frac{7}{3} = x$ //

19) $3x + 2y = 12$

$2y = 12 - 3x$

$y = 6 - \frac{3}{2}x$



$x^2 + y^2 = 100$

$\therefore\ x^2 + \left(\frac{12 - 3x}{2}\right)^2 = 100$

$x^2 + \frac{144 - 72x + 9x^2}{4} = 100$

$4x^2 + 9x^2 - 72x + 144 = 400$

$13x^2 - 72x - 256 = 0$

$\boxed{x_1 = 8}$

$x_2 = -\frac{32}{13}$ X

$\therefore$ when $x = 8$

$y = -6$

$\therefore\ \overrightarrow{OB} = 8i - 6j$

20) a) $\left(1 + \frac{2}{x}\right)^2$

$= \left(1 + \frac{2}{x}\right)\left(1 + \frac{2}{x}\right)$

$= 1 + \frac{2}{x} + \frac{2}{x} + \frac{4}{x^2}$

$= 1 + \frac{4}{x} + \frac{4}{x^2}$ //

$\left(1 + \frac{2}{5}x\right)^6$

$= 1 + 6(1)^5\left(\frac{2}{5}x\right) + 15(1)^4\left(\frac{2}{5}x\right)^2 + 20(1)^3\left(\frac{2}{5}x\right)^3$

$= 1 + \frac{12}{5}x + \frac{12}{5}x^2 + \frac{32x^3}{25}$

b) $\boxed{1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1}$

c) $\frac{48x}{5} + \frac{12}{5}x + \frac{128}{25}x = \frac{428}{25}x$ //

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21) when $x = 3$

a)

$2y - 9 = 5(3)$

$2y - 9 = 15$

$2y = 24$

$y = 12$

$\therefore \quad A = (3, 12)$

$l_1: \ 2y = 5x + 9$

$y = \boxed{\frac{5}{2}}x + \frac{9}{2}$

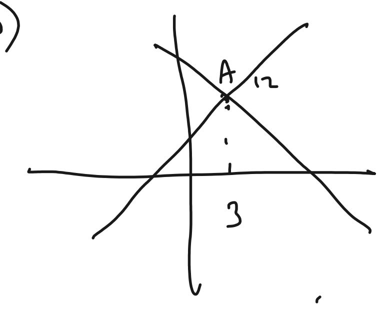
$m_1 = \frac{5}{2}$

$\therefore m_2 = -\frac{2}{5}$

$\therefore \quad y - y_1 = m(x - x_1)$

$y - 12 = -\frac{2}{5}(x - 3)$ //

b)



when $y = 0$ for l_1 $x = -\frac{9}{5}$

when $y = 0$ for l_2 $x = 33$

$\therefore$ base $= 33 - -\frac{9}{5}$

$= \frac{174}{5}$

height $= 12$

$\therefore \ \frac{1}{\cancel{2}} \times \frac{174}{5} \times \cancel{12}^{6} = 208.8$

22)

a) for $x \leq -1$

b) for $x = -\frac{1}{2}$

using discriminant:

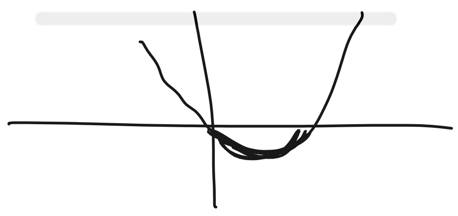
23) $25k^2 - 20k < 0$

CV

$25k^2 - 20k = 0$

$5k(5k - 4) = 0$

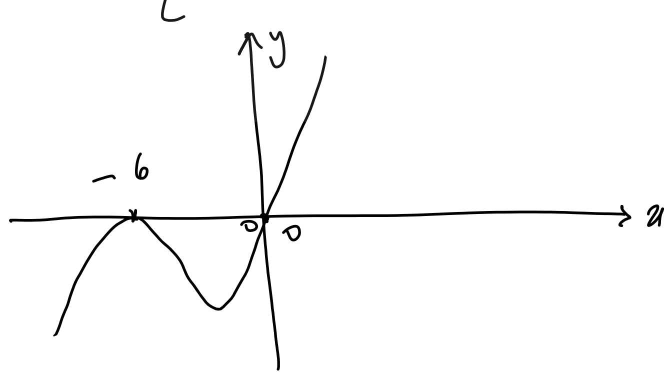
$\therefore \ \boxed{0 < k < \frac{4}{5}}$



24) a) $x(x^2 + 12x + 36)$

$= x(x+6)^2$

b)



25) $6e^{6x} - 38e^{3x} + 40 = 0$

let $e^{3x} = y$

$\therefore \ 6y^2 - 38y + 40 = 0$

$y_1 = 5$ $\quad e^{3x} = 5 \quad x = \frac{\ln 5}{3}$

$y_2 = \frac{4}{3}$ $\quad e^{3x} = \frac{4}{3} \quad x = \frac{\ln \frac{4}{3}}{3}$

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26) $\int_9^k \frac{8}{\sqrt{x}} \, dx = 32$

$\int_9^k 8x^{-\frac{1}{2}} \, dx = 32$

$\left[16x^{\frac{1}{2}}\right]_9^k = 32$

$16\sqrt{k} - 48 = 32$

$16\sqrt{k} = 80$

$\sqrt{k} = 5$

$k = 25$

27) $f(x) = ax^3 + bx^2 + cx$

$f'(x) = 3ax^2 + 2bx + c$

when $x = -1$, $f'(x) = 7$

$3a - 2b + c = 7$ — (i)

when $x = 2$, $f'(x) = 5$

$12a + 4b + c = 5$ — (ii)

$f''(x) = 6ax + 2b$

when $x = 1$, $f''(x) = 2$

$\therefore 6a + 2b = 2$ — (iii)

use equation solver : $a = \frac{8}{9}$, $b = -\frac{5}{3}$, $c = 1$

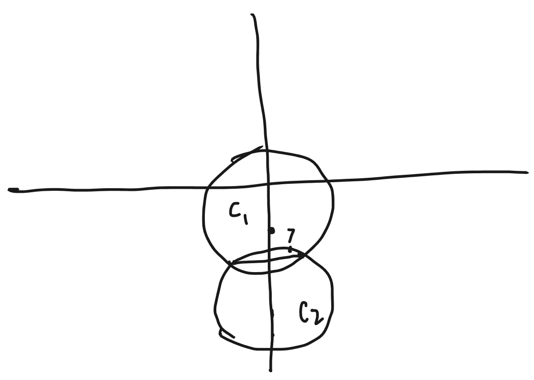
28) a) C_1 : radius $= \sqrt{16} = 4$

centre $= (0, -2)$

C_2 : radius $= \sqrt{16} = 4$

centre $= (0, -5)$

b)



c) $C_1 = C_2$

$\cancel{x^2} + (y+2)^2 - \cancel{16} = \cancel{x^2} + (y+5)^2 - \cancel{16}$

$y^2 + 4y + 4 = y^2 + 10y + 25$

$-6y = 21$

$y = \frac{21}{-6}$

$\therefore x = \frac{\sqrt{55}}{2} \times 2 = \sqrt{55}$

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29 a) $\binom{n}{r} q^{100} + \binom{n}{r+1} q^{99} p + \binom{n}{r+2} q^{98} p^{2} + \binom{n}{r+3} q^{97} p^{3}$

$n = 100$, $r = 0$ } use n choose r formula / button on calculator: $\boxed{nCr}$

$= q^{100} + 100 q^{99} p + 4950 q^{98} p^{2} + 161700 q^{97} p^{3}$

b) if $p = 0.02$
then $q = 1 - 0.02$
$= 0.98$

$\therefore$ by subbing into part(a), ans $= 0.8589 6$

30) a) let $p, q \in \mathbb{R}$

$\therefore p^2, q^2 \in \mathbb{R}$

$(p-q)^2 = p^2 + q^2 - 2pq$

$= p^2 - q^2 + 2q^2 - 2pq$

$\therefore 2pq > 2q^2$

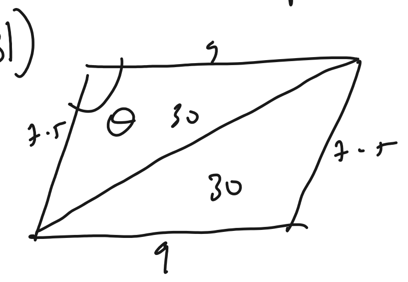
since $p > q$. $\therefore \underline{2q^2 - 2pq < 0}$

$\therefore (p-q)^2 < p^2 - q^2$

$\therefore p - q < \sqrt{p^2 - q^2}$ //

b) when p and $q = 0$ $\quad 0 \not< 0$

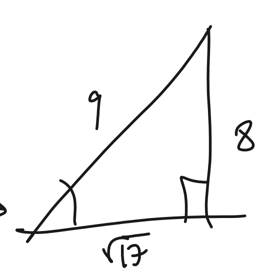
31)



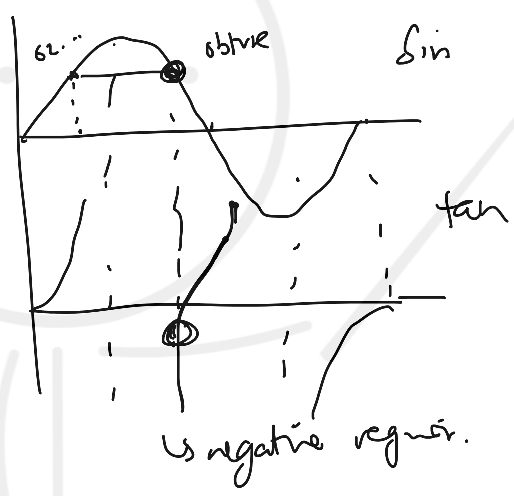
$\therefore \frac{1}{2}$ closure $= 30$

$\frac{1}{2} \cdot 9 \cdot 7.5 \cdot \sin\theta = 30$

$\sin\theta = \frac{8}{9} \rightarrow$



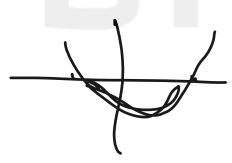
$\therefore \tan\theta = \underline{\underline{-\frac{8}{\sqrt{17}}}}$ //



