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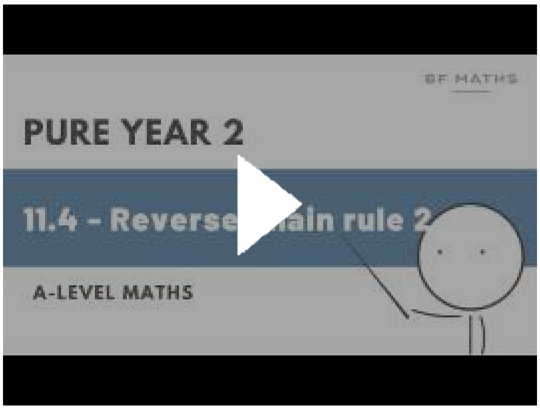
If you need help on this chapter:
A-Level Maths | Pure Year 2 | 11.4 - Reverse chain rule 2 Walkthrough | Edexcel



11.4 - Reverse Chain Rule (RCR)

1ai) $\frac{d}{dx}(x^2-4)^3$

$= 3(x^2-4)^2 \times (2x)$

$= \underline{\underline{6x(x^2-4)^2}}$

1aii) $\frac{d}{dx}(x^2+4x-5)^4$

$= 4(x^2+4x-5)^3 \times (2x+4)$

$= 4(2x+4)(x^2+4x-5)^3$

$= \underline{\underline{8(x+2)(x^2+4x-5)^3}}$

this step is not necessary

Recap Differentiating with chain rule

[A-Level Maths | Pure Year 2 | 9.3.5 - Chain rule (Shortcut) Walkthrough | Edexcel]

1bi) $\int 6x(x^2-4)^2\,dx$

$= \underline{\underline{(x^2-4)^3 + c}}$

(use the ans. from ai)

1bii) $\int 15x(x^2-4)^2\,dx$

$= \frac{5}{2}\int 6x(x^2-4)^2\,dx$

$= \underline{\underline{\frac{5}{2}(x^2-4)^3 + c}}$

1biii) $\int -8(x+2)(x^2+4x-5)^2\,dx$

be careful

$= -8\int (x+2)(x^2+4x-5)^2\,dx$

Let $y = (x^2+4x-5)^3$

$y' = 3(x^2+4x-5)^2(2x+4)$

$y' = 6(x+2)(x^2+4x-5)^2$

$= -8 \times \frac{1}{6}(x^2+4x-5)^3 + c$

$= \underline{\underline{-\frac{4}{3}(x^2+4x-5)^3 + c}}$

2a) $\int 6x(x^2-7)^{\frac{1}{2}}\,dx$

$= 6\int x(x^2-7)^{\frac{1}{2}}\,dx$

Let $y = (x^2-7)^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}(x^2-7)^{\frac{1}{2}} \times 2x = 3(x^2-7)^{\frac{1}{2}}$

$= 6 \times \frac{1}{3}(x^2-7)^{\frac{3}{2}} + c = \underline{\underline{2(x^2-7)^{\frac{3}{2}} + c}}$

2b) $\int 5\sin 2x\, e^{\cos 2x}\,dx = 5\int \sin 2x\, e^{\cos 2x}\,dx$

Let $y = e^{\cos 2x}$

$y' = (-2\sin 2x)e^{\cos 2x}$

$5\int \sin 2x\, e^{\cos 2x}\,dx = 5 \times -\frac{1}{2}e^{\cos 2x} + c$

$= \underline{\underline{-\frac{5}{2}e^{\cos 2x} + c}}$

11.4 - Reverse Chain Rule (RCR)

2c) $\int (6x^2+8)(4x^3+16x-5)^{\frac{3}{2}}\,dx$

$= 2\int (3x^2+4)(4x^3+16x-5)^{\frac{3}{2}}\,dx$

Let $y = (4x^3+16x-5)^{\frac{5}{2}}$

$y' = \frac{5}{2}(4x^3+16x-5)^{\frac{3}{2}}(12x^2+16)$

$= \frac{5}{2}(4x^3+16x-5)^{\frac{3}{2}} \times 4(3x^2+4)$

$= 10(4x^3+16x-5)^{\frac{3}{2}}(3x^2+4)$

$2\int (3x^2+4)(4x^3+16x-5)^{\frac{3}{2}} = 2 \times \frac{1}{10}(4x^3+16x-5)^{\frac{5}{2}} + c$

