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EXAM QUESTION BANK

① $(6+4)^2+(3-K)^2+(-3-0)^2 = (5\sqrt{5})^2$

$100+(3-K)^2+9 = 125$

$(3-K)^2 = 16$

$3-K=4$ or $3-K=-4$

$K=-1$ $\quad K=7$

② a) $f(2) = -2$

$f(2.5) = 1$

b) $x=\frac{7}{3}$ is an asymptote, a change of sign is caused by the asymptote.

③ $m = \frac{2}{\sin x}$ $\quad \sin x = 5n$

$m = \frac{2}{5n}$

④ a) $\frac{dy}{dx} = 4x+2-e^x$

b) at $x=0 \rightarrow m_T = 2-1 = 1$

$y = x+c \rightarrow y = x-1$

$-1 = 0+c$

$c=-1$

⑤

$$\begin{array}{r|l} & 2x^2+11 \\ x^2-4 & 2x^4+3x^2-5x+2 \\ & -\underline{2x^4-8x^2} \\ & \quad 11x^2-5x+2 \\ & -\underline{11x^2+\cancel{0x}-44} \\ & \quad -5x+46 \end{array}$$

$2x^2+11+\frac{-5x+46}{x^2-4}$

$A(x+2)+B(x-2) = -5x+46$

$x=2 \rightarrow 4A = 36$

$A = 9$

$x=-2 \rightarrow -4B = 56$

$B=-14$

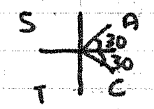
$\therefore 2x^2+11-\frac{14}{x+2}+\frac{9}{x-2}$

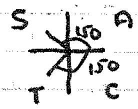
$a=2 \quad b=0 \quad c=11 \quad d=-14 \quad e=9$

⑥ $4\cos x = \frac{3}{\cos x}$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$ or $\cos x = -\frac{\sqrt{3}}{2}$





$x = 30^\circ, -30^\circ$ $\quad x = -150^\circ, 150^\circ$

$\therefore x = -150^\circ, -30^\circ, 30^\circ, 150^\circ$

⑦ a) $u = \ln y$ $\quad v = y$

$u' = \frac{1}{y}$ $\quad v' = 1$

$\frac{dx}{dy} = \frac{1-\ln y}{y^2}$

$\frac{dy}{dx} = \frac{y^2}{1-\ln y}$

b) $\frac{dy}{dx} = \frac{e^{-8}}{1-\cancel{\ln e}^{-4}} = \frac{e^{-8}}{5} = \frac{1}{5e^{-8}}$

⑧ $\int_0^{\pi/4} 6\sec^2 x - 4(\sec^2 x - 1)\,dx$

$\int_0^{\pi/4} 6\sec^2 x - 4\sec^2 x + 4\,dx$

$= \int_0^{\pi/4} 2\sec^2 x + 4\,dx = \left[2\tan x + 4x\right]_0^{\frac{\pi}{4}}$

$\left[2\tan(\frac{\pi}{4}) + 4(\frac{\pi}{4})\right] - 0 = 2+\pi$

⑨ $\frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$A(x+1)^2 + B(x+1)(x+2) + C(x+2) = 7x^2+12x$

$x=-1 \rightarrow C = -5$

$x=-2 \rightarrow A = 4$

$x=0 \rightarrow A+2B+2C = 0$

$4+2B-10 = 0$

$2B = 6$

$B = 3$

$\therefore \frac{4}{\;} + \frac{3}{\;} + \frac{5}{\;}$

(10) $t = x-1$

$$y = \big(2(x-1)+2\big)\big(2-(x-1)\big)$$
$$= 2x(3-x)$$
$$= 6x - 2x^2$$

$$3x+2 = 6x-2x^2$$
$$2x^2-3x+2=0$$

$$b^2-4ac = (9)-4(2)(2)$$
$$= -7 < 0 \quad \text{NO SOLUTIONS.}$$

(11) a) $\sin 2x \cos x + \cos 2x \sin x$

$$(2\sin x\cos x)\cos x + (1-2\sin^2 x)(\sin x)$$
$$2\sin x(1-\sin^2 x) + (1-2\sin^2 x)\sin x$$

$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$
$$= 3\sin x - 4\sin^3 x$$

b) $3\sin x - 4\sin^3 x = \sin x$

$$0 = 4\sin^3 x - 2\sin x$$
$$0 = 2\sin x(2\sin^2 x - 1)$$

$\sin x = 0$ or $\sin x = \frac{1}{\sqrt{2}}$ or $\sin x = \frac{-1}{\sqrt{2}}$

$x = 0, \pi$ $\quad x = \frac{1}{4}\pi, \frac{3}{4}\pi$ $\quad$ NOT IN RANGE

$\therefore x = 0, \frac{1}{4}\pi, \frac{3}{4}\pi, \pi$

(12) a) $\frac{1}{3}x^2 + 1 = 3\sin x$

$$\sin x = \frac{1}{9}x^2 + \frac{1}{3}$$

$$x = \arcsin\left(\frac{1}{9}x^2 + \frac{1}{3}\right)$$

b) $x_{n+1} = \arcsin\left(\frac{1}{9}x_n^2 + \frac{1}{3}\right)$

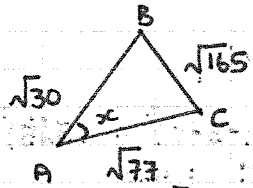
$x_0 = 0.5$
$x_1 = 0.369$
$x_2 = 0.356$
$x_3 = 0.355$

c) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$= 0.4 - \left[\frac{\frac{1}{3}(0.4)^2 - 3\sin(0.4) + 1}{\frac{2}{3}(0.4) - 3\cos(0.4)}\right]$$

$= 0.354$ (3 d.p.)

(13)



$$\overrightarrow{AC} = -2i - 8j + 3k$$

$$|AC| = \sqrt{77}$$

$$(\sqrt{165})^2 = (\sqrt{30})^2 + (\sqrt{77})^2 - 2(\sqrt{30})(\sqrt{77})\cos x$$

$$\frac{165-30-77}{-2(\sqrt{30})(\sqrt{77})} = \cos x$$

$$\cos x = -0.603\ldots$$
$$x = 127.1^\circ$$

(14) a) $9^{\frac{1}{2}}\left(1+\frac{x}{9}\right)^{\frac{1}{2}} = 3\left(1+\frac{x}{9}\right)^{\frac{1}{2}}$

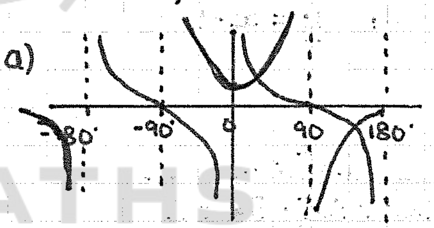
$$3\left[1 + \left(\tfrac{1}{2}\right)\left(\tfrac{x}{9}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{x}{9}\right)^2}{2!} + \ldots\right]$$

$$= 3\left(1 + \frac{x}{18} - \frac{x^2}{648} + \ldots\right)$$

$$= 3 + \frac{x}{6} - \frac{x^2}{216} + \ldots \qquad \therefore k = -\frac{1}{216}$$

b) $|x| < 9$, So it is Valid For $x = 1$.

(15) a)



b) 2 $\quad$ c) $\frac{1}{6}$ $\quad$ $\boxed{3}\sec x$ max = 3
+3 max = 6

(16) a) $a = \frac{F}{m} = \left(-\frac{2}{3}i - \frac{8}{3}j + \frac{4}{3}k\right) ms^{-2}$

b) $\cos x = \dfrac{-8}{3} \div \left(\dfrac{14}{\sqrt{21}}\right) = \dfrac{-4\sqrt{21}}{21}$

$$x = 151^\circ$$

(17) LHS $= \dfrac{1+(\sec^2 x - 1)}{1-(\sec^2 x - 1)} = \dfrac{\sec^2 x}{2-\sec^2 x}$

$$= \frac{1}{\cos^2 x} \div \left(2 - \frac{1}{\cos^2 x}\right) = \frac{1}{\cos^2 x} \div \left(\frac{2\cos^2 x - 1}{\cos^2 x}\right)$$

$$= \frac{1}{2\cos^2 x - 1}$$

RHS $= \sec 2x = \dfrac{1}{\cos 2x} = \dfrac{1}{2\cos^2 x - 1}$ $\quad$ LHS = RHS

(18) a) $\alpha = \frac{4}{5}\sqrt{\alpha+2}$

$$0 = \frac{4}{5}\sqrt{\alpha+2} - \alpha$$

$\alpha = 1 \rightarrow 0.3856 > 0$
$\alpha = 2 \rightarrow -0.4 < 0$

Sign change implies at least one root in interval.

b) $x_1 = 1.47,\ x_2 = 1.49,\ x_3 = 1.49$
converges towards α.
Iteration is suitable.