32) a) $3x - 7 > 3 - 7x$

$10x > 10$

$\underline{x > 1}$



b) $x^2 - 4x - 21 \leq 0$

$\underline{\text{cv}}$

$(x-7)(x+3)$

$-3 \leq x \leq 7$

c) $1 < x \leq 7$

33) a) $(5, -4.5)$

b) $(10, 0)$

c) $x \geq 5$

d) $k \leq 1.5$

34) a) when $n = -1$

$5^{-1} \not> 2^{-1}$

$0.2 \not> 0.5$

b) $\frac{1}{2}\left[(n+1)^2 + (n+3)^2\right]$

$\frac{1}{2}\left[n^2 + 2n + 1 + n^2 + 6n + 9\right]$

$\frac{1}{2}\left[2n^2 + 8n + 10\right]$

$= n^2 + 4n + 5$

$= (n+2)^2 - 2 + 5$

$= (n+2)^2 + 3$

any number + odd number = odd $\therefore$ always odd.

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35) $x = 2-y$

$\therefore\ 3y^2 - (2-y)^2 = 12$

$3y^2 - (4-4y+y^2) = 12$

$2y^2 + 4y - 4 = 12$

$2y^2 + 4y - 16 = 0$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

$y = -4,\quad y = 2$

$x = 6 \quad\ \ x = 0$

36) let $4^{x-1} = y$

then $y^2 - py + 16 = 0$

$(y-2)(y-8) = 0$

$y = 2$

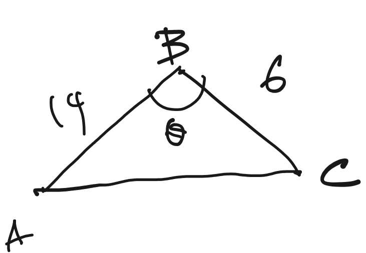
$y = 8$

$\therefore\ \frac{4^x}{4} = 2 \qquad \frac{4^x}{4} = 8$

$4^x = 8 \qquad 4^x = 32$

$x = \frac{3}{2}$ or $x = \frac{5}{2}$

37 a)



$\frac{1}{2}ab\sin C = 24$

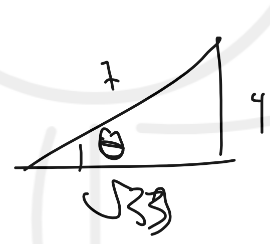
$\frac{1}{2} \cdot 6 \cdot 14 \cdot \sin C = 24$

$\sin C = \frac{4}{7}$

$\underline{C_1 = 34.8^\circ}$

$\therefore\ C_2 = 180 - 34.8$

$= \underline{145.2^\circ}$



b) $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 14^2 + 6^2 - 2(14)(6)\underbrace{\cos A} \longrightarrow -\frac{\sqrt{33}}{7}$ as we want largest positive side length

$a^2 = 14^2 + 6^2 + \ldots$

$a = \underline{19.2\text{ cm}}$

38) $16 - 6x - x^2$

a) $= -(x^2 + 6x) + 16$

$= -\left[(x+3)^2 - 9\right] + 16$

$= -(x+3)^2 + 9 + 16$

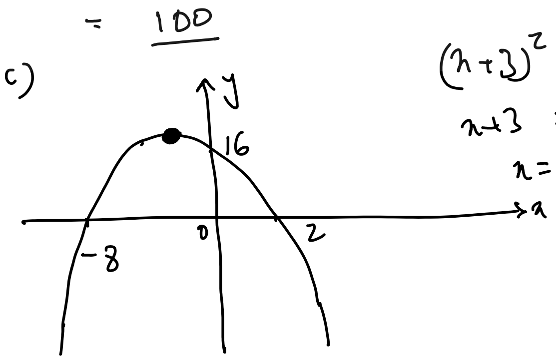
$= \underbrace{25}_{q} - (\underbrace{x+3}_{p})^2$

b) $b^2 - 4ac$

$= 36 + 64$

$= \underline{100}$

c)



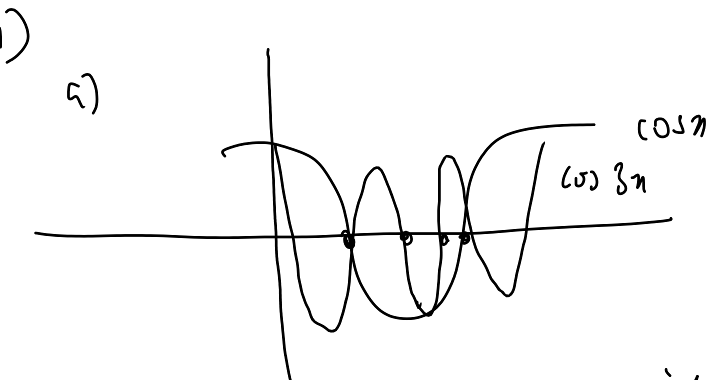
$(x+3)^2 = 25$

$x+3 = \pm 5$

$x = \pm 5 - 3$

39)

a)



→ check desmos for desmos !!

b) stretch parallel to x axis by scale factor $\frac{1}{3}$

c) $x = 20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ, 340^\circ$

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40 a) $\left(2 - \frac{x}{15}\right)^{10}$

$\binom{n}{0} 2^{10} + \binom{n}{1} 2^9 \left(\frac{-x}{15}\right) + \binom{n}{2} 2^8 \left(\frac{-x}{15}\right)^2$

$1024 - \frac{1024}{3}x + \frac{256}{5}x^2$

b) $1024p = 512$

$p = \frac{1}{2}$ //

c) $\therefore \frac{1}{2} \cdot -\frac{1024}{3}x = -\frac{512}{3}x$

$1024q\cancel{x} - \frac{512}{3}\cancel{x} = -\frac{256}{3}\cancel{x}$

$1024q - \frac{512}{3} = \frac{-256}{3}$

$q = \frac{1}{12}$ //

41 a) when $x = -1$

$f(-1) = 0$

$\therefore$

	$6x^2$	$-x$	-2
x	$6x^3$	$-x^2$	$-2x$
$+1$	$6x^2$	$-x$	-2

$\therefore (x+1)(6x^2 - x - 2)$

$(x+1)(3x-2)(2x+1)$ $3\sqrt{2}$

b) $\frac{(x+1)(3x-2)(2x+1)}{x(2x^2+3x+1)}$

$\frac{\cancel{(2x+1)}\cancel{(x+1)}(3x-2)}{x\cancel{(2x+1)}\cancel{(x+1)}}$

$= \frac{3x-2}{x}$

$= 3 - \frac{2}{x}$

$A = 3$

$B = -2$ //

42) $(x-a)^2 + (y-b)^2 = r^2$

a) $(x-2)^2 + (y+1)^2 = r^2$

using: $(8,5)$

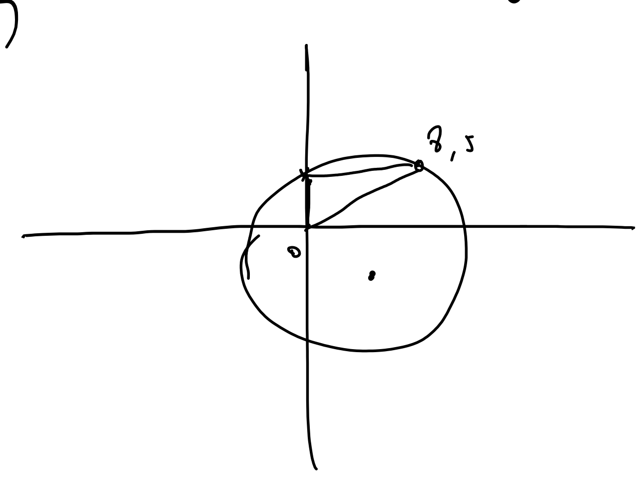
$(8-2)^2 + (5+1)^2 = r^2$

$36 + 36 = r^2$

$72 = r^2$

$\therefore (x-2)^2 + (y+1)^2 = 72$ //

b)



$4 + y^2 + 2y + 1 = 72$

$y^2 + 2y + 5 = 72$

$y^2 + 2y - 67 = 0$

$y_1 = -1 + 2\sqrt{17}$

$y_2 \times -1 - 2\sqrt{17}$

$\therefore \frac{1}{\cancel{2}} \times (-1 + 2\sqrt{17}) \cdot \cancel{8}^{4}$

$= -4 + 8\sqrt{17}$ //

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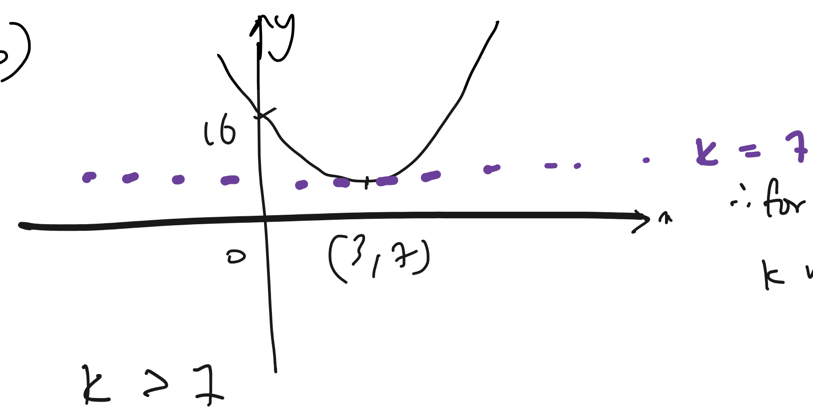
43) a) $f(x) = (x-3)^2 - 9 + 16$