$= \frac{1}{5}(4x^3+16x-5)^{\frac{5}{2}} + c$

2d) $\int \sec^2 x\sqrt{1+\tan x}\,dx = \int \sec^2 x\,(1+\tan x)^{\frac{1}{2}}\,dx$

Let $y = (1+\tan x)^{\frac{3}{2}}$

$y' = \frac{3}{2}(1+\tan x)^{\frac{1}{2}} \times \sec^2 x$

$\int \sec^2 x\,(1+\tan x)^{\frac{1}{2}}\,dx = \frac{2}{3}(1+\tan x)^{\frac{3}{2}} + c$

2e) $\int \operatorname{cosec}^2 3x\, e^{\cot 3x}\,dx$

Let $y = e^{\cot 3x}$

$\frac{dy}{dx} = (-3\operatorname{cosec}^2 3x)e^{\cot 3x}$

$\int \operatorname{cosec}^2 3x\, e^{\cot 3x}\,dx = -\frac{1}{3}e^{\cot 3x} + c$

2f) $\int \cos^4 5x \sin 5x\,dx$

Let $y = \cos^5 5x$ or $(\cos 5x)^5$

$y' = 5(\cos 5x)^4 \times (-5\sin 5x)$

$= -25\cos^4 5x \sin 5x$

$\int \cos^4 5x \sin 5x\,dx = -\frac{1}{25}\cos^5 5x + c$

11.4 - Reverse Chain Rule (RCR)

3a) $\int \frac{4x}{2-x^2}\,dx$

Let $y = \ln|2-x^2|$

$y' = \frac{1}{2-x^2} \times (-2x)$

$y' = \frac{-2x}{2-x^2}$

$\int \frac{4x}{2-x^2}\,dx = \underline{\underline{-2\ln|2-x^2| + c}}$

3b) $\int \frac{\sin x}{4-3\cos x}\,dx$

Let $y = \ln|4-3\cos x|$

$y' = \frac{1}{4-3\cos x}(3\sin x)$

$y' = \frac{3\sin x}{4-3\cos x}$

$\int \frac{\sin x}{4-3\cos x}\,dx = \underline{\underline{\frac{1}{3}\ln|4-3\cos x| + c}}$

3c) $\int \frac{6x^2-9}{14+9x-2x^3}\,dx$

Let $y = \ln|14+9x-2x^3|$

$y' = \frac{1}{14+9x-2x^3} \times (9-6x^2)$

$\int \frac{6x^2-9}{14+9x-2x^3}\,dx = \underline{\underline{-\ln|14+9x-2x^3| + c}}$

3d) $\int \frac{\sin 2x}{\cos^2 x}\,dx$

$= \int \frac{2\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x}\,dx = \int \frac{2\sin x}{\cos x}\,dx$

$= 2\int \tan x\,dx$

from FB

$= \underline{\underline{2\ln|\sec x|}} + c$

Different to the answer in the book, but this is perfectly valid.

$2\ln|\sec x| = 2\ln\left|\frac{1}{\cos x}\right| = 2\ln|\cos x|^{-1}$

$= -2\ln|\cos x|$

3e) $\int \frac{\frac{1}{2}\sin x\cos x}{4-\cos 2x}\,dx = \frac{1}{2}\int \frac{\sin x\cos x}{4-\cos 2x}\,dx$

Let $y = \ln|4-\cos 2x|$

$y' = \frac{1}{4-\cos 2x}(2\sin 2x)$

$y' = \frac{4\sin x\cos x}{4-\cos 2x}$

$\frac{1}{2}\int \frac{\sin x\cos x}{4-\cos 2x}\,dx = \frac{1}{2} \times \frac{1}{4}\ln|4-\cos 2x| + c$

$= \underline{\underline{\frac{1}{8}\ln|4-\cos 2x| + c}}$

3f) $\int \frac{e^{-3x}}{1+e^{-3x}}\,dx$

Let $y = \ln|1+e^{-3x}|$

$\frac{dy}{dx} = \frac{1}{1+e^{-3x}} \times (-3e^{-3x})$

$= \frac{-3e^{-3x}}{1+e^{-3x}}$

$\int \frac{e^{-3x}}{1+e^{-3x}}\,dx = \underline{\underline{-\frac{1}{3}\ln|1+e^{-3x}| + c}}$

11.4 - Reverse Chain Rule (RCR)