(19) $(2t)^2 + (4t)^2 + (-2t)^2 = (9\sqrt{6})^2$
$4t^2 + 16t^2 + 4t^2 = 486$
$24t^2 = 486$
$t^2 = \frac{81}{4}$
$t = \pm 4.5$

(20) $u = \ln(x^2+1)$, $v' = x^3$
$u' = \frac{2x}{x^2+1}$, $v = \frac{x^4}{4}$

$$= \frac{x^4}{4}\ln(x^2+1) - \int \frac{x^5}{2(x^2+1)}\,dx$$

$$= \frac{x^4}{4}\ln(x^2+1) - \frac{1}{2}\int \frac{x^5}{x^2+1}\,dx$$

$$\begin{array}{r} x^3 \qquad\qquad -x \qquad\qquad \\ x^2+1 \,\big)\, x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 0 \\ -\;\underline{x^5 + 0x^4 + x^3} \qquad\qquad\qquad \\ -x^3 + 0x^2 + 0x \qquad \\ -\;\underline{-x^3 + 0x^2 - x} \qquad \\ x \qquad \end{array}$$

$$\frac{x^5}{x^2+1} = x^3 - x + \frac{x}{x^2+1}$$

$$-\frac{1}{2}\int x^3 - x + \frac{x}{x^2+1}\,dx = -\frac{1}{2}\left(\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2}\ln(x^2+1)\right)$$

$$= -\frac{x^4}{8} + \frac{x^2}{4} - \frac{1}{4}\ln(x^2+1)$$

$\ln(x^2+1) \longrightarrow \frac{2x}{x^2+1}$

$\frac{1}{2}\ln(x^2+1) \longleftarrow \frac{x}{x^2+1}$

$$= \frac{x^4}{4}\ln(x^2+1) - \frac{x^4}{8} + \frac{x^2}{4} - \frac{1}{4}\ln(x^2+1) + C$$

$$= \frac{x^4}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2+1) - \frac{1}{8}x^2(x^2-2) + C$$

(21) a) $\operatorname{cosec}^2 P = 1 + \cot^2 P$

$$\frac{1}{\sin^2 P} = 1 + (-2)^2$$

$$\sin P = \pm\frac{1}{\sqrt{5}}$$

b) $\frac{\cos P}{\sin P} = -2$ $\quad \times \pm\frac{1}{\sqrt{5}}$

$$\cos P = \pm\frac{2}{\sqrt{5}} \rightarrow \sec P = \pm\frac{\sqrt{5}}{2}$$

(22) a) $h(9) = 0.71111\ldots > 0$
$h(10) = -2.784\ldots < 0$

Sign change implies at least one root in interval between 9 and 10.

b) $x - \frac{f(x)}{f'(x)} = 9 - \frac{h(9)}{h'(9)}$

$$= 9 - \left(\frac{0.7111620183}{-9e^{-0.5(9)} + \frac{10}{3}\cos(3) - \frac{1}{10}}\right)$$

-3.499955958

$$= 9.203$$

c) $h(9.2035) = -0.0028\ldots < 0$
$h(9.2025) = 0.0007\ldots > 0$

Sign change implies at least one root in interval, so 9.203 is correct to 3 d.p.

(23) $2\ln(e^x+1) \rightarrow \frac{e^x}{e^x+1}$ $(\times 2)$
$(\times 2)$

$$\left[\ln(e^x+1)^2\right]_{\ln 2}^{\ln a} = \ln 4$$

$\ln(a+1)^2 - \ln(3)^2 = \ln 4$
$\ln(a+1)^2 = \ln 4 + \ln 9$
$\ln(a+1)^2 = \ln 36$

$a + 1 = \sqrt{36}$

$a+1 = 6$ or $a+1 = -6$
$a = 5$ $\quad a = -7$

24 a) $a = 2 \quad d = 6$

$U_{20} = 2 + (19 \times 6) = 116$

b) $\frac{n}{2}(4 + (n-1)6) = 7450$

$\frac{n}{2}(6n - 2) - 7450 = 0$

$3n^2 - \cancel{2}n - 7450 = 0$

$3n^2 - n - 7450 = 0$

$n = 50$ ✓ or -49.66 Rej.

25 a) $\sin\theta \approx \theta \quad \cos\theta \approx 1 - \frac{\theta^2}{2}$

$\tan\theta \approx \theta$

$$\frac{1 + 2\theta + 2\theta}{2\left(1 - \frac{(2\theta)^2}{2}\right) - 1} = \frac{4\theta + 1}{2 - 4\theta^2 - 1} = \frac{4\theta + 1}{1 - 4\theta^2}$$

$$\approx \frac{4\theta + 1}{(1 - 2\theta)(1 + 2\theta)}$$

b) $\theta = 0.03$ rad $\rightarrow \approx 1.124046568$ (Estimate)

≈ 1.124 (3d.p.)

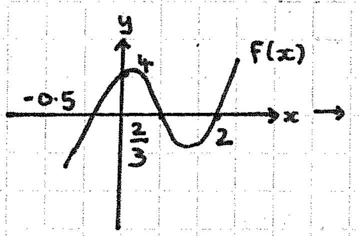
c) Exact $= 1.12408159$

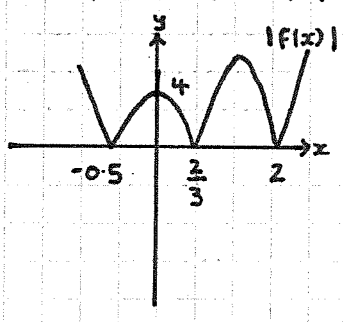
$$\frac{\text{Exact - Estimate}}{\text{Exact}} \times 100 = 3.12 \times 10^{-3} \%$$

26

$$\begin{array}{r} 6x^2 - x - 2 \\ x - 2 \overline{\big) 6x^3 - 13x^2 + 0x + 4} \\ -\underline{6x^3 - 12x^2} \\ -x^2 + 0x + 4 \\ -\underline{-x^2 + 2x} \\ -2x + 4 \\ -\underline{-2x + 4} \\ 0 \end{array}$$

$(x-2)(6x^2 - x - 2) = (x-2)(3x-2)(2x+1)$





27 a) $a_2 = 3K - 5$

b) $a_3 = 3(3K - 5) - 5 = 9K - 20$

c) $a_4 = 3(9K - 20) - 5 = 27K - 65$

$K + 3K - 5 + 9K - 20 + 27K - 65 = -30$

$40K - 90 = -30$

$40K = 60$

$K = \frac{3}{2} = 1.5$

28 a) $y = \cos 2x - (\sin 2x)^2$

$$\frac{dy}{dx} = -2\sin 2x - 2.2.\cos 2x \sin 2x$$

$$= -2\sin 2x - 4\cos 2x \sin 2x$$

$$= -2\sin 2x - 2\sin 4x$$

$$\frac{d^2y}{dx^2} = -4\cos 2x - 8\cos 4x$$

b) $2\sin 2x + 4\cos 2x \sin 2x = 0$

$\sin 2x + 2\cos 2x \sin 2x = 0$

$\sin 2x (1 + 2\cos 2x) = 0$

$\sin 2x = 0$ or $\cos 2x = -\frac{1}{2}$

$x = -\frac{\pi}{2}, -\pi \qquad x = -\frac{2\pi}{3}, -\frac{\pi}{3}$

c) $x = -\frac{\pi}{2}$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = -4 \leq 0$

Implies local max.

29 a) $x^3 = 0 \qquad \ln x = 0$

$x = 0 \qquad x = e^0 = 1$

$A(0,0) \qquad B(1,0)$

b) $U = x^3 \quad V = \ln x$

$U' = 3x^2 \quad V' = \frac{1}{x}$

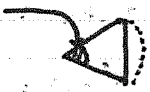
$f'(x) = x^2 + 3x^2 \ln x$

c) $f'(0.7165) = -0.00006\ldots < 0$

$f'(0.7175) = 0.0020\ldots > 0$

Sign change implies at least one turning point in interval, so x coordinate of P is 0.717 to 3d.p.

30 a) $\text{Acute} = 2\pi - 4.2$



$\frac{1}{2}r^2(4.2) = 88.725$

$r^2 = \frac{169}{4} \rightarrow r = \frac{13}{2}$

$l = r\theta = \frac{13}{2}(2\pi - 4.2) = (13\pi - 27.3)\text{ cm}$

b) $S = \frac{1}{2}\left(\frac{169}{4}\right)(2\pi - 4.2) - \frac{1}{2}\left(\frac{169}{4}\right)\sin(2\pi - 4.2)$

$= 25.6\text{ cm}^2$

31 a) $\cos t = \frac{x+4}{5} \quad \sin t = \frac{y-1}{5}$

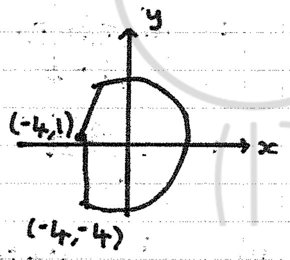
$\left(\frac{x+4}{5}\right)^2 + \left(\frac{y-1}{5}\right)^2 = 1$

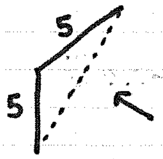
$(x+4)^2 + (y-1)^2 = 5^2$

$(x+4)^2 + (y-1)^2 = 25$

b) $-4 \le x \le -4 + \frac{5}{2}\sqrt{2}$

$-4 \le y \le 1 + \frac{5}{2}\sqrt{2}$





$\sqrt{\left(-4+\frac{5}{2}\sqrt{2}\right)^2 + \left(1+\frac{5}{2}\sqrt{2}\right)^2}$