$f(x) = (x-3)^2 + 7$

b)



$\therefore$ for distinct roots k must be > 7.

c) $k > 7$

44) a) $x + 2 = -x^2 + 2x + 4$

$x^2 - x - 2 = 0$

$x_1 = 2 \longrightarrow y = 4$

$x_2 = -1 \longrightarrow y = 1$

$\therefore A = (-1, 1)$

$B = (2, 4)$

b) $I = \int_{-1}^{2} -x^2 + 2x + 4 \, dx$

$I = \left[\frac{-x^3}{3} + \frac{2x^2}{2} + 4x \right]_{-1}^{2}$

$I = \left[-\frac{1}{3}x^3 + x^2 + 4x \right]_{-1}^{2}$

$I = \left(-\frac{1}{3} \cdot 8 + 4 + 8 \right) - \left(\frac{1}{3} + 1 - 4 \right)$

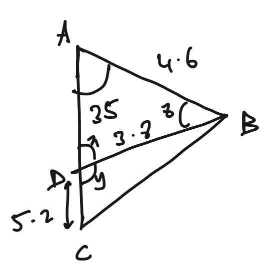
$I = 12$

Area of trapezium $= \frac{1}{2}(1 + 4) \times 3$

$= \frac{15}{2}$

$R = \therefore 12 - \frac{15}{2} = \frac{9}{2}$ units^2 //

45)



$\frac{\sin 35}{3.7} = \frac{\sin x}{4.6}$

$x = 43.97$

$\therefore y = 180 - x$

$= 136.03$

$\Delta BDC = \frac{1}{2} \times 3.7 \times 5.2 \sin 136.03$

$= 6.86$

$z = 180 - 35 - 43.97$

$= 101.03$

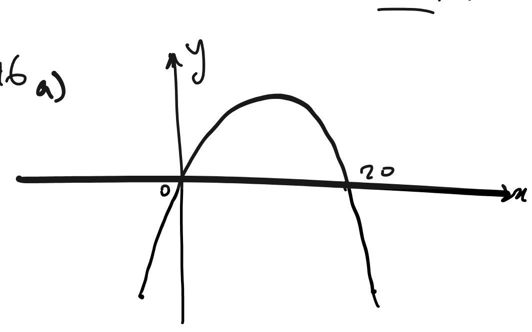
$\therefore \Delta ABD = \frac{1}{2} \times 4.6 \times 3.7 \times \sin 101.03$

$= 8.579$

$\therefore ABC = 8.579 + 6.86$

$= \underline{15.4}$ cm^3

46 a)



b) $0 = x - 0.05x^2$

$0 = 20x - x^2$

$0 = 0.05x(20 - x)$

$\underline{x = 20m}$

c) $-0.05(x^2 - 20x) = y$

$-0.05\left[(x-10)^2 - 100\right] = y$

$-0.05(x-10)^2 + 5 = y$

model suggests $\underline{5m}$ as max height

47) $6\tan^2 x - 11\tan x - 10 = 0$

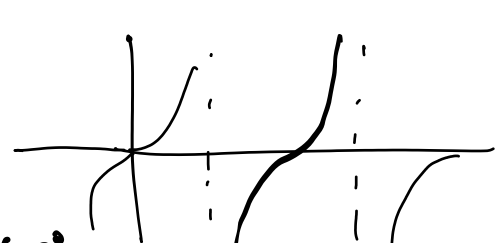
$\tan x_1 = \frac{5}{2}$

$\tan x_2 = -\frac{2}{3}$

$\therefore x_1 = \underline{68.2^\circ}, \underline{248.2^\circ}$

$x_2 = -33.7,$

$\underline{\underline{146.3^\circ, 326.3^\circ}}$



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48) a) $\frac{dy}{dx} = 3x^2 + 6x - 9$

b) $3x^2 + 6x - 9 > 0$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$

$x < -3, \quad x > 1$

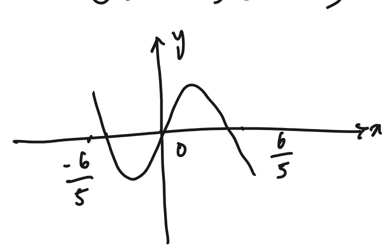
$\therefore \{x \in \mathbb{R} : x < -3\} \cup \{x \in \mathbb{R} : x > 1\}$

49) a) $36x - 25x^3$

$= x(36 - 25x^2)$

$= x(6+5x)(6-5x)$

b)



c) when $x = -1$, $y = -11$

when $x = 1$, $y = 11$

$\therefore$ $d = \sqrt{(2)^2 + (22)^2}$

$d = 2\sqrt{122}$

50) $f(x) = 4 - 2\sqrt{x} + \frac{1}{2}x^2$

a) $\therefore f'(x) = -x^{-\frac{1}{2}} + x$

$\therefore$ when $f'(x) = 0$

$-\frac{1}{\sqrt{x}} + x = 0$

$\frac{1}{\sqrt{x}} = x$

$x \cdot \sqrt{x} = 1$

$\underline{x = 1}$, $y = \frac{5}{2}$ $\quad (1, \frac{5}{2})$

b) when $x = 4$

$m_T = -\frac{1}{\sqrt{4}} + 4 \longrightarrow$ sub into gradient function.

$= -\frac{1}{2} + 4$

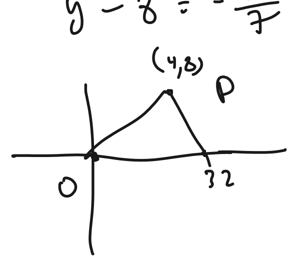
$= \frac{7}{2}$

$\therefore m_N = -\frac{2}{7}$

$\therefore \quad y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{7}(x - 4)$ //

c)



$\therefore \frac{1}{\cancel{2}} \times 32 \times \cancel{8}^{4} = \underline{128}$ units^2

51) a) $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
$\boxed{1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1}$

b) i) let $x = \sqrt{5}$

then $1 + 5\sqrt{5} + 50 + 50\sqrt{5} + 125 + 25\sqrt{5}$

$= 176 + 80\sqrt{5}$ //

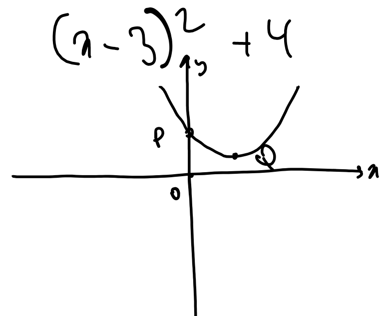
ii) $\log_2 (176 + 80\sqrt{5})$

$= \log_2 16(11 + 5\sqrt{5})$

$= 4 + \log_2 (11 + 5\sqrt{5})$ //

52) a) $(x-3)^2 + 4$

b)



$Q = (3, 4)$

$P = (0, 13)$ //

c) $d = \sqrt{3^2 + 9^2}$

$= 3\sqrt{10}$ //

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53) let $u = \sqrt{x}$

a) then $u^3 + 2u^2 - 8u = 0$

$u(u^2 + 2u - 8) = 0$

$u(u+4)(u-2) = 0$

$u = 0$

$u = -4$ ×

$u = 2 \longrightarrow \sqrt{x} = 2$

$x = 4$ //

b) $\int_0^4 x^{\frac{3}{2}} + 2x - 8x^{\frac{1}{2}} \, dx$

$= \left[\frac{2x^{\frac{5}{2}}}{5} + x^2 - \frac{16x^{\frac{3}{2}}}{3} \right]_0^4$

$= \left(\frac{64}{5} + 16 - \frac{128}{3} \right) - (0)$

$= \frac{-208}{15} = \frac{208}{15}$ → as area is scalar ∴ consider magnitude only

54) a) $\log_3(7x-18) - \log_3(x-2)^2 = 1$

$\log_3\left(\frac{7x-18}{(x-2)^2}\right) = 1$

$\frac{7x-18}{(x-2)^2} = 3$

$3(x-2)^2 = 7x - 18$

$3(x^2 - 4x + 4) - 7x + 18 = 0$

$= 3x^2 - 12x + 12 - 7x + 18 = 0$

$= 3x^2 - 19x + 30 = 0$ //

b) ∴ $x_1 = \frac{10}{3}$

$x_2 = 3$

55) $2x - 4 = x^2 + px - 2p$

a) $\underset{(a)}{x^2} + \underset{(b)}{(p-2)}x \; \underset{(c)}{-2p+4} = 0$

using discriminant $b^2 - 4ac < 0$

$(p-2)^2 - 4(1)(-2p+4) < 0$

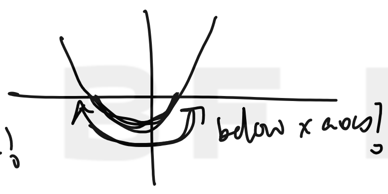
$p^2 - 4p + 4 + 8p - 16 < 0$

$p^2 + 4p - 12 < 0$ //

b) $(p+6)(p-2) < 0$

$-6 < p < 2$

draw graph!



56) a) Show each side has different magnitude!