4a) $\displaystyle\int_{\frac{3\pi}{8}}^{\frac{\pi}{2}} \sin^2 4x \cos 4x \, dx$

Let $y = \sin^3 4x$ or $(\sin 4x)^3$

$y' = 3(\sin 4x)^2(4\cos 4x)$

$= 12\sin^2 4x \cos 4x$

$$\int_{\frac{3\pi}{8}}^{\frac{\pi}{2}} \sin^2 4x \cos 4x \, dx = \left[\frac{1}{12}\sin^3 4x\right]_{\frac{3\pi}{8}}^{\frac{\pi}{2}}$$

$$= 0 - \left(-\frac{1}{12}\right) = \underline{\underline{\frac{1}{12}}}$$

4b) $\displaystyle\int_0^{\frac{\pi}{9}} \frac{\cancel{\sec^2 3x} \tan 3x}{\cancel{\sec^2 3x}} \, dx$

$= \displaystyle\int \tan 3x \, dx$

FB

$= \left[\frac{1}{3}\ln|\sec 3x|\right]_0^{\frac{\pi}{9}}$

$= \left(\frac{1}{3}\ln\left|\sec \frac{3\pi}{9}\right|\right) - \left(\frac{1}{3}\ln|\sec 0|\right)$

don't put the whole sum into Calculator because of the "ln"

$= \frac{1}{3}\ln 2 - \cancel{\frac{1}{3}\ln(1)}$

$= \underline{\underline{\frac{1}{3}\ln 2}}$

4c) $\displaystyle\int_{\ln 2}^{\ln 3} \frac{6e^{3x}}{e^{3x}+1} \, dx = 6\int \frac{e^{3x}}{e^{3x}+1} \, dx$

Let $y = \ln|e^{3x}+1|$

$y' = \dfrac{1}{e^{3x}+1} \times 3e^{3x}$

$$6\int_{\ln 2}^{\ln 3} \frac{e^{3x}}{e^{3x}+1} \, dx = \left[6 \times \frac{1}{3}\ln|e^{3x}+1|\right]_{\ln 2}^{\ln 3}$$

$= \left[2\ln|e^{3x}+1|\right]_{\ln 2}^{\ln 3}$

$= 2\ln|e^{3\ln 3}+1| - 2\ln|e^{3\ln 2}+1|$

$= 2\ln|\cancel{e}^{\ln 27}+1| - 2\ln|\cancel{e}^{\ln 8}+1|$

$= 2\ln 28 - 2\ln 9$

$= \underline{\underline{2\ln\left(\frac{28}{9}\right)}}$

4d) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} 3x \cot 3x \, e^{\operatorname{cosec} 3x} \, dx$

Let $y = e^{\operatorname{cosec} 3x}$

$\dfrac{dy}{dx} = 3\operatorname{cosec} 3x \cot 3x \, e^{\operatorname{cosec} 3x}$

$$\int \ldots \, dx = \left[\frac{1}{3}e^{\operatorname{cosec} 3x}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$= \frac{1}{3}e^{\operatorname{cosec}\frac{3\pi}{4}} - \frac{1}{3}e^{\operatorname{cosec}\frac{3\pi}{6}}$

$= \underline{\underline{\frac{1}{3}e^{\sqrt{2}} - \frac{1}{3}e}}$

11.4 - Reverse Chain Rule (RCR)

5) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1+\cot x)^3 \operatorname{cosec}^2 x \, dx$

Let $y = (1+\cot x)^4$ ← I chose this term because differentiating $\cot x$ gives $\operatorname{cosec}^2 x$

$y' = 4(1+\cot x)^3 \times -\operatorname{cosec}^2 x$

$$\int \ldots dx = \left[-\frac{1}{4}(1+\cot x)^4\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{1}{4}\left[(1+\cot x)^4\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= -\frac{1}{4}\left[\left(1+\cot\frac{\pi}{4}\right)^4 - \left(1+\cot\frac{\pi}{6}\right)^4\right]$$

$$= -\frac{1}{4}\left[2^4 - (1+\sqrt{3})^4\right]$$

Tips: Type $(1+\sqrt{3})^2$ then square again in the calculator to give exact value