$= \sqrt{50 + 25\sqrt{2}}$

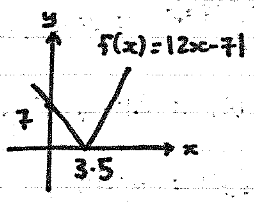
$50 + 25\sqrt{2} = 25 + 25 - 2(5)(5)\cos x$

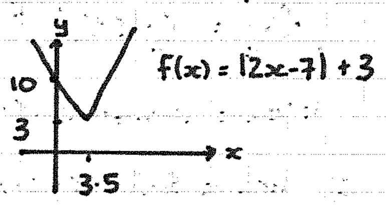
$\cos x = -\frac{\sqrt{2}}{2}$

$x = \frac{3}{4}\pi \rightarrow l = r\theta = 5\left(\frac{3\pi}{4}\right)$

$= \frac{15}{4}\pi$

32 a)





b) $f(x) \ge 3$

c) $2x - 7 + 3 > x + 2 \qquad -2x + 7 + 3 > x + 2$

$x > 6 \qquad -3x > -8$

$\{x : x < \frac{8}{3}\} \cup \{x : x > 6\}$

33 a) $(4 - 3x)^{-\frac{1}{2}}$

$= 4^{-\frac{1}{2}}\left(1 - \frac{3}{4}x\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left(1 - \frac{3}{4}x\right)^{-\frac{1}{2}}$

$\frac{1}{2}\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{3}{4}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{4}x\right)^2}{2} + \ldots\right]$

$= \frac{1}{2}\left(1 + \frac{3}{8}x + \frac{27}{128}x^2 + \ldots\right)$

$= \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \ldots$

b) $(x+8)\left(\frac{1}{2} + \frac{3x}{16} + \frac{27x^2}{256}\right)$

$= \frac{1}{2}x + \frac{3x^2}{16} + 4 + \frac{3x}{2} + \frac{27x^2}{32}$

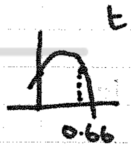
$= 4 + 2x + \frac{33}{32}x^2$

34 a) $x = 0 \rightarrow t = 0 \rightarrow y = 1.9$

b) $2.8 = -4.9t^2 + 4.6t + 1.9$

$4.9t^2 - 4.6t + 0.9 = 0$

$t = 0.66\text{ s} \quad \text{or} \quad t = 0.28$



c) $x = 3.8(0.66) = 2.51\text{m}$

35 a) She used $\frac{\cos\theta}{\sin\theta} = \tan\theta$

She should have used

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

b) $\sqrt{3}\sin\left(-\frac{\pi}{6}\right) \ne \cos\left(-\frac{\pi}{6}\right)$

So can't be a solution.
Error was caused by squaring.

36) $y = \cos t \cos \frac{\pi}{3} - \sin t \sin \frac{\pi}{3}$

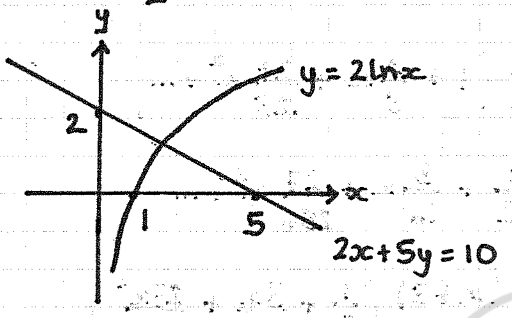
$$y = \frac{1}{2}\cos t - \frac{\sqrt{3}}{2}\sin t$$

$\cos t = x \qquad \sin^2 t = 1 - \cos^2 t$
$\qquad\qquad\qquad\quad = 1 - x^2$

$$\sin t = \sqrt{1-x^2}$$

$$y = \frac{1}{2}x - \frac{\sqrt{3}}{2}\sqrt{1-x^2}, \quad -1<x<1$$

37) a)



b) $2\ln x = 2 - \frac{2}{5}x$

$2\ln x + \frac{2}{5}x - 2 = 0 \quad (\times 5)$

$10\ln x + 2x - 10 = 0$

As the curve and line intersect once, f(x) has exactly one root.

c) $f(1{\cdot}8) = -0{\cdot}5221\ldots < 0$
$f(1{\cdot}9) = 0{\cdot}2185\ldots > 0$

Sign change implies the root is in this interval.

38) $\left(\frac{4}{5}\right)\ln(5x+k) \rightleftarrows \frac{5\left(\frac{4}{5}\right)}{5x+k}$

$$\left[\frac{4}{5}\ln(5x+k)\right]_{3k}^{4k}$$

$$= \frac{4}{5}\ln 21k - \frac{4}{5}\ln 16k = \frac{4}{5}\ln\left(\frac{21k}{16k}\right)$$

$$= \frac{4}{5}\ln\left(\frac{21}{16}\right)$$

Answer is independent of k, as k does not appear in the solution.

39) $f'(x) = 2 - \ln 4 \cdot 4^{-x}$

$f'(-1) = 2 - \ln 4 \cdot 4^{1}$
$= 2 - 4\ln 4 = 2 - 4\ln 2^2$
$= 2 - 8\ln 2 = m_T$

$f(-1) = 2(-1) + 4^{--1} = -2 + 4 = 2$

$m_T = 2 - 8\ln 2 \quad (-1, 2)$

$y = (2 - 8\ln 2)x + c$

$2 = -2 + 8\ln 2 + c$
$c = 4 - 8\ln 2$

$y = (2-8\ln 2)x + 4 - 8\ln 2$

$0 = (2-8\ln 2)x - y + 4 - 8\ln 2$

40) a) $\dfrac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$

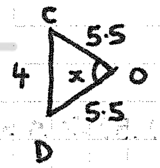
$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}+3+1+\sqrt{3}}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$$

b) $\dfrac{1}{-2-\sqrt{3}} \cdot \dfrac{(-2+\sqrt{3})}{(-2+\sqrt{3})} = \dfrac{-2+\sqrt{3}}{4-3} = -2+\sqrt{3}$

41) a)

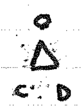


$4^2 = (5{\cdot}5)^2 + (5{\cdot}5)^2 - 2(5{\cdot}5)(5{\cdot}5)\cos x$

$$\cos x = \frac{4^2 - (5{\cdot}5)^2 - (5{\cdot}5)^2}{-2(5{\cdot}5)(5{\cdot}5)}$$

$$x = 0{\cdot}744 \quad (3\text{ d.p.})$$

b)



$= \frac{1}{2}(5{\cdot}5)(5{\cdot}5)\sin(0{\cdot}744)$
$= 10{\cdot}246\text{ m}^2$

$= \frac{1}{2}(5{\cdot}5)^2\left(\frac{\pi - 0{\cdot}744}{2}\right) = 18{\cdot}131\text{ m}^2$

Area $= 10{\cdot}246 + 18{\cdot}131 = 28{\cdot}4\text{ m}^2$ (1 d.p.)

c) Arc $= 5{\cdot}5\left(\frac{\pi - 0{\cdot}744}{2}\right) = 6{\cdot}59$

$P = 5{\cdot}5 + 5{\cdot}5 + 6{\cdot}59 = 17{\cdot}6\text{m}$

(42) a) $(\sec^2x - \tan^2x)(\sec^2x + \tan^2x)$

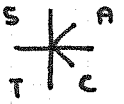
$= (1)(\sec^2x + \sec^2x - 1)$
$= 2\sec^2x - 1$

b) $2\sec^2x - 1 = 24$
$2\sec^2x = 25$
$\sec^2x = \frac{25}{2}$

$\cos^2x = \frac{2}{25} \rightarrow \cos x = \pm\frac{\sqrt{2}}{5}$

$\cos x = \frac{2}{\sqrt{5}}$ and $\cos x = -\frac{2}{\sqrt{5}}$

$x = 73.57^\circ, 286.4^\circ$ $\quad$ $x = 106.4^\circ, 253.6^\circ$



(43) a) $12x - 12y.\frac{dy}{dx} - 4x.\frac{dy}{dx} - 4y = 0$

$12x - 4y = 12y.\frac{dy}{dx} + 4x.\frac{dy}{dx}$ $(\div 4)$

$3x - y = 3y.\frac{dy}{dx} + x.\frac{dy}{dx}$

$3x - y = \frac{dy}{dx}(x + 3y)$

$\frac{dy}{dx} = \frac{3x - y}{x + 3y}$

b) $-\frac{3}{5} = \frac{3x - y}{x + 3y}$

$-3(x + 3y) = 5(3x - y)$
$-3x - 9y = 15x - 5y$
$0 = 18x + 4y$ $(\div 2)$
$0 = 9x + 2y$

(44) a)

$$\begin{array}{r|l} & -2x^2 + 2x - 1 \\ x - 3 & -2x^3 + 8x^2 - 7x + 3 \\ & -\underline{(-2x^3 + 6x^2)} \\ & 2x^2 - 7x \\ & -\underline{(2x^2 - 6x)} \\ & -x + 3 \\ & -\underline{(-x + 3)} \\ & 0 \end{array}$$