$\overrightarrow{EF} = \overrightarrow{ED} + \overrightarrow{DF}$

$= -3i - 7j + 8i - j$

$= 5i - 8j$

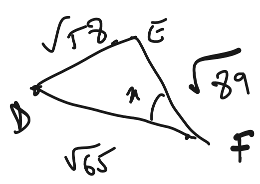
∴ $|\overrightarrow{EF}| = \sqrt{5^2 + (-8)^2} = \sqrt{89}$

$|\overrightarrow{DE}| = \sqrt{3^2 + 7^2} = \sqrt{58}$

$|\overrightarrow{DF}| = \sqrt{8^2 + (-1)^2} = \sqrt{65}$

∴ as magnitude of each length is different the sides are not same length ∴ scalene. //

b)



∴ $\cos x = \frac{65 + 89 - 58}{2\sqrt{65}\sqrt{89}}$

$x = 50.9°$ (3sf) //

57) a) if $g(x)$ is divisible by $(x+3)$ then $g(-3)$ must be 0.

∴ $g(-3) = 4(-3)^3 - 16(-3)^2 - 35(-3) + 147$

$= 0$

∴ as $g(-3) = 0$, $(x+3)$ must be a factor of $g(x)$. //

b)

	$4x^2$	$-28x$	49
x	$4x^3$	$-28x^2$	$49x$
$+3$	$12x^2$	$-84x$	147

$(x+3)(4x^2 - 28x + 49)$

$(x+3)(2x-7)^2$

c) i) $x \leq -3$

ii) $x \leq -\frac{3}{2}$

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58) a) i) $x^2 - 8x + y^2 + 12y = -16$

$(x-4)^2 - 16 + (y+6)^2 - 36 = -16$

$(x-4)^2 + (y+6)^2 = 36$

$\therefore$ centre : $(4,-6)$

ii) radius = 6

b) $x^2 + k^2x^2 - 8x + 12kx + 16 = 0$

$\underset{a}{(1+k^2)x^2} + \underset{b}{(12k-8)} + \underset{c}{16} = 0$

$\therefore b^2 - 4ac < 0$

$\therefore (12k-8)^2 - 4(1+k^2)(16) < 0$

$144k^2 - 192k + \cancel{64} - \cancel{64} - 64k^2 < 0$

$80k^2 - 192k < 0$

$10k^2 - 24k < 0$

$k(10k - 24) < 0$

$0 < k < \frac{12}{5}$ //

59) $A = Pe^{\frac{rt}{100}}$

a) $A = 700 \times e^{\frac{3.5\times 4}{100}}$

$A = £805.19$

b) $\frac{dA}{dt} = \frac{Pr}{100} e^{\frac{rt}{100}} = 24.5e^{0.035t}$

c) $1400 = 700e^{\frac{3.5t}{100}}$

$2 = e^{\frac{3.5t}{100}}$

$\ln 2 = \frac{3.5t}{100}$

$\frac{\ln 2 \times 100}{3.5} = t$

19.8 years $= t$

60) a) $1 - \cos^2 x = 8\cos^2 x - 6\cos x$

$9\cos^2 x - 6\cos x - 1 = 0$

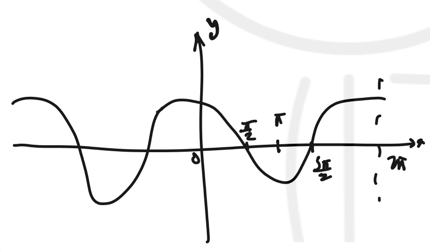
$(3\cos x - 1)^2 - 2 = 0$

$(3\cos x - 1)^2 = 2$ //

b) $3\cos x - 1 = \pm\sqrt{2}$

$\cos x = \frac{1\pm\sqrt{2}}{3}$

$\cos x_1 = \frac{1-\sqrt{2}}{3}$ $\qquad$ $\cos x_2 = \frac{1+\sqrt{2}}{3}$



$x_1 : 97.9°$, $262.1°$ $\qquad$ $x_2 : 36.4°$, $323.6°$

61) a) 14641

$1(1)^4 + 4(1)^3(3x) + 6(1)^2(3x)^2 + 4(1)(3x)^3 + 1(3x)^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b) $(1-3x)^4 = 1 - \cancel{12x} + 54x^2 - \cancel{108x^3} + 81x^4$

$= 2 + 108x^2 + 162x^4$

c) $f'(x) = 216x + 648x^3$

$\therefore$ when $f'(x) = 0$

$216x(1 + 3x^2) = 0$

$\boxed{x=0}$ or $x^2 \cancel{=} -\frac{1}{3}$

when $x = 0$, $y = 2$ $\quad (0,2)$ //

62) $mx - y - 3 = 0$

$y = mx - 3$

$\therefore x^2 - 4x + (mx-3)^2 - 6(mx-3) = 7$

$x^2 - 4x + m^2x^2 - 6mx + 9 - 6mx + 18 = 7$

$\underset{a}{(1+m^2)x^2} + \underset{b}{(-4-12m)x} + \underset{c}{20} = 0$

$b^2 - 4ac = 0$

$\therefore (-4-12m)^2 - 4(1+m^2)(20) = 0$

$16 + 96m + 144m^2 - 80 - 80m^2 = 0$

$64m^2 + 96m - 64 = 0$

$4m^2 + 6m - 4 = 0$

$2m^2 + 3m - 2 = 0$

$2m^2 + 4m - m - 2 = 0$

$2m(m+2) - 1(m+2) = 0$

$(m+2)(2m-1) = 0$

$m = -2$

$m = \frac{1}{2}$ //

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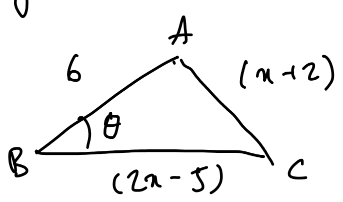
63) a)



$$\cos\theta = \frac{(2x-5)^2 + 36 - (x+2)^2}{2 \times 6 \times (2x-5)}$$

$$\frac{3}{4} = \frac{4x^2 - 20x + 25 + 36 - (x^2+4x+4)}{12(2x-5)}$$

$$\frac{3}{4} = \frac{3x^2 - 24x + 57}{24x - 60}$$

$$72x - 180 = 12x^2 - 96x + 228$$

$$12x^2 - 168x + 408 = 0$$

$$x^2 - 14x + 34 = 0 \quad //$$

b)

$$(x-7)^2 - 49 + 34 = 0$$

$$(x-7)^2 = 15$$

$$x - 7 = \pm\sqrt{15}$$

$$x = 7 \pm \sqrt{15}$$

$$x = 7 + \sqrt{15},\ 7 - \sqrt{15} \quad //$$

64) a) $(1+x)^2 = 1 + 2x + x^2$

$\therefore\ 1 + x^2 < 1 + 2x + x^2$

$0 < 2x$

$0 < x \quad //$

b) if $x = -1$ then $(1-1)^2 = 0$

however, $1 + (-1)^2 = 2$

$2 > 0$

$\therefore$ statement not true for all values of x.

65 $\quad k - x = x^2 + 5x - 2$

$\underset{a}{x^2} + \underset{b}{6x} \underbrace{-2 - k}_{c} = 0$

$b^2 - 4ac = 0$ as it's a tangent to circle $\therefore$ touches $\underline{1}$.

$36 - 4(1)(-2-k) = 0$

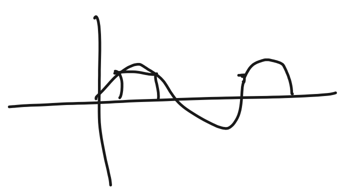
$36 + 8 + 4k = 0$

$44 + 4k = 0$

$k = -11$

66) a) if $\sin\alpha = \frac{\sqrt{3}}{2}$

$\alpha = 60°$ or $120°$

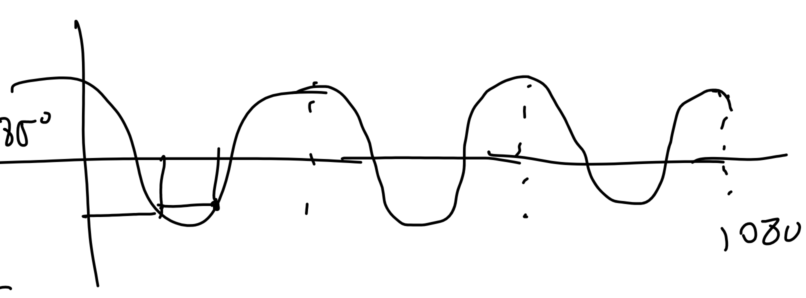


$\therefore\ x + 15 = 60$ or $x + 15 = 120$

$x = 45°$ $\quad x = 105°$

b) $\cos\beta = -\frac{1}{\sqrt{2}}$

$\beta = 135°, 225°, 495°, 585°$



$\therefore\ 2x = 135 \quad 2x = 225$

$2x = 495 \quad 2x = 585$

$\therefore\ x = 67.5°, 112.5°, 247.5°, 292.5°$

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67) $P = 6.25\,(30x - x^2) - 1256.25$

ai) $P = -6.25\left[(x-15)^2 - 225\right] - 1256.25$

$P = -6.25\,(x-15)^2 + 1406.25 - 1256.25$

$P = 150 - 6.25\,(x-15)^2$ //

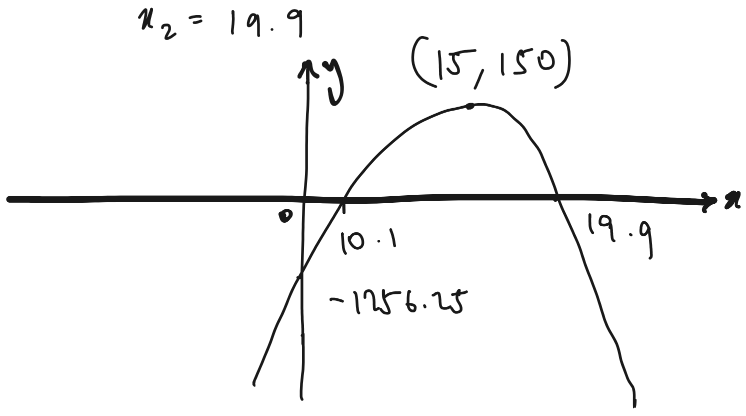
ii) $\therefore$ when $P = 0$

$x = 15 \pm \sqrt{24}$

$x_1 = 10.1$

$x_2 = 19.9$

iii)



b) when $x = 25$

$P = -475{,}000$

$\therefore$ not appropriate as company will make a loss.