$$= -\frac{1}{4}\left(16 - (28+16\sqrt{3})\right)$$

$$= \underline{\underline{3+4\sqrt{3}}}$$

6a) LHS $= \sin^5 x$

$= \sin x(\sin^2 x)(\sin^2 x)$

$= \sin x(1-\cos^2 x)(1-\cos^2 x)$

$= \sin x(1-2\cos^2 x+\cos^4 x)$

$= \sin x - 2\sin x\cos^2 x + \sin x\cos^4 x$

$= \underline{\underline{\text{RHS}}}$

6b) $\int \sin^5 x \, dx$

$= \int \sin x - 2\sin x\cos^2 x + \sin x\cos^4 x \, dx$

$-\cos x$

Let $y = \cos^3 x$

$y' = 3\cos^2 x(-\sin x)$

$= -3\sin x\cos^2 x$

$\int -2\sin x\cos^2 x \, dx = \frac{2}{3}\cos^3 x$

Let $y = \cos^5 x$

$y' = -5\sin x\cos^4 x$

$\int \sin x\cos^4 x \, dx = -\frac{1}{5}\cos^5 x$

$$= \underline{\underline{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c}}$$

11.4 - Reverse Chain Rule (RCR)

7) $\int_0^4 (x+1)(2x^2+4x)^{\frac{1}{2}}\,dx \rightarrow$ let $y=(2x^2+4x)^{\frac{3}{2}}$

$y' = \frac{3}{2}(2x^2+4x)^{\frac{1}{2}} \times (4x+4)$

$y' = \frac{3}{2}(2x^2+4x)^{\frac{1}{2}} \times 4(x+1)$

$y' = 6(2x^2+4x)^{\frac{1}{2}}(x+1)$

$\int \cdots dx = \frac{1}{6}\left[(2x^2+4x)^{\frac{3}{2}}\right]_0^4$

$= \frac{1}{6}\left[(2\times4^2+4\times4)^{\frac{3}{2}} - (2\times0^2+0)^{\frac{3}{2}}\right]$

$= \frac{1}{6}\left(48^{\frac{3}{2}}\right)$

$48^{\frac{3}{2}} = 48^1 \times 48^{\frac{1}{2}}$

$= 48 \times \sqrt{48}$

$= 48 \times 4\sqrt{3}$

$= \frac{1}{6}(192\sqrt{3})$

$= \underline{\underline{32\sqrt{3}}}$

8) $\int_0^{\frac{\pi}{8}} \sin 4x\left(e^{\cos 4x}\right)dx \rightarrow$ let $y = e^{\cos 4x}$

$y' = -4\sin 4x\, e^{\cos 4x}$

$= \left[-\frac{1}{4}e^{\cos 4x}\right]_0^{\frac{\pi}{8}}$

$= -\frac{1}{4}e^{\cos(\frac{4\pi}{8})} - \left(-\frac{1}{4}e^{\cos 0}\right)$

$= -\frac{1}{4}e^0 + \frac{1}{4}e$

$= \underline{\underline{-\frac{1}{4} + \frac{1}{4}e}}$

11.4 - Reverse Chain Rule (RCR)

9) $$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{(3-\tan x)^4}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \sec^2 x\,(3-\tan x)^{-4}\,dx$$

Let $y = (3-\tan x)^{-3}$

$y' = -3(3-\tan x)^{-4} \times -\sec^2 x$

$= 3\sec^2 x\,(3-\tan x)^{-4}$

$$= \left[\frac{1}{3}(3-\tan x)^{-3}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{24} - \frac{1}{81} = \underline{\underline{\frac{19}{648}}}$$

10) $$\int_1^k \frac{x}{x^2+3}\,dx = \ln\frac{3}{2}$$

Let $y = \ln|x^2+3|$

$y' = \frac{1}{x^2+3} \times 2x$

$$\left[\frac{1}{2}\ln|x^2+3|\right]_1^k = \ln\frac{3}{2}$$

$$\frac{1}{2}\ln|k^2+3| - \frac{1}{2}\ln|1+3| = \ln\frac{3}{2}$$

$$\frac{1}{2}\ln|k^2+3| - \frac{1}{2}\ln 4 = \ln\frac{3}{2}$$

$$\frac{1}{2}\ln|k^2+3| = \ln\frac{3}{2} + \frac{1}{2}\ln 4$$

$\ln\frac{3}{2} + \ln(4^{\frac{1}{2}})$

$= \ln\left(\frac{3}{2} \times 2\right)$

$= \ln 3$

$$\frac{1}{2}\ln|k^2+3| = \ln 3$$

$$\ln|k^2+3| = 2\ln 3$$

$2\ln 3 = \ln 3^2 = \ln 9$

$$k^2+3 = 9$$

$$k^2 = 6$$

$$k = \underline{\underline{\sqrt{6}}}$$