$(x - 3)(-2x^2 + 2x - 1)$

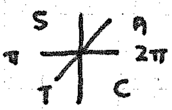
b) $y^2 = x$

$y^2 - 3 = 0$
$y^2 = 3$
$y = \pm\sqrt{3}$

$b^2 - 4ac = 4 - 4(-2)(-1)$
$= 4 - 8$
$= -4 < 0$
No real solutions

2 real solutions

c) $\tan\theta = 3$ $\quad$ $0 \le \theta < 3\pi$



3 solutions

(45) a) $t^{\frac{1}{2}} = x$
$t = x^2$
$t^2 = x^4$

$y = 2x - 4x^4, \; 0 \le x \le \sqrt{3}$

b) $\frac{dy}{dt} = t^{-\frac{1}{2}} - 8t = 0$

$\frac{1}{t^{1/2}} = 8t$

$\frac{1}{8} = t^{3/2}$

$t = \left(\frac{1}{8}\right)^{2/3} = \frac{1}{4}$

$y = 2\left(\frac{1}{4}\right)^{\frac{1}{2}} - 4\left(\frac{1}{4}\right)^2 = \frac{3}{4}$

$y = 2(3)^{\frac{1}{2}} - 4(3)^2 = 2\sqrt{3} - 36$

$2\sqrt{3} - 36 \le f(x) \le \frac{3}{4}$

(46) a) $2\sqrt{3} = k\sin\left(\frac{\pi}{3}\right)$

$k = \frac{2\sqrt{3}}{\sin\left(\frac{\pi}{6}\right)} = 4$

b) $y = 4\sin\left(x + \frac{\pi}{3}\right)$

when $y = 0 \rightarrow$ $\sin\left(x + \frac{\pi}{3}\right) = 0$

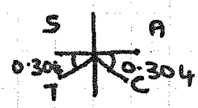
$x + \frac{\pi}{3} = \pi, 2\pi$

$x = \frac{2}{3}\pi, \frac{5}{3}\pi \rightarrow p = \frac{2}{3}\pi$

$q = \frac{5}{3}\pi$

c) $-1.2 = 4\sin\left(x + \frac{\pi}{3}\right)$

$\sin\left(x + \frac{\pi}{3}\right) = -0.3$



$x + \frac{\pi}{3} = 3.446, 5.978$

$x = 2.40, 4.93$

(47) $u = \text{cosec}\,x$ $\qquad 1+\cot^2 x = \text{cosec}^2 x$

$\frac{du}{dx} = -\text{cosec}\,x \cot x \qquad 1+\cot^2 x = u^2$

$\qquad \cot^2 x = u^2 - 1$

$$\frac{dx}{du} = \frac{-du}{\text{cosec}\,x \cot x}$$

when $x = \frac{\pi}{2} \to u = 0+1 = 1$

when $x = \frac{\pi}{4} \to u = \sqrt{2}$

$$\int_{\sqrt{2}}^{1} \cot^{3} x \cdot \frac{du}{-\cancel{\cot x}\,\text{cosec}\,x} = -\int_{\sqrt{2}}^{1} \frac{\cot^2 x}{\text{cosec}\,x} \cdot du$$

$$= -\int_{\sqrt{2}}^{1} \frac{u^2-1}{u} \cdot du = -\int_{\sqrt{2}}^{1} u - \frac{1}{u} \cdot du$$

$$= -\left[\frac{u^2}{2} - \ln u \right]_{\sqrt{2}}^{1} = -\left[\frac{1}{2} - 0 - \left(1 - \ln\sqrt{2}\right)\right]$$

$$= -\left(-\frac{1}{2} + \ln 2^{\frac{1}{2}}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\ln 2$$

$$= \frac{1}{2}(1 - \ln 2)$$

(48) a) $y = 0 \to t = \frac{1}{3} \to x = -\frac{5}{3} \quad A\left(-\frac{5}{3}, 0\right)$

$x = 0 \to t = 2 \to y = -\frac{5}{2} \quad B\left(0, -\frac{5}{2}\right)$

b) $m = \frac{5}{2} \div -\frac{5}{3} = -\frac{3}{2}$

$y = -\frac{3}{2}x - \frac{5}{2} \quad (\times 2)$

$2y = -3x - 5$

$3x + 2y + 5 = 0$

(49) a) $A(x+1)(x+2) + B(3x+1)(x+2) + C(3x+1)(x+1) = 12x^2 + 20x + 12$

$x = -1 \to \quad -2B = 4 \quad \to \quad B = -2$

$x = -2 \to \quad 5C = 20 \quad \to \quad C = 4$

$x = 0 \to \quad 2A + 2B + C = 12$

$2A - 4 + 4 = 12$

b) $$\int_0^2 \frac{6}{3x+1} - \frac{2}{x+1} + \frac{4}{x+2} \cdot dx$$

$$\left[2\ln(3x+1) - 2\ln(x+1) + 4\ln(x+2) \right]_0^2$$

$$\ln 7^2 - \ln 3^2 + \ln 4^2 - (\ln 1 - \ln 1 + \ln 2^4)$$

$$= \ln \frac{7^2 \times 4^2}{3^2 \times 2^4} = \ln \frac{784}{9}$$

(50) a) $\text{LHS} = \frac{2\sin\theta\cos\theta}{1} - \frac{\sin\theta}{\cos\theta}$

$$= \frac{2\sin\theta\cos^2\theta - \sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}(2\cos^2\theta - 1)$$

$$= \tan\theta\cos 2\theta = \text{RHS}$$

b) $(\sec^2 x + 3)(\tan x \cos 2x) = 4\tan^2 x \cos 2x$

$(\sec^2 x + 3)(\sin 2x - \tan x) - 4\tan x \tan x \cos 2x$

$(\tan x \cos 2x)$

$(\tan x)(\cos 2x)(\sec^2 x + 3 - 4\tan x) = 0$

$\tan x = 0 \qquad \cos 2x = 0 \qquad \tan^2 x - 4\tan x + 4 = 0$

$(\tan x - 2)^2 = 0$

$\tan x = 2$

$x = 0$

$x = \frac{1}{4}\pi, -\frac{\pi}{4} \qquad x = 1.107$

$\therefore x = -\frac{\pi}{4}, 0, \frac{\pi}{4}, 1.107$

(51) a) $9^{\frac{1}{2}}\left(1 + \frac{7}{9}x\right)^{\frac{1}{2}} = 3\left(1 + \frac{7}{9}x\right)^{\frac{1}{2}}$

$$3\left[1 + \left(\tfrac{1}{2}\right)\left(\tfrac{7}{9}x\right) + \frac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(\tfrac{7}{9}x\right)^2}{2} + \dots\right]$$

$$3\left(1 + \frac{7}{18}x - \frac{49}{648}x^2 + \dots\right)$$

$$3 + \frac{7}{6}x - \frac{49}{216}x^2 + \dots$$

(51) b) i) $x = \frac{9}{25} \rightarrow \left(9+7\left(\frac{9}{25}\right)\right)^{\frac{1}{2}} = \frac{12\sqrt{2}}{5}$

$\cancel{5}\sqrt{2} = \frac{12\sqrt{2}}{5} \times \frac{5}{12}$

$\frac{5}{12}\left(3 + \frac{7}{6}\left(\frac{9}{25}\right) - \frac{49}{216}\left(\frac{9}{25}\right)^2\right)$

$\frac{5}{12}\left(3 + \frac{21}{50} - \frac{147}{5000}\right) = 1.41275$

$\frac{141275}{100000} \approx \frac{5651}{4000}$

ii) Valid when $|x| < \frac{9}{7}$

and $\frac{9}{25} < \frac{9}{7}$

(52) a)

$u = x^2 \quad v = (2-5x)^5$
$u' = 2x \quad v' = (-5)(5)(2-5x)^4$
$\quad = -25(2-5x)^4$

$f'(x) = -25x^2(2-5x)^4 + 2x(2-5x)^5$

$u = -25x^2 \quad v = (2-5x)^4 \qquad u = 2x \quad v = (2-5x)^5$
$u' = -50x \quad v' = -20(2-5x)^3 \qquad u' = 2 \quad v' = -25(2-5x)^4$

$f''(x) = 500x^2(2-5x)^3 - 50x(2-5x)^4 - 50x(2-5x)^4 + 2(2-5x)^5$
$= 500x^2(2-5x)^3 - 100x(2-5x)^4 + 2(2-5x)^5$

b) $2(2-5x)^3\left(250x^2 - 50x(2-5x) + (2-5x)^2\right) = 0$

$2(2-5x)^3(250x^2 - 100x + 250x^2 + 4 - 20x + 25x^2) = 0$
$2(2-5x)^3(525x^2 - 120x + 4) = 0$

$x = 0.4 \qquad x = 0.19$ or $x = 0.04$

c) $f'(0.4) = 0$
$f'(0.35) = -0.011... < 0$
$f'(0.45) = -0.0206... < 0$

-ve 0 -ve

So there is a point of inflection.