c) i) when $x = £15$ $= £150{,}000$

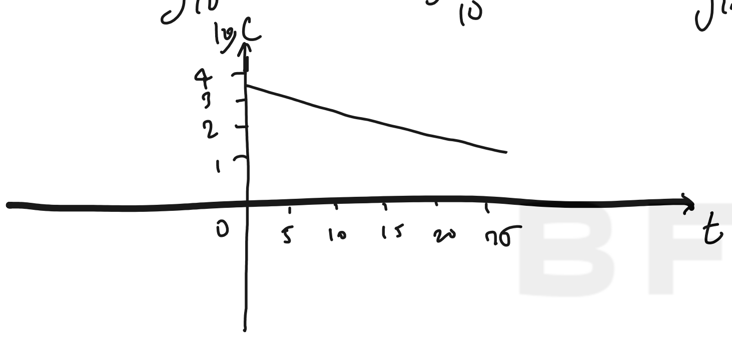
ii) £15

68) a) $\log_{10} C = \log_{10} ab^t$

$\log_{10} C = \log_{10} a + \log_{10} b^t$

$\log_{10} C = \log_{10} a + t\log_{10} b$ //

b)



c) $a = 5100$

$b = 0.95$

69) a) 7

b) as $(x-1)$ is a factor of $f(x)$, $f(1)$ must be 0.

$\therefore (1+k)(2+5) + 7 = 0$

$(1+k)7 + 7 = 0$

$7 + 7k + 7 = 0$

$k = -2$ //

c) $(x^2 - 2)(2x+5) + 7$

$= 2x^3 + 5x^2 - 4x - 10 + 7$

$= 2x^3 + 5x^2 - 4x - 3$

$\therefore$

	$2x^2$	$7x$	3
x	$2x^3$	$7x^2$	$3x$
-1	$-2x^2$	$-7x$	-3

$(x-1)(2x^2 + 7x + 3)$

$(x-1)(2x+1)(x+3)$

$x = 1$

$x = -3$

$x = -\frac{1}{2}$ //

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70) a i) $x^2+y^2-10x+6y-11=0$

$(x-5)^2-25+(y+3)^2-9-11=0$

$(x-5)^2+(y+3)^2=45$

$\therefore$ centre: $(5,-3)$

ii) radius $=\sqrt{45}$

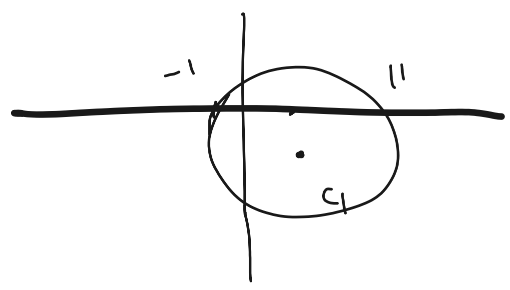
$=3\sqrt{5}$ //

b) $x^2-10x-11=0$

$(x-11)(x+1)=0$

$x=11$

$x=-1$



$\therefore$ diameter of $C_2=12$

radius of $C_2=6$

midpoint of diameter = centre

$\therefore$ $(5,0)$ = centre

$\therefore$ C_2: $(x-5)^2+y^2=36$ //

71) a) $(x-4)^2-16+17>0$

$(x-4)^2+1>0$

since $(x-4)^2\geq 0$

$(x-4)^2+1>0$

b) $(n+2)^2>n^2$

if $n=-2$

then $(-2+2)^2$

$=0^2$

$=0$

$(-2)^2$

$=4$

$\therefore$ not true for $n=-2$

if $n=1$

$(1+2)^2=9$

$(1)^2=1$

$\therefore$ true for $n=1$

$\therefore$ statement is sometimes true.

72) a) when $y=0$

$-x^3+2x^2+15x=0$

$x(-x^2+2x+15)=0$

$x=0$

$-x^2+2x+15$

$(5-x)(x+3)=0$

$\therefore$ $x=5$

$x=-3$ //

$\therefore$ $A=(-3,0)$

$B=(5,0)$ //

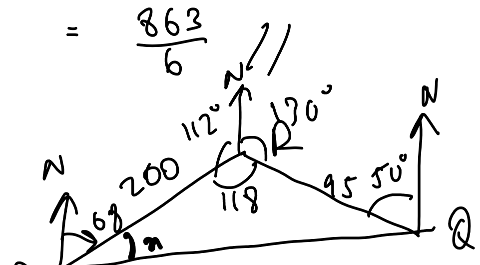
b) $\int_{-3}^{0}-x^3+2x^2+15x\,dx+\int_{0}^{5}-x^3+2x^2+15x$

$=-\left(-\frac{117}{4}\right)+\frac{1375}{12}$

$=\frac{863}{6}$ //

73)

a)



$a^2=200^2+95^2-2(200)(95)\cos 118$

$a=258.58$

$\therefore |\overrightarrow{PQ}|=258.58\text{ m}$

b) $\frac{\sin x}{95}=\frac{\sin 118}{258.58}$

$x=18.9$

$\therefore$ $68+18.9=86.9°$

$=087°$

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74) $2\pi r + 2y = 300$

a) $2\pi \frac{x}{2} + 2y = 300$

$x\pi + 2y = 300$

$y = 150 - \frac{1}{2}x\pi$

$\therefore \pi r^2 + xy$

$A = \frac{x^2}{4}\pi + x150 - \frac{1}{2}x^2\pi$

$A = 150x - \frac{x^2}{4}\pi$

b) $\frac{dA}{dx} = 150 - \frac{1}{2}x\pi$

$150 - \frac{1}{2}x\pi = 0$

$\frac{1}{2}x\pi = 150$

$x = \frac{300}{\pi}$

$\therefore A = 150\left(\frac{300}{\pi}\right) - \frac{\left(\frac{300}{\pi}\right)^2}{4}\pi$

$A = \frac{45000}{\pi} - \frac{22500}{\pi}$

$A = \frac{22500}{\pi}\ m^2$ //

75 a) $f(x) = 2x^3 + kx^2 - 24x + 45$

since $(x-5)$ is a factor $f(5) = 0$

$\therefore 2(5)^3 + k(5)^2 - 24(5) + 45 = 0$

$250 + 25k - 120 + 45 = 0$

$k = -7$ //

b)

	$2x^2$	$3x$	-9
x	$2x^3$	$3x^2$	$-9x$
-5	$-10x^2$	$-15x$	45

$(x-5)(2x^2 + 3x - 9)$

$(x-5)(2x^2 + 6x - 3x - 9)$

$(x-5)(x+3)(2x-3)$

c) $x_1 = 5 \quad x_2 = -3 \quad x_3 = \frac{3}{2}$

$\therefore 5^y = 5 \quad 5^y \neq -3 \quad 5^y = \frac{3}{2}$

$y = 1 \quad\quad y = 0.25$

76) let $\alpha = 3x - 20$

$\therefore \cos\alpha = -0.6$

$\alpha_1 = 126.87$

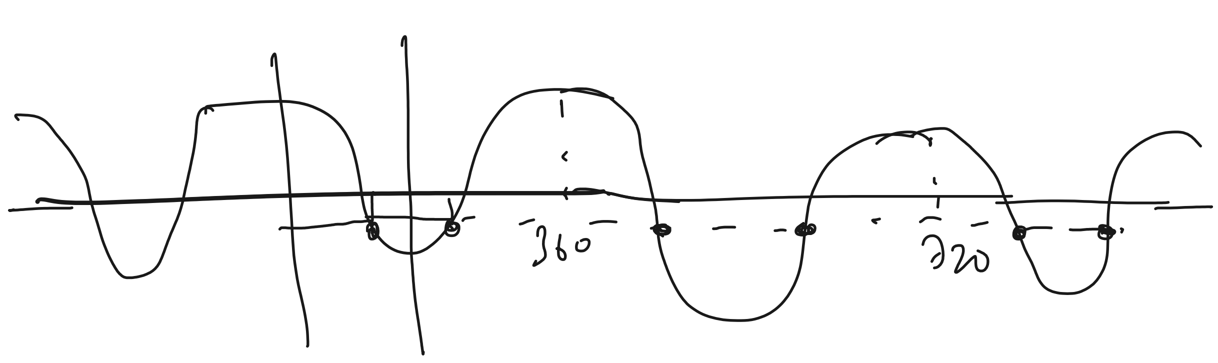
$\alpha_2 = 233.13$

$\alpha_3 = 486.87$

$\therefore x_1 = 49.0°$

$x_2 = 84.4°$

$x_3 = 169.0°$



$\alpha = 3x - 20$

$\frac{\alpha + 20}{3} = x$

77) $y - y_1 = m(x - x_1)$

a) $2 - 4 = m(6 - 0)$

$-2 = m6$

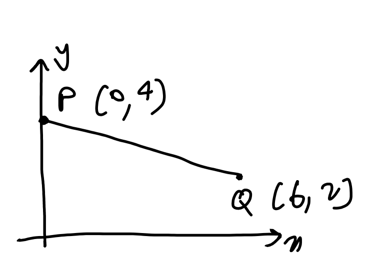
$-\frac{1}{3} = m$

$\therefore m_p = 3$

$\therefore y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 0)$

$y = 3x$



b) $y - 4 = -\frac{1}{3}x$

$\therefore 3x - 4 = -\frac{1}{3}x$

$\frac{10}{3}x = 4$

$x = \frac{12}{10} \quad \therefore y = \frac{36}{10} \quad \left(\frac{12}{10}, \frac{36}{10}\right)$ //

c) $d = \sqrt{\left(\frac{12}{10}\right)^2 + \left(\frac{36}{10}\right)^2} = \frac{6\sqrt{10}}{5}$

d) $\sqrt{6^2 + 2^2} = 2\sqrt{10}$

e) $\frac{1}{2} \times 4 \times 6 = 12\ \text{unit}^2$

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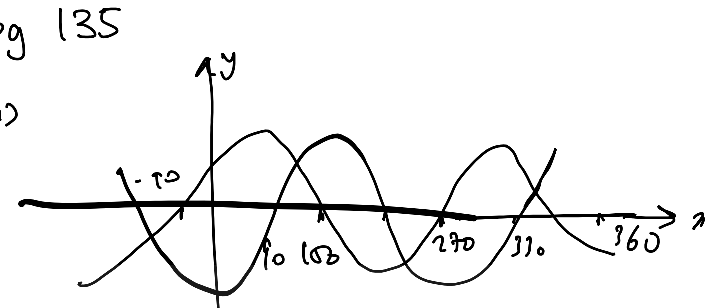
78) a)



b) $(0, 0.5)$, $(150, 0)$
$(-30, 0)$, $(330, 0)$

c) $x = -60, 120, 300$

79) a) $f'(x) = 6x^{\frac{1}{2}} - \frac{3}{2}x^2$

$\therefore$ let $f'(x) = 0$ $\therefore$ $6x^{\frac{1}{2}} - \frac{3}{2}x^2 = 0$

$6\sqrt{x} = \frac{3}{2}x^2$

$\frac{12}{3} = x^{\frac{3}{2}}$

$4 = x^{\frac{3}{2}}$

$\sqrt[3]{16} = x$

$\therefore$ when $x = \sqrt[3]{16}$, $y = 8$

$(\sqrt[3]{16}, 8)$

b) when $y = 0$, $4x^{\frac{3}{2}} - \frac{1}{2}x^3 = 0$

$4x^{\frac{3}{2}} = \frac{1}{2}x^3$

$8 = x^{\frac{3}{2}}$

$4 = x$

$\therefore B = (4, 0)$ //

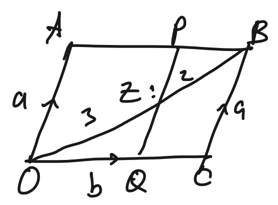
c) $\int_0^4 4x^{\frac{3}{2}} - \frac{1}{2}x^3 \, dx$ $\longrightarrow$ you can check integrals on your calculator!

$\left[\frac{8x^{\frac{5}{2}}}{5} - \frac{1}{8}x^4\right]_0^4$

$= \frac{256}{5} - 32$

$= \frac{96}{5}$ //

80)



a) $\vec{OB} = \vec{OC} + \vec{CB}$

$= \underset{\sim}{b} + \underset{\sim}{a}$

$\therefore$ $\vec{OZ} = \frac{3}{5}\vec{OB}$

$= \frac{3}{5}\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{a}$

$\therefore$ $\vec{CZ} = \vec{CO} + \vec{OZ}$

$= -\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{a}$

$= -\frac{2}{5}\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{a}$

b) $\vec{CA} = \underset{\sim}{a} - \underset{\sim}{b}$ $\therefore$ $\vec{CZ}$ will not pass through $\vec{CA}$ as they are not parallel $\therefore$ not collinear.

c) $\vec{OZ} = \vec{OA} + \vec{AP} + \vec{PZ}$

$\frac{3}{5}\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{a} = \underset{\sim}{a} + \lambda\underset{\sim}{b} - \mu\underset{\sim}{a}$

$1 - \mu = \frac{3}{5}$ | $\frac{3}{5} = \lambda$

$\mu = \frac{2}{5}$

$\therefore$ $\vec{AP} : \vec{PB}$ | $\vec{PZ} : \vec{ZQ}$

$= \frac{3}{5} : \frac{2}{5}$ | $\frac{2}{5} : \frac{3}{5}$

$= 3 : 2$ // | $= 2 : 3$ //

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81) $\therefore$ POI

a) $4x + 3(2x+5) - 35 = 0$

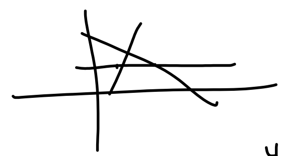
$4x + 6x + 15 - 35 = 0$

$10x - 20 = 0$

$x = 2$

$\therefore$ when $x = 2$, $y = 9$ $\quad \therefore P = (2, 9)$

b)



$\therefore$ when $y = 1$, $x_{l_4} = -2$

$y = 1$, $x_{l_2} = 8$

$\therefore \frac{1}{2} \times \cancel{8}^{4} \times 10 = 40 \text{units}^2$

c) $\sqrt{(10-2)^2 + (k-9)^2} < 10$

$64 + k^2 - 18k + 81 < 100$

$k^2 - 18k + 45 < 0$

$(k-15)(k-3) < 0$

$\therefore 3 < k < 15$ //

82) a) $(x-5)^2 + (y-3)^2 = r^2$

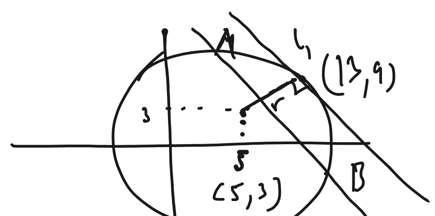
at Q : $(13-5)^2 + (9-3)^2 = r^2$

$8^2 + 6^2 = r^2$

$r = 10$

$\therefore (x-5)^2 + (y-3)^2 = 100$

b)



mp

(9,6)

radius

$\therefore y - y_1 = m(x - x_1)$

$9 - 3 = m(13-5)$

$6 = m8$

$\frac{3}{4} = m$

$\therefore m = -\frac{4}{3}$ $\quad l_1$

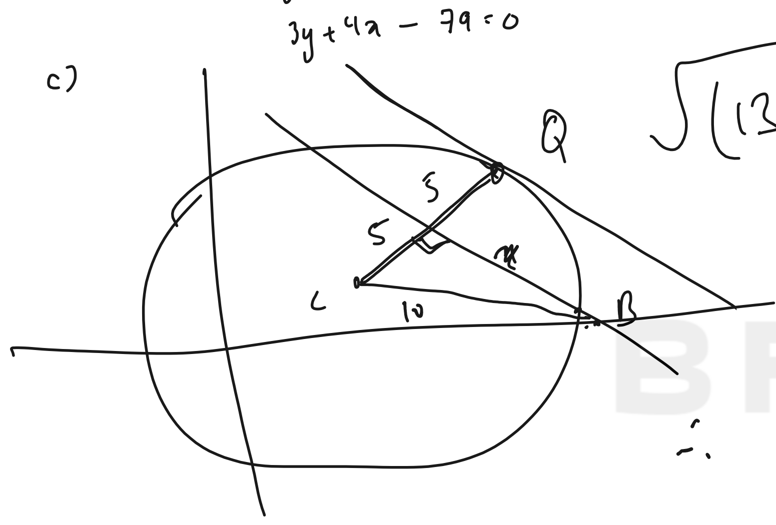
$\therefore y - y_1 = m(x - x_1)$

$y - 9 = -\frac{4}{3}(x - 13)$

$3y - 27 = -4x + 52$

$3y + 4x - 79 = 0$

c)



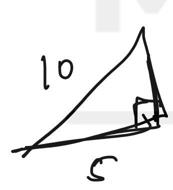
$\sqrt{(13-5)^2 + (9-3)^2}$

$= 10$

$\therefore$ each half $= 5$

$\overrightarrow{CB}$ = radius = 10

$\therefore$



$\therefore x = \sqrt{10^2 - 5^2}$

$x = 10\sqrt{3}$

83) $f(x) = x^3 - 2x^2 + x + 18$

a) $(-2)^3 - 2(-2)^2 + (-2) + 18 = 0$ $\quad \therefore (x+2)$ is a factor of $f(x)$ since $f(-2) = 0$.

b)

	x^2	$-4x$	9
x	x^3	$-4x^2$	$9x$
$+2$	$2x^2$	$-8x$	18

$(x+2)\underbrace{(x^2 - 4x + 9)}_{\text{using discriminant}}$

$b^2 - 4ac$

$= 16 - 36$

$= -20 < 0$

$\therefore$ no real roots as $b^2 - 4ac < 0$ //

c) $x(x^2 - 2x + 1) + 18$

$x(x-1)^2 + 18$

since $x \geq 0$

$x(x-1)^2 \geq 0$

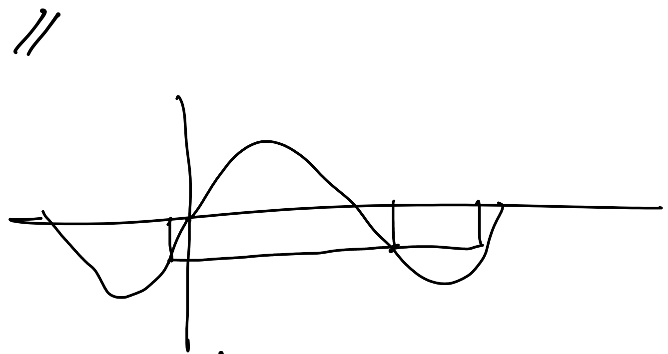
$\therefore x(x-1)^2 + 18 \geq 18$

$\therefore f(x) \geq 18$ //

84)

a)