(53) a) $x = 1.5 \rightarrow y = 3(1.5)^2 e^{-2(1.5)} = 0.3361$

b) $A \approx \frac{1}{2}(0.5)\left[0 + 0.0669 + 2(0.2759 + 0.4060 + 0.3361 + 0.2198 + 0.1263)\right]$

≈ 0.699 (3 s.f.)

c)

$u = 3x^2 \quad v' = e^{-2x}$
$u' = 6x \quad v = -\frac{1}{2}e^{-2x}$

$-\frac{3}{2}x^2e^{-2x} + 3\int xe^{-2x}\,dx$

$u = x \quad v' = e^{-2x}$
$u' = 1 \quad v = -\frac{1}{2}e^{-2x}$

$-\frac{3}{2}x^2e^{-2x} + 3\left(-\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx\right)$

$= \left[-\frac{3}{2}x^2e^{-2x} - \frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x}\right]_0^3$

$= -\frac{27}{2}e^{-6} - \frac{9}{2}e^{-6} - \frac{3}{4}e^{-6} - \left(-\frac{3}{4}\right)$

$= \frac{3}{4} - \frac{75}{4}e^{-6} = \frac{3}{4} - \frac{75}{4e^6}$

(54) a)

$\cos t = 6x$
$\cos^2 t = 36x^2$
$\cos^3 t = 216x^3$

$y = \cos(2t + t)$
$= \cos 2t \cos t - \sin 2t \sin t$
$= (2\cos^2 t - 1)\cos t - (2\sin t \cos t)\sin t$
$= 2\cos^3 t - \cos t - 2\sin^2 t \cos t$
$= 2\cos^3 t - \cos t - 2(1 - \cos^2 t)\cos t$
$= 2\cos^3 t - \cos t - 2\cos t + 2\cos^3 t$
$= 4\cos^3 t - 3\cos t$
$= 4(216x^3) - 3(6x)$
$= 864x^3 - 18x$
$= 18x(48x^2 - 1)$

b) $\frac{\pi}{6} \le t \le \frac{5\pi}{6} \rightarrow -\frac{\sqrt{3}}{12} \le x \le \frac{\sqrt{3}}{12}$

c)

$u = 18x \quad v = 48x^2 - 1$
$u' = 18 \quad v' = 96x$

$\frac{dy}{dx} = 1728x^2 + 864x^2 - 18 = 0$

$2592x^2 = 18$
$x^2 = \frac{18}{2592} = \frac{1}{144}$

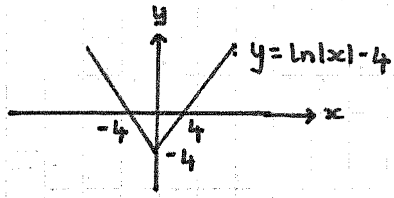
$x = \pm\sqrt{\frac{1}{144}} = \frac{1}{12}$ or $-\frac{1}{12}$

$x = \frac{1}{12} \rightarrow y = 864\left(\frac{1}{12}\right)^3 - 18\left(\frac{1}{12}\right) = -1 \quad \left(\frac{1}{12}, -1\right)$

$x = -\frac{1}{12} \rightarrow y = 864\left(-\frac{1}{12}\right)^3 - 18\left(-\frac{1}{12}\right) = 1 \quad \left(-\frac{1}{12}, 1\right)$

(55) a) $2x+8-2=x+3$ $\quad x=-3$ $\qquad -2x-8-2=x+3$ $\quad -3x=13$ $\quad x=-\frac{13}{3}$

b) i)



ii) $|x-4|$ is always positive
$|x|-4$ is negative when $|x|<4$

for $x\geq 4$, $|x|-4 = |x-4|$

for $x\leq -4$, $|x-4| = 4-x$ and $|x|-4=-4-x$

So $\quad |x|-4 < |x-4|$

(56) a) $\frac{dV}{dt} = 200 - \frac{1}{5}V \qquad (\times 5)$

$5\frac{dV}{dt} = 1000 - V \qquad (\times -1)$

$-5\frac{dV}{dt} = V-1000$

b) $-5.dV = (V-1000).dt$

$\int \frac{1}{V-1000}.dV = \int -\frac{1}{5}.dt$

$\ln(V-1000) = -\frac{1}{5}t + c$

$V=9000,\ t=0 \rightarrow \quad c = \ln 8000$

$\ln(V-1000) = -\frac{1}{5}t + \ln 8000$

$\ln\left(\frac{V-1000}{8000}\right) = -\frac{1}{5}t$

$\frac{V-1000}{8000} = e^{-\frac{1}{5}t}$

$V = 8000e^{-\frac{1}{5}t} + 1000$

c) As $t\to\infty$, $8000e^{-\frac{1}{5}t}\to 0$, $V\to 1000$

Min Volume $= 1000\text{m}^3$

(57) a) $(1+6x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$

$\left(1+\left(\frac{1}{2}\right)(6x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(6x)^2}{2}\right)\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-x)^2}{2}\right)$

$\left(1+3x-\frac{9}{2}x^2\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2\right)$

$\approx 1+\frac{1}{2}x+\frac{3}{8}x^2+3x+\frac{3}{2}x^2-\frac{9}{2}x^2$

$\approx 1+\frac{7}{2}x-\frac{21}{8}x^2$

b) Expansion is valid for $|x|<\frac{1}{6}$ and $\frac{1}{5}>\frac{1}{6}$

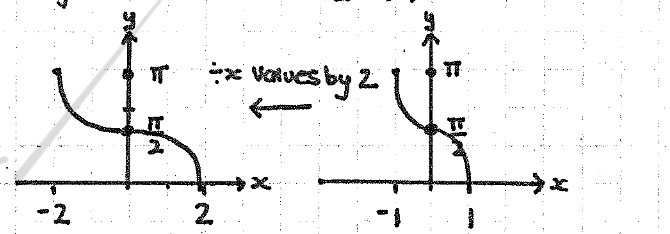
c) $x=\frac{2}{65} \qquad \sqrt{\frac{1+6\left(\frac{2}{65}\right)}{1-\left(\frac{2}{65}\right)}} = \frac{\sqrt{11}}{3}$

$\approx 3\left[1+\frac{7}{2}\left(\frac{2}{65}\right)-\frac{21}{8}\left(\frac{2}{65}\right)^2\right]$

≈ 3.31562

(58) a) $gh(x) = \arccos\left(\frac{1}{2}x\right),\ -2\leq x\leq 2$

b)



$y=\arccos\left(\frac{1}{2}x\right)$ $\qquad y=\arccos(x)$

c) $x = \arccos\left(\frac{1}{2}y\right)$

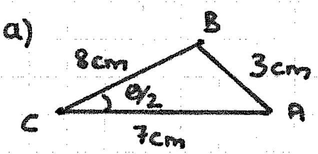
$\frac{1}{2}y = \cos x$

	D	R
$\cos x$		$0\leq g(x)\leq\pi$
$\arccos x$	$0\leq x\leq\pi$	

$y = 2\cos x$

$(gh)^{-1}(x) = 2\cos x \quad , \quad 0\leq x\leq\pi$

(59) a)



$3^2 = 8^2+7^2-2(8)(7)\cos\left(\frac{\theta}{2}\right)$

$\cos\left(\frac{\theta}{2}\right) = \frac{13}{14}$

b) $\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)-1 = 2\left(\frac{13}{14}\right)^2-1 = \frac{71}{98}$

$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{71}{98}\right)^2-1 = \frac{239}{4802}$

60 a) $(3-4x)^{-2} = (3)^{-2}\left(1-\frac{4}{3}x\right)^{-2} = \frac{1}{9}\left(1-\frac{4}{3}x\right)^{-2}$

$= \frac{1}{9}\left[1+(-2)\left(-\frac{4}{3}x\right)+\frac{(-2)(-3)}{2}\left(-\frac{4}{3}x\right)^2\right]$

$= \frac{1}{9}\left(1+\frac{8}{3}x+\frac{16}{3}x^2\right) = \frac{1}{9}+\frac{8}{27}x+\frac{16}{27}x^2$

b) $(2+kx)\left(\frac{1}{9}+\frac{8}{27}x+\frac{16}{27}x^2\right)$

$= \frac{2}{9}+\frac{16}{27}x+\frac{32}{27}x^2+\frac{k}{9}x+\frac{8k}{27}x^2+\dots$

$\left(\frac{16}{27}+\frac{k}{9}\right)x = \left(\frac{7}{27}\right)x$

$\frac{k}{9} = \frac{7}{27}-\frac{16}{27} = -\frac{1}{3}$

$k = -3$

c) $\frac{32}{27}+\frac{8}{27}(-3) = \frac{32-24}{27} = \frac{8}{27}$

61 a) $\sum_{r=1}^{12} 2 + \sum_{r=1}^{12} 4r + \sum_{r=1}^{12} 3^r$

$= (2\times 12) + \frac{12}{2}(4+48) + \frac{3(3^{12}-1)}{3-1}$

$= 24 + 312 + 797160 = 797496$

b) $U_1 = \frac{3}{4}$ $U_2 = -\frac{4}{3}$ $U_3 = \frac{3}{4}$ $U_4 = -\frac{4}{3}$

$100\times(U_1+U_2) = 100\times -\frac{7}{12} = -\frac{175}{3}$

62 a) $p=\sqrt{3}$ $q = \frac{1}{\sqrt{3}}$, $pq = \sqrt{3}\times\frac{1}{\sqrt{3}} = 1$

Irrational   Rational

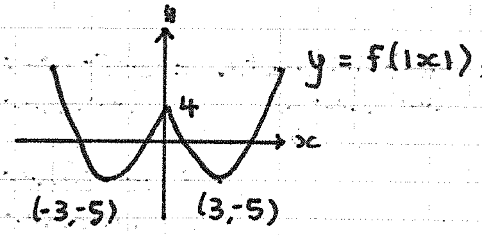
b) Assume that $\sqrt{2}$ is rational

$\sqrt{2} = \frac{a}{b}$, where a and b are integers $b \neq 0$ and HCF of a and b is 1.

$a = \sqrt{2}b \rightarrow a^2 = 2b^2$

So a^2 is divisible by 2. As a and b are both divisible by 2, then the HCF is not 1. This contradiction implies that $\sqrt{2}$ is irrational.

63 a) i) $(6,-10)$
ii) $(-3,5)$

b) 

c) $f(x) = (x-3)^2-5 \quad \underset{3}{\rightarrow} \quad \downarrow 5$

d) The function is not one-to-one.

64 a) $S_{15} = \frac{30\left(1-\left(\frac{5}{8}\right)^{30}\right)}{1-\frac{5}{8}} = 80.0$

b) $S_\infty = \frac{30}{1-\frac{5}{8}} = 80$

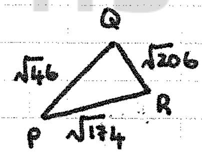
$80 - \frac{30\left(1-\left(\frac{5}{8}\right)^N\right)}{1-\frac{5}{8}} < 0.5$

$-30\left(1-\left(\frac{5}{8}\right)^N\right) < -\frac{477}{16}$

$1-\left(\frac{5}{8}\right)^N > \frac{159}{160}$

$\left(\frac{5}{8}\right)^N < \frac{1}{160} \rightarrow N\log\left(\frac{5}{8}\right) < \log\left(\frac{1}{160}\right)$

$N > 10.798\dots$
$N = 11$

65 a) 

$|PQ| = \sqrt{3^2+6^2+1^2} = \sqrt{46}$

$|PR| = \sqrt{11^2+7^2+2^2} = \sqrt{174}$

$\overrightarrow{QR} = -(-3i+6j+k)+(11i+7j-2k) = 14i+j-3k$

$|QR| = \sqrt{14^2+1^2+3^2} = \sqrt{206}$

$(\sqrt{174})^2 = (\sqrt{46})^2+(\sqrt{206})^2-2(\sqrt{46})(\sqrt{206})\cos Q$

$\cos Q = \frac{-78}{-194} \rightarrow Q = 66.4^\circ$

Area $= \frac{1}{2}(\sqrt{46})(\sqrt{206})\sin 66.4^\circ = 44.6$ (3sf)

b) $L_{SF} = 2 \rightarrow A_{SF} = 2^2 = 4$

$\Delta PST = 4\times 44.6$
$= 178.4$ (1 d.p)

66 a) $f(x) = \dfrac{(2x-7)(x-2)+1}{(x-3)(x-2)}$

$= \dfrac{2x^2-11x+14+1}{(x-3)(x-2)} = \dfrac{2x^2-11x+15}{(x-3)(x-2)}$

$= \dfrac{(2x-5)(x-3)}{(x-3)(x-2)}$

$= \dfrac{2x-5}{x-2}$

b) $$\begin{array}{r|l} & 2 \\ \hline x-2 & 2x-5 \\ & \underline{2x-4} \\ & -1 \end{array}$$ $f(x) = 2 - \dfrac{1}{x-2}$

c) T_1 is a translation by $\begin{pmatrix}2\\0\end{pmatrix}$

T_2 is a reflection in the x-axis

T_3 is a translation by $\begin{pmatrix}0\\2\end{pmatrix}$

67 a) $\dfrac{dx}{d\theta} = -4\sin 2\theta \quad \dfrac{dy}{d\theta} = \dfrac{1}{2}\cos\theta$

$\dfrac{dy}{dx} = \dfrac{\cos\theta}{-8\sin 2\theta} = \dfrac{\cos\theta}{-16\sin\theta\cancel{\cos\theta}}$

$= -\dfrac{1}{16}\operatorname{cosec}\theta$

b) $m_T = \dfrac{-1}{16\sin\left(\frac{\pi}{6}\right)} = -\dfrac{1}{8}$

$m_N = 8 \quad P\left(1, \frac{1}{4}\right)$

$y - \frac{1}{4} = 8(x-1) \quad (\times 4)$

$4y - 1 = 4(8x-8) \rightarrow 0 = 32x - 4y - 31$

c) $x = 0 \rightarrow y = -\dfrac{31}{4}$

$y = 0 \rightarrow x = \dfrac{31}{32}$

Area $= \dfrac{1}{2} \times \dfrac{31}{32} \times \dfrac{31}{4} = \dfrac{961}{256}$

68 a) $U_{20} = 10 \times \left(\frac{3}{5}\right)^{19} = 0.000609$

b) $S_\infty = \dfrac{10}{1-\frac{3}{5}} = 25$

c) $S_k > 24.99$

$\dfrac{10(1-0.6^k)}{1-0.6} > 24.99$

$1 - 0.6^k > 0.9996$

$-0.6^k > -0.0004$

$0.6^k < 0.0004$

$k\log 0.6 < \log 0.0004$

$k > \dfrac{\log 0.0004}{\log 0.6}$

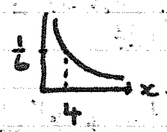
d) $k > 15.316\ldots$

$k = 16$

69 a) $f(x) = \dfrac{(2x-2)-(x+2)}{(x-4)(x+2)} = \dfrac{(x-4)}{(x-4)(x+2)}$

$= \dfrac{1}{x+2}$

b) $x = 4 \rightarrow f(4) = \frac{1}{6}$



$0 < f(x) < \frac{1}{6}$

c) $x = \dfrac{1}{y+2} \rightarrow xy + 2x = 1$

$xy = 1 - 2x$

$y = \dfrac{1-2x}{x}$

	D	R
$f(x)$		$0 < f(x) < \frac{1}{6}$
$f^{-1}(x)$	$0 < x < \frac{1}{6}$	

$f^{-1}(x) = \dfrac{1-2x}{x}$

Domain: $0 < x < \frac{1}{6}$

d) $\dfrac{1}{2x^2+1+2} = \dfrac{1}{43}$

$2x^2 + 3 = 43$

$2x^2 = 40$

$x^2 = 20$

$x = \sqrt{20}$

$x = \pm 2\sqrt{5}$

as $x > 4$, $x = 2\sqrt{5}$

70 a) $5(3+4x)^{-1} - 3(2-5x)^{-1}$

$$\frac{5}{3}\left(1+\frac{4}{3}x\right)^{-1} - \frac{3}{2}\left(1-\frac{5}{2}x\right)^{-1}$$

$$= \frac{5}{3}\left[1+(-1)\left(\frac{4}{3}x\right)+\frac{(-1)(-2)}{2}\left(\frac{4}{3}x\right)^2\right] - \frac{3}{2}\left[1+(-1)\left(-\frac{5}{2}x\right)+\left(-\frac{5}{2}x\right)^2\right]$$

$$= \frac{5}{3}\left(1-\frac{4}{3}x+\frac{16}{9}x^2\right) - \frac{3}{2}\left(1+\frac{5}{2}x+\frac{25}{4}x^2\right)$$

$$= \frac{5}{3} - \frac{20}{9}x + \frac{80}{27}x^2 - \frac{3}{2} - \frac{15}{4}x - \frac{75}{8}x^2$$

$$= \frac{1}{6} - \frac{215}{36}x - \frac{1385}{216}x^2$$

b) $f(0.01) = 0.1062753$ (Exact)

c) Estimate $= \frac{1}{6} - \frac{215}{36}(0.01) - \frac{1385}{216}(0.01)^2$

$= 0.1063032407$

% Error $= \frac{0.1063032407 - 0.1062753}{0.1062753} \times 100$

$= 0.026\%$

71 a) $R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = \frac{3}{2}$ $\quad R\sin\alpha = \frac{5}{2}$

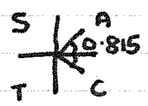
$\tan\alpha = \frac{5}{2} \div \frac{3}{2} = \frac{5}{3}$