	$6x^2$	$-x$	-1
x	$6x^3$	$-x^2$	$-x$
$+2$	$12x^2$	$-2x$	-2



$(x+2)(6x^2 - x - 1)$

$= (x+2)(2x-1)(3x+1)$

b) $\sin y + 2 = 0$ $\quad \sin y = \frac{1}{2}$ $\quad \sin y = -\frac{1}{3}$

$\cancel{\sin y = -2}$

$30°, 150°, 199.5°, 340.5°$ //

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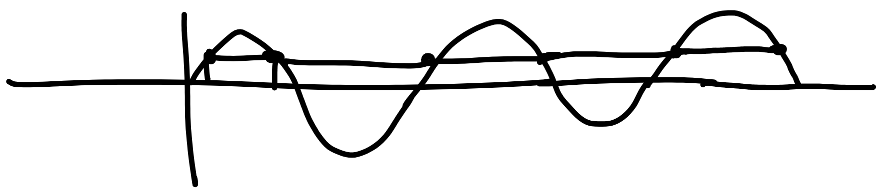
85) let $\alpha = 2\theta + 60$

a) 

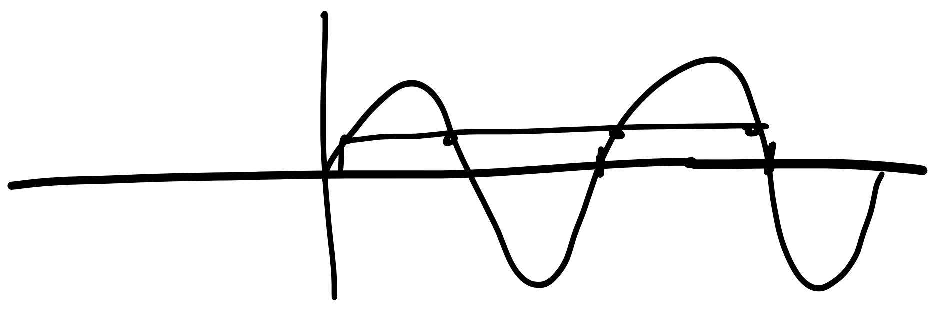
$5\sin\alpha = 4$

$\sin\alpha = \frac{4}{5}$

$\alpha = \cancel{53.1°}, 126.9°, 413.1°, 486.9°, 773.1°$

$\therefore \frac{\alpha - 60}{2} = \theta$

$\theta = 33.4°, 176.6°, 213.4°, 356.6°$

b) 

$8\tan x - 3\cos x = 0$

$\frac{8\sin x}{\cos x} = 3\cos x$

$8\sin x = 3\cos^2 x$

$8\sin x = 3 - 3\sin^2 x$

$3\sin^2 x + 8\sin x - 3 = 0$

$\sin x_1 = \frac{1}{3}$ $\quad \cancel{\sin x_2 = -3}$

$x = 379.5°, 520.5°$

86) a)

$y - y_1 = m(x - x_1)$

$7 - (-1) = m(-2-4)$

$8 = m \cdot -6$

$-\frac{4}{3} = m$

$\therefore y - 7 = -\frac{4}{3}(x+2)$

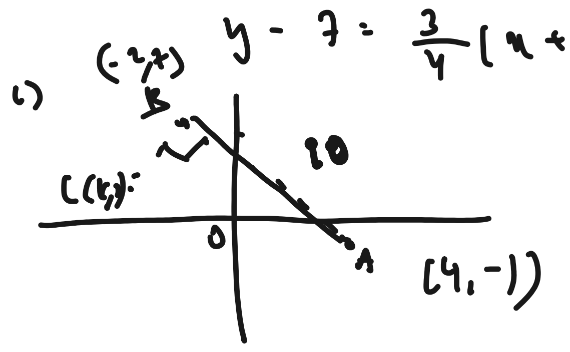
$3y - 21 = -4x - 8$

$3y + 4x - 13 = 0$

b) $m = \frac{3}{4}$

$\therefore y - y_1 = m(x - x_1)$

$y - 7 = \frac{3}{4}(x+2)$ //

c) 

$\therefore$ when $y = 1$ in L_2:

$1 - 7 = \frac{3}{4}(x+2)$

$-6 = \frac{3}{4}(x+2)$

$-\frac{24}{3} = x+2$

$-8 = x+2$

$-10 = x$ //

$\therefore k = -10$

d) circle through A, B, C

midpoint of BC : $\left(\frac{-10-2}{2}, \frac{1+7}{2}\right) = (-6, 4)$

midpoint of AB : $\left(\frac{-2+4}{2}, \frac{7-1}{2}\right) = (1, 3)$

$\therefore$ line through BC : $y = -\frac{4}{3}x + c$

$4 = -\frac{4}{3}(-6)^2 + c$

$4 = 8 + c$

$-4 = c$

line through AB : $y = \frac{3}{4}x + c$

$3 = \frac{3}{4} + c$

$2.25 = c$

$y_1 = -\frac{4}{3}x - 4$

$y_2 = \frac{3}{4}x + \frac{9}{4}$

$\therefore -\frac{4}{3}x - 4 = \frac{3}{4}x + \frac{9}{4}$

$-\frac{25}{12}x = \frac{25}{4}$

$x = -3, \; y = 0$

$\therefore$ centre $= (-3, 0)$

$\therefore (x-a)^2 + (y-b)^2 = r^2$

$(x+3)^2 + y^2 = r^2$

when $(4,-1)$

$7^2 + 1 = r^2$

$50 = r^2$

circle: $(x+3)^2 + y^2 = 50$ //

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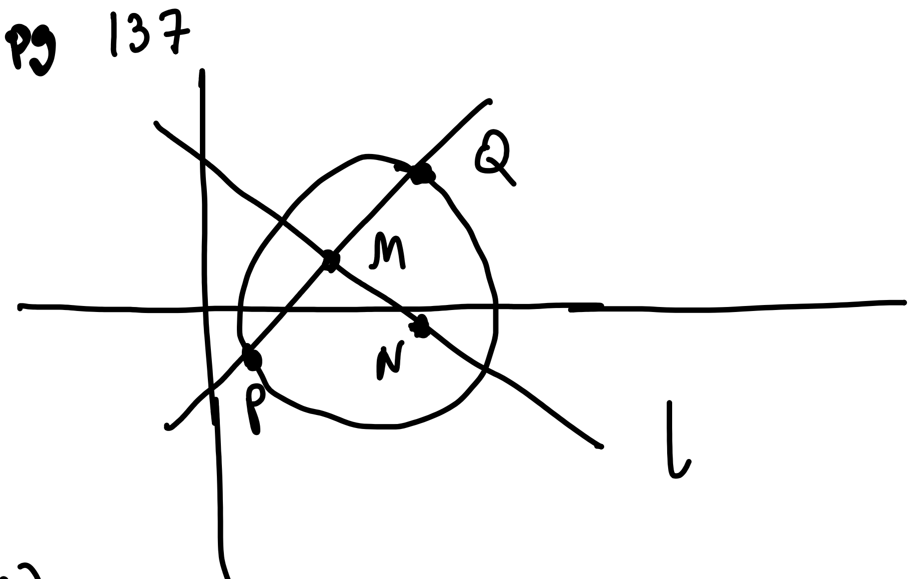
87)



a) gradient of PM :

$y - y_1 = m(x - x_1)$

$1 + 3 = m(5 - 2)$

$4 = m\,3$

$\frac{4}{3} = m$

$\therefore$ gradient of $l = -\frac{3}{4}$

$\therefore \quad y - y_1 = m(x - x_1)$

$y - 1 = -\frac{3}{4}(x - 5)$ //

b) if $x = 13$

then $y - 1 = -\frac{3}{4}(8)$

$y = -5$

$\therefore N = (13, -5)$ //

c) $(x-a)^2 + (y-b)^2 = r^2$

$(x-13)^2 + (y+5)^2 = r^2$

$\therefore$ using Point P :

$(2-13)^2 + (-3+5)^2 = r^2$

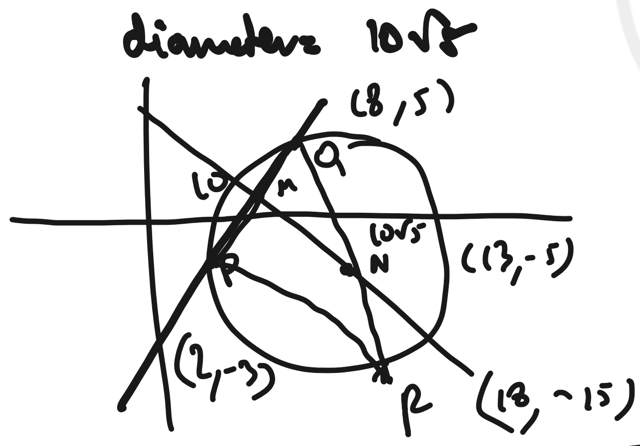
$r^2 = 125$

$\therefore (x-13)^2 + (y+5)^2 = 125$ //

d)

radius = $5\sqrt{5}$

diameter = $10\sqrt{5}$



use centre to work out R !