$\alpha = 1.03$

$R = \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2} = \frac{\sqrt{34}}{2} \rightarrow \frac{\sqrt{34}}{2}\cos(\theta - 1.03)$

b) $\frac{16}{5} + \left(\frac{\sqrt{34}}{2}\right)^2 = \frac{117}{10} = 11.7$

c) $\frac{\sqrt{34}}{2}\cos(2\theta - 1.03) = 2$



$2\theta - 1.03 = 0.815, -0.815, 5.468, -7.098$

$\theta = -3.03, -2.22, 0.11, 0.92$

Range : $-\pi < \theta < \frac{\pi}{2}$

New Range : $-2\pi < 2\theta < \pi$

$-7.31 < 2\theta - 1.03 < 2.11$

72 a) $y = e^{2t} + 3$

$e^{2t} + 3 = 6e^t - 2$

$e^{2t} - 6e + 5 = 0$

$(e^t - 5)(e^t - 1) = 0$

$e^t = 5$ or $e^t = 1$

$t = \ln 5 \qquad t = 0$

$\downarrow \qquad \downarrow$

$x = e^{\ln 5^2} = 25 \qquad x = e^0 = 1$

$\downarrow \qquad \downarrow$

$y = 6e^{\ln 5} - 2 \qquad y = 6e^0 - 2$

$= 30 - 2 = 28 \qquad = 4$

$(25, 28) \qquad (1, 4)$

b) $t = \ln 2 \rightarrow x = e^{\ln 2^2} \rightarrow y = 6e^{\ln 2} - 2$

$x = 4 \qquad y = 12 - 2 = 10$

$t = \ln 3 \rightarrow x = e^{\ln 3^2} \rightarrow y = 6e^{\ln 3} - 2$

$x = 9 \qquad y = 18 - 2 = 16$

$(4, 10) \quad (9, 16)$

$m = \frac{16-10}{9-4} = \frac{6}{5}$

$y - 10 = \frac{6}{5}(x - 4)$

$5y - 50 = 6x - 24$

$0 = 6x - 5y + 26$

73 a) $U_n = 60{,}000r^{n-1}$

b) $60{,}000r^{n-1} > 240{,}000$

$r^{n-1} > 4$

$(n-1)\log r > \log 4$

$n - 1 > \frac{\log 4}{\log r}$

$n > \frac{\log 4}{\log r} + 1$

c) $r = 1.08 \rightarrow n > 19.01\ldots$

$n = 20$ (Year 20)

d) $S_{10} = \frac{60{,}000(1.08^{10} - 1)}{1.08 - 1}$

$= £869,193$

$= £870,000$

(74) a) $\frac{dx}{dt} = -6\sin t \quad \frac{dy}{dt} = 4\cos 2t$

$$\frac{dy}{dx} = \frac{-4\cos 2t}{6\sin t} = \frac{-2\cos 2t}{3\sin t}$$

at $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{-2\cos(\frac{\pi}{3})}{3\sin(\frac{\pi}{6})} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$

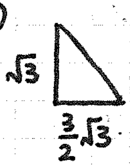
b) $t = \frac{\pi}{6} \rightarrow x = 3\sqrt{3} \rightarrow y = \sqrt{3}$

$$y - \sqrt{3} = -\frac{2}{3}(x - 3\sqrt{3})$$

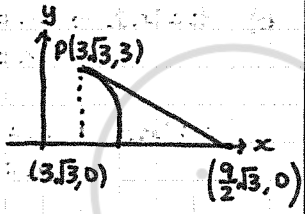
$$3y - 3\sqrt{3} = -2x + 6\sqrt{3}$$

$$2x + 3y - 9\sqrt{3} = 0$$

c)



$A = \frac{1}{2} \times \frac{3}{2}\sqrt{3} \times \sqrt{3} = \frac{9}{4}$



$\int_0^{\frac{\pi}{6}} y \cdot \frac{dx}{dt} \cdot dt$, when $y = 0$: $0 = 2\sin 2t$

$t = 0, \frac{\pi}{2}, \pi$

Sub t into x : $x = 6, 0, -6$

$$= \int_0^{\frac{\pi}{6}} (2\sin 2t)(-6\sin t) \cdot dt$$

$$= \int_0^{\frac{\pi}{6}} -12\underbrace{\sin 2t}_{(2\sin t\cos t)}\sin t \cdot dt = -24\int_0^{\frac{\pi}{6}} \sin^2 t\cos t \cdot dt$$

Reverse Chain

Let $y = (\sin t)^3$

$\frac{dy}{dt} = 3\cos t\sin^2 t$ $\times -8$

$$-8\left[\sin^3 t\right]_0^{\frac{\pi}{6}} = -8\left(\frac{1}{8} - 0\right) = -1 \rightarrow 1$$

(Area is +ve)

Shaded area =  - 

$$= \frac{9}{4} - 1$$

$$= \frac{5}{4}$$

(75) a) $\frac{x\cancel{(5x-4)}}{\cancel{(5x-4)}(x+2)} - \frac{10}{(x-3)(x+2)}$

$$\frac{x(x-3) - 10}{(x-3)(x+2)} = \frac{x^2 - 3x - 10}{(x-3)(x+2)}$$

$$= \frac{(x-5)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}}$$

$$= \frac{x-5}{x-3}$$

b)

$$\begin{array}{r|l} & \ \ 1 \\ x-3 & \overline{x-5} \\ & \ \ x-3 \\ \hline & \ \ -2 \end{array}$$

$$1 - \frac{2}{x-3}$$

c) $f(x) = 1 - 2(x-3)^{-1}$

$$f'(x) = 2(x-3)^{-2}$$

$$= \frac{2}{(x-3)^2} > 0$$

$f(x)$ is increasing for all $x > 3$

(76) a) $ar = 270 \quad ar^3 = 120$

$$r^2 = \frac{120}{270} = \frac{4}{9} \rightarrow r = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$a\left(\frac{2}{3}\right) = 270 \rightarrow a = 405$$

$$S_\infty = \frac{405}{1 - \frac{2}{3}} = 1215$$

b) $\frac{405\left(1 - \left(\frac{2}{3}\right)^N\right)}{1 - \frac{2}{3}} > 1200$

$$1 - \frac{2}{3}^N > \frac{80}{81}$$

$$\frac{2}{3}^N < \frac{1}{81}$$

$$N\log\left(\frac{2}{3}\right) < \log\left(\frac{1}{81}\right)$$

$$N > 10.83\ldots$$

$$N = 11$$

77 a) $U = x^2 - 4x \qquad V = e^{-x}$
$U' = 2x - 4 \qquad V' = -e^{-x}$

$$\frac{dy}{dx} = -(x^2-4x)e^{-x} + e^{-x}(2x-4)$$

at $x=1 \rightarrow m_T = 3e^{-1} + (-2e^{-1}) = e^{-1}$

When $x=1 \rightarrow y = -3e^{-1} \quad (1, -3e^{-1})$

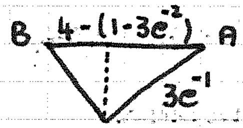
$$y + 3e^{-1} = e^{-1}(x-1)$$
$$y = e^{-1}x - e^{-1} - 3e^{-1}$$
$$y = e^{-1}x - 4e^{-1} \quad (\times e^{1})$$
$$ey = x - 4$$
$$4 = x - ey$$

b) $m_N = -e^{+1}$

$$y + 3e^{-1} = -e^{+1}(x-1)$$
$$y = -e^{+1}x + e^{+1} - 3e^{-1}$$

at $y=0 \rightarrow \quad 0 = -e^{+1}x + e^{1} - 3e^{-1} \quad (\times e^{1})$
$$0 = -e^2x + e^2 - 3$$
$$x = \frac{e^2-3}{e^2}$$
$$x = 1 - 3e^{-2}$$

$B(1-3e^{-2}, 0) \qquad A(4,0)$



$$\text{Area} = \frac{(3+3e^{-2})(3e^{-1})}{2}$$
$$= \frac{9e^{-1} + 9e^{-3}}{2} \quad \frac{(\times e^3)}{(\times e^3)}$$
$$= \frac{9e^2+9}{2e^3} = \frac{9(e^2+1)}{2e^3}$$

78 a) $A(x+1)(x-2) + B(x-2) + C(x+1) = 3x^2 - x + 2$

$x=-1 \rightarrow \quad -3B = 6$
$B = -2$

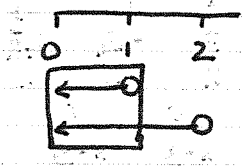
$x=2 \rightarrow \quad 3C = 12$
$C = 4$

$x=0 \rightarrow \quad -2A - 2B + C = 2$
$-2A + 4 + 4 = 2$
$-2A = -6$
$A = 3$

b) $3 - 2(x+1)^{-1} + 4(x-2)^{-1}$

$$= 3 - 2(1+x)^{-1} + 4(2)^{-1}(1-\tfrac{1}{2}x)^{-1}$$
$$= 3 - 2(1+x)^{-1} - 2(1-\tfrac{1}{2}x)^{-1}$$
$$= 3 - 2\left[1 + (-1)(x) + \frac{(-1)(-2)}{2}(x)^2\right] - 2\left[1 + (-1)(-\tfrac{1}{2}x) + \frac{(-1)(-2)}{2}(-\tfrac{1}{2}\right.$$
$$= 3 - 2(1 - x + x^2) - 2(1 + \tfrac{1}{2}x + \tfrac{1}{4}x^2)$$
$$= 3 - 2 + 2x - 2x^2 - 2 - x - \tfrac{1}{2}x^2$$
$$= -1 + x - \frac{5}{2}x^2$$

c) $|x| < 1$



79 a) $x = \dfrac{2-3y}{y-4}$

$$x(y-4) = 2-3y$$
$$xy - 4x = 2 - 3y$$
$$xy + 3y = 4x + 2$$
$$y(x+3) = 2(2x+1)$$
$$y = \frac{2(2x+1)}{x+3}$$

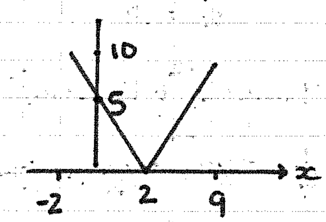
$$f^{-1}(x) = \frac{2(2x+1)}{x+3}, \quad x \in \mathbb{R}, \ x \neq 3$$

b) $-10 \leq g(x) \leq 5$

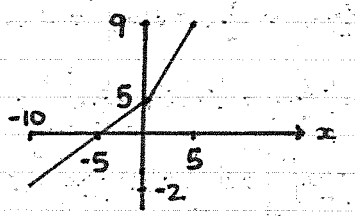
c) $g(2) = 0$
$g(0) = -5$

d) $g(9) = 5$
$$f(5) = \frac{2-3(5)}{5-4} = -13$$

e) i)



ii)



f) $-10 \leq x \leq 5$

80 a) $\dfrac{4K-5}{6K-5} = \dfrac{5-K}{4K-5}$

$(4K-5)(4K-5) = (6K-5)(5-K)$

$16K^2 - 40K + 25 = 30K - 6K^2 - 25 + 5K$

$22K^2 - 75K + 50 = 0$

$(11K-10)(2K-5) = 0$

$K = \dfrac{10}{11}$ or $K = \dfrac{5}{2}$

As $K<1$, $K = \dfrac{10}{11}$

b) $U_1 \quad U_2 \quad U_3$

$\dfrac{5}{11} \quad -\dfrac{15}{11} \quad \dfrac{45}{11}$

$x-3 \quad x-3$

i) $U_4 = -3 \times \dfrac{45}{11} = -\dfrac{135}{11}$

ii) $S_{10} = \dfrac{\frac{5}{11}(1-(-3)^{10})}{1--3} = \dfrac{-26840}{4} = -6710$

81 a) $A(3x-2)(2-x) + B(2-x) + C(3x-2)^2$
$= 36x^2 + 15x - 14$

$x=2 \rightarrow 16C = 160$
$C = 10$

$x = \dfrac{2}{3} \rightarrow \dfrac{4}{3}B = 12$
$B = 9$

$x=0 \rightarrow -4A + 2B + 4C = -14$
$-4A + 18 + 40 = -14$
$-4A = -72$
$A = 18$

b) $18(3x-2)^{-1} + 9(3x-2)^{-2} + 10(2-x)^{-1}$

$18.(-2)^{-1}\left(1-\frac{3}{2}x\right)^{-1} + \frac{9}{4}\left(1-\frac{3}{2}x\right)^{-2} + \frac{10}{2}\left(1-\frac{1}{2}x\right)^{-1}$

$= -9\left(1-\frac{3}{2}x\right)^{-1} + \frac{9}{4}\left(1-\frac{3}{2}x\right)^{-2} + 5\left(1-\frac{1}{2}x\right)^{-1}$

$= -9\left[1+(-1)\left(-\frac{3}{2}x\right) + \frac{(-1)(-2)}{2}\left(-\frac{3}{2}x\right)^2\right] + \frac{9}{4}\left[1+(-2)\left(-\frac{3}{2}x\right) + \frac{(-2)(-3)}{2}\left(-\frac{3}{2}x\right)^2\right] + 5\left[1+(-1)\left(-\frac{1}{2}x\right) + \frac{(-1)(-2)}{2}\left(-\frac{1}{2}x\right)^2\right]$

$= -9 - \frac{27}{2}x - \frac{81}{4}x^2 + \frac{9}{4} + \frac{27}{4}x + \frac{243}{16}x^2 + 5 + \frac{5}{2}x + \frac{5}{4}x^2$

c) $f(0.1) = \dfrac{36(0.1)^2 + 15(0.1) - 14}{(3(0.1)-2)^2(2-0.1)}$

$= -2.210890548$ (Exact)

Estimate $= -\dfrac{7}{4} - \dfrac{17}{4}(0.1) - \dfrac{61}{16}(0.1)^2$

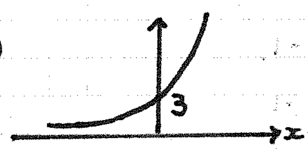
$= -2.213125$

% Error $= \dfrac{2.213125 - 2.210890548}{2.210890548} \times 100$

$= 0.10\%$

82 a) $g(3x + \ln 3) = e^{3x+\ln 3}$
$= e^{3x}e^{\ln 3}$
$= 3e^{3x}$

b)



c) $g(x) > 0$

d) $9e^{3x} = 4$

$e^{3x} = \dfrac{4}{9}$

$3x = \ln\left(\dfrac{4}{9}\right) \rightarrow x = \dfrac{1}{3}\ln\left(\dfrac{4}{9}\right)$

$x = -0.270$ (3 s.f.)

83 a) $R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\cos\alpha = 12.5 \quad R\sin\alpha = 6.5$

$\tan\alpha = \dfrac{13}{25} \rightarrow \alpha = 0.5$

$R = \sqrt{(12.5)^2 + (6.5)^2} = \dfrac{\sqrt{794}}{2}$

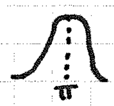
$\dfrac{\sqrt{794}}{2}\cos(x - 0.5)$

b) i) $\dfrac{\sqrt{794}}{2} = 14.1$ (3 s.f.)

ii) 0.5

c) $H = 16 - \dfrac{\sqrt{794}}{2}\cos(1.05t - 0.5)$

Max $= 16 + 14.1 = 30.1$



$t = \dfrac{\pi + 0.5}{\ldots} = 3.5\ldots$

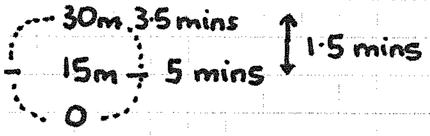
83 d) $t=4 \rightarrow H=27.9$

e) $15 = 16 - \frac{\sqrt{794}}{2}\cos(1.05t - 0.5)$

$1.05t - 0.5 = 1.499...$
$t = 1.9$ minutes

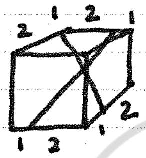
$1.05t - 0.5 = 2\pi - 1.499...$
$t = 5.0$ minutes

f)



$4 \times 1.5 = 6$ minutes

84 $\underset{(\text{pass } \overrightarrow{KR})}{\overrightarrow{OP}} = \frac{1}{3}a + \lambda(\overrightarrow{KR})$



$\overrightarrow{KR} = \frac{2}{3}a + b + c - \frac{1}{3}a$

$= \frac{1}{3}a + b + c$

$\underset{(\text{pass } \overrightarrow{KR})}{\overrightarrow{OP}} = \frac{1}{3}a + \lambda\left(\frac{1}{3}a + b + c\right)$

$\underset{(\text{pass } \overrightarrow{LS})}{\overrightarrow{OP}} = a + \frac{1}{3}b + \mu(\overrightarrow{LS})$

$\overrightarrow{LS} = \frac{2}{3}b + c - a - \frac{1}{3}b = \frac{1}{3}b + c - a$

$\underset{(\text{pass } \overrightarrow{LS})}{\overrightarrow{OP}} = a + \frac{1}{3}b + \mu\left(-a + \frac{1}{3}b + c\right)$

$$\begin{pmatrix} \frac{1}{3} + \frac{1}{3}\lambda \\ \lambda \\ \lambda \end{pmatrix} = \begin{pmatrix} 1 - \mu \\ \frac{1}{3} + \frac{1}{3}\mu \\ \mu \end{pmatrix}$$

$\lambda = \mu$ $\quad \frac{1}{3} + \frac{1}{3}\lambda = 1 - \lambda$

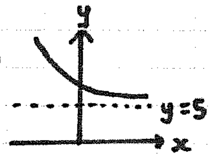
$\frac{4}{3}\lambda = \frac{2}{3}$

$\lambda = \frac{1}{2}$

CHECK : $\frac{1}{2} = \frac{1}{3} + \frac{1}{3}\mu \rightarrow \frac{1}{6} = \frac{1}{3}\mu \rightarrow \mu = \frac{1}{2}$

Lines do intersect as P lies half-way

85 a)

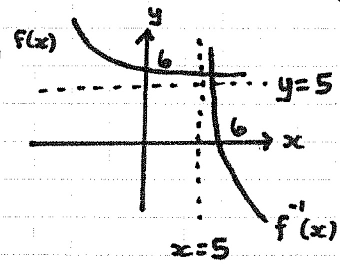


$f(x) > 5$

b) $y = e^{-x} + 5$
$\ln(y-5) = -x$
$x = -\ln(y-5)$

$f^{-1}(x) = -\ln(x-5)$, $x \in \mathbb{R}$, $x > 5$

c)



d) $x = e^{-x} + 5$ Graph meets at $y=x$

$\frac{1}{e^x} = x - 5$

$1 = e^x(x-5)$

86 a) $y=0 \rightarrow 20t(4-t) = 0$
$t=0$ or $t=4$
$T=4$

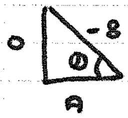
b) $\frac{dy}{dt} = 80 - 40t = 0$

$40t = 80$
$t = 2$

Sub t into y $\rightarrow$ $y = 80(2) - 20(2)^2 = 80m$

c) i) $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{80 - 40t}{20t^{-1/2}}$

End of path at $t=4 \rightarrow \frac{dy}{dx} = -8$



$\theta = \tan^{-1}(-8)$
$\theta = 82.9°$

c) ii) The model is unlikely to be valid, as it shows the rollercoaster would be heading into the ground.

d) $t = \frac{x^2}{1600} \rightarrow y = \frac{x^2}{20} - \frac{x^4}{128000}$

$\int_0^{80} y.dx = \left[\frac{x^3}{60} - \frac{x^5}{640000}\right]_0^{80}$

$= 8533.\dot{3} - 5120$
$= 3413.3$

$= 3410\ m^2$ (3 s.f.)