$\therefore |\vec{PR}| = \sqrt{(18-2)^2 + (-15 - -3)^2}$

$= \sqrt{733}$

$\therefore \angle PQR =$

$\cos A = \dfrac{10^2 + (10\sqrt{5})^2 - 733}{2(10)(10\sqrt{5})}$

$A = 68.07..$

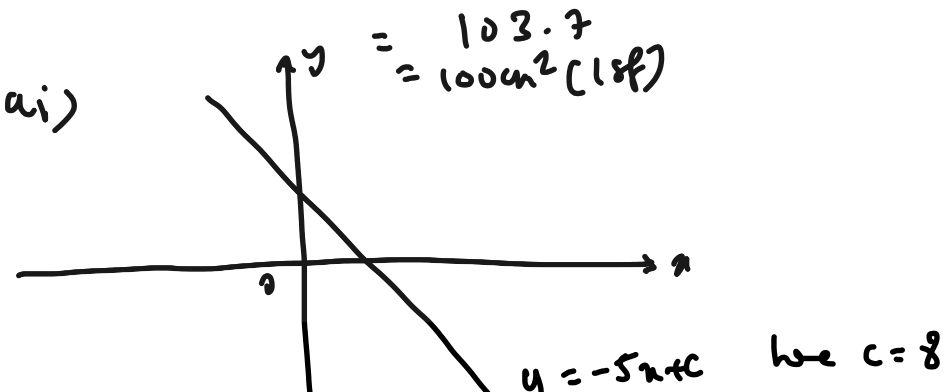
$\therefore$ area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \cdot 10 \cdot 10\sqrt{5} \cdot \sin 68.07..$

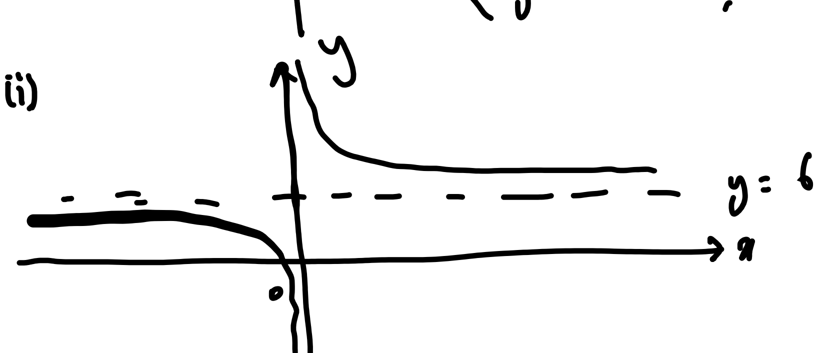
$= 103.7$

$= 100\text{cm}^2$ (1sf)

88) ai)



(i)



b) $\frac{1}{x} + 6 = -5x + c$

$5x^2 + (6 - c)x + 1 = 0$

$\therefore$ as it intersects $2 \Rightarrow b^2 - 4ac > 0$

$\therefore (6-c)^2 - 4(5)(1) > 0$

$c < 6 - 2\sqrt{5}$ or $c > 6 + 2\sqrt{5}$

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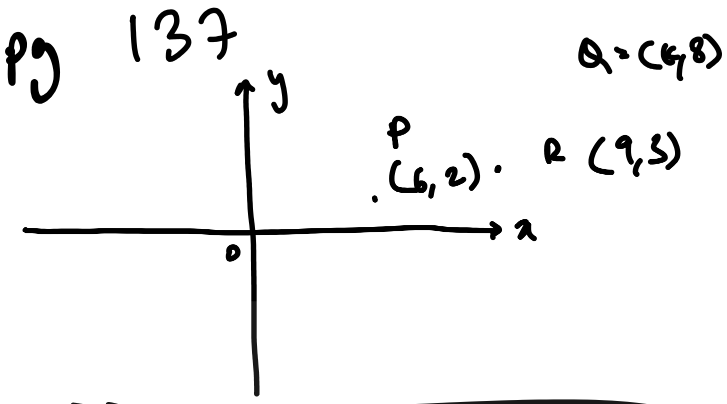
89)

a)



$|\vec{PQ}| = \sqrt{(k-6)^2 + (8-2)^2}$

$|\vec{PR}| = \sqrt{(9-6)^2 + (3-2)^2}$

$\therefore$ since $|\vec{PQ}| = 2|\vec{PR}|$

$\therefore \sqrt{(k-6)^2 + (8-2)^2} = 2\sqrt{(9-6)^2 + (3-2)^2}$

$(k-6)^2 + 36 = 4[(3)^2 + 1]$

$(k-6)^2 = 40 - 36$

$(k-6)^2 = 4$

$k - 6 = \pm 2$

$k = 6 \pm 2$

$\therefore k = 8$ or 4

b) $3x - y - 16 = 0$

$y = 3x - 16$

$\therefore y - y_1 = m(x - x_1)$

$8 - 2 = 3(x - 6)$

$6 = 3(x-6)$

$2 = x - 6$

$8 = x$

$\therefore Q = (8, 8)$

$P = (6, 2)$

$\therefore$ mid point $= (7, 5)$

90) a) $x(36-2x)(60-2x)$

$V = 4x^3 - 192x^2 + 2160x$

b) $\frac{dV}{dx} = 12x^2 - 384x + 2160$

let $\frac{dV}{dx} = 0$: $\frac{384 \pm 48\sqrt{19}}{24}$

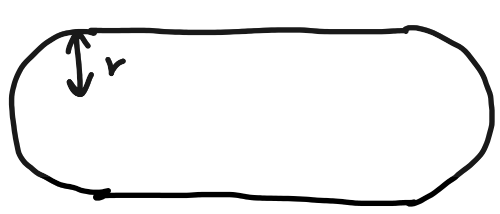
$\therefore x = 16 \pm 2\sqrt{19}$

$= 16 - 2\sqrt{19}$

c) $\therefore$ sub in x into V : $7092.4\,cm^3$

91)

a)



$2\pi r + 2x = 500$

$x = 250 - \pi r$

$\therefore$ area $= (250 - \pi r)2r + \pi r^2$

$= 500r - 2\pi r^2 + \pi r^2$

$A = 500r - \pi r^2$

b) $\frac{dA}{dr} = 500 - 2\pi r$

$\therefore$ let $\frac{dA}{dr} = 0$

$r = \frac{250}{\pi}$

$\therefore A = \frac{62500}{\pi}$

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92) $(x-3)^2+(y-k)^2=r^2$

a) $(8-3)^2+(10-k)^2=r^2$

$25+k^2-20k+100=r^2$

$k^2-20k+125=r^2$

$(16-3)^2+(-2-k)^2=r^2$

$13^2+4+4k+k^2=r^2$

$k^2+4k+173=r^2$

$\therefore\ k^2-20k+125=k^2+4k+173$

$24k=-48$

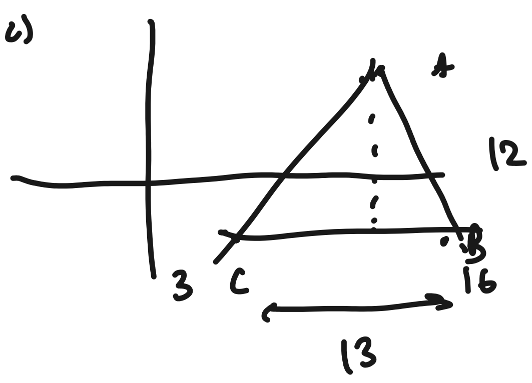
$k=\underline{-2}$ //

b) $\therefore\ (x-3)^2+(y+2)^2=r^2$

sub in a point e.g $(8,10)$ to get $r^2=169$

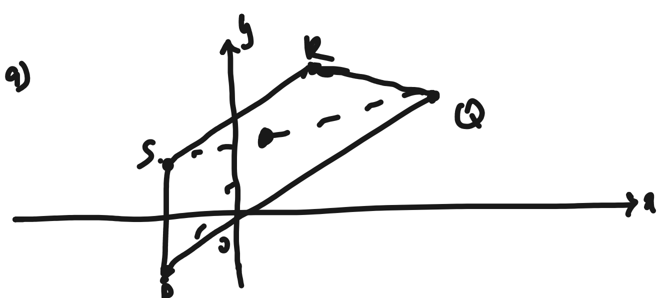
$\therefore\ (x-3)^2+(y+2)^2=169$ //

c)



$\therefore\ \frac{1}{2}\times\cancel{12}^{6}\times 13=78$ units^2

93 a)



if PQ and SR are parallel then gradient must be equal.

$\therefore\ y-y_1=m(x-x_1)$

$16-7=m(10+5)$

$9=m15$

$\frac{9}{15}=m_{SR}$

$\frac{3}{5}=m_{SR}$

$y-y_1=m(x-x_1)$

$9+6=m(15+10)$

$\frac{15}{25}=m_{PQ}$

$\frac{3}{5}=m_{PQ}$

$\therefore$ as $m_{SR}=m_{PQ}=\frac{3}{5}$, lines PQ and SR are parallel.

b) $|\overrightarrow{SP}|=\sqrt{(-5)^2+(13)^2}$

$=\sqrt{194}$

$|\overrightarrow{RQ}|=\sqrt{(5)^2+(7)^2}$

$=\sqrt{74}$

$\therefore\ |\overrightarrow{SP}|\neq|\overrightarrow{RQ}|$ $\therefore$ no pair of sides equal length $\therefore$ ⏢ is not isosceles.

c) SQ:

$y-y_1=m(x-x_1)$

$9-7=m(15+5)$

$2=m20$

$\frac{1}{10}=m$

$y=\frac{1}{10}x+c$

$9=\frac{15}{10}+c$

$\frac{75}{10}=c$

$y=\frac{1}{10}x+\frac{75}{10}$

PR:

$y-y_1=m(x-x_1)$

$16+6=m(10+10)$

$22=m20$

$\frac{11}{10}=m$

$y=\frac{11}{10}x+c$

$16=11+c$

$5=c$

$\therefore\ y=\frac{11}{10}x+5$

$\therefore$ POI

$\frac{1}{10}x+\frac{75}{10}=\frac{11}{10}x+5$

$\frac{25}{10}=x$

$x=\frac{5}{2}$ $\therefore\ y=\frac{31}{4}$: $\left(\frac{5}{2},\frac{31}{4}\right)$ //

d) midpoint of PR : $(0,5)$
midpoint of SQ : $(5,8)$ } not same $\therefore$ neither diagonal bisects other.