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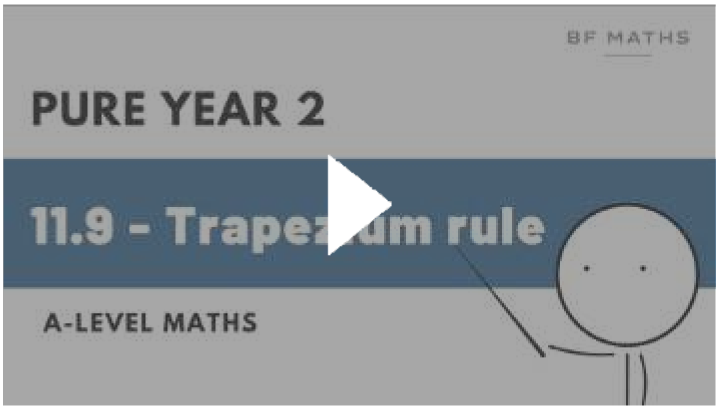
If you need help on this chapter:
A-Level Maths | Pure Year 2 | 11.9 - Trapezium rule Walkthrough | Edexcel



11.9 - The trapezium rule

1a) $y = 2x^2 - x^3$

x	0	0.5	1	1.5	2
y	0	0.375	1	1.125	0

$2(0.5)^2 - (0.5)^3$

1b) $h = \frac{b-a}{n} = \frac{2-0}{4} = \underline{\underline{0.5}}$

Because the area is divided into 4 trapezium strips.

1c) Area $\approx \frac{0.5}{2}\left[0 + 2(0.375 + 1 + 1.125) + 0\right]$

approx. equal

equal

$= \underline{\underline{1.25}}$

1d) $\int_0^2 (2x^2 - x^3)\,dx = \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$

$= \left[\frac{2 \times 2^3}{3} - \frac{2^4}{4}\right] - \left[\frac{0}{3} - \frac{0}{4}\right]$

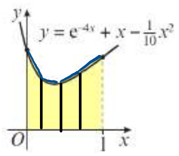
$= \underline{\underline{\frac{4}{3}}}$

2a) $y = e^{-4x} + x - \frac{1}{10}x^2$

x	0	0.25	0.5	0.75	1
y	1	0.6116	0.6103	0.7435	0.9183

2b) Area $\approx \frac{0.25}{2}\left[1 + 2(0.6116 + 0.6103 + 0.7435) + 0.9183\right]$

$= \underline{\underline{0.731}}$ (3dp)



2c) $\int_0^1 \left(e^{-4x} + x - \frac{1}{10}x^2\right) dx$

$= \left[\frac{1}{-4}e^{-4x} + \frac{x^2}{2} - \frac{x^3}{30}\right]_0^1$

$= \left[-\frac{1}{4}e^{-4} + \frac{1}{2} - \frac{1}{30}\right] - \left[\frac{1}{-4} + 0 - 0\right]$

$= \underline{\underline{-\frac{1}{4e^4} + \frac{43}{60}}}$

2d) The graph is convex, meaning the estimated area using trapezium rule included extra space, hence overestimation.

2e) Increase the number of strips.

11.9 - The trapezium rule

3a) $y = x^2 \cos x$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	0.142	0.436	0.531	0

3b) $I \approx \frac{\frac{\pi}{8}}{2}\left[0 + 2(0.142 + 0.436 + 0.531) + 0\right]$

$= \underline{\underline{0.436}}$ (3sf)

3c) $I = \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ IBP (ILATE)

$u = x^2 \qquad v' = \cos x$

$u' = 2x \qquad v = \sin x$

$= \left[x^2 \sin x - \int \sin x (2x) \, dx\right]_0^{\frac{\pi}{2}}$

IBP (ILATE)

$u = 2x \qquad v' = \sin x$

$u' = 2 \qquad v = -\cos x$

$\int \ldots dx = -2x\cos x - \int -\cos x (2) \, dx$

$= -2x\cos x - -2\sin x$

$= -2x\cos x + 2\sin x$

$= \left[x^2 \sin x - (-2x\cos x + 2\sin x)\right]_0^{\frac{\pi}{2}}$

$= \left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\frac{\pi}{2}}$

$= \left[\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - 2\sin\left(\frac{\pi}{2}\right)\right] - \left[0 + 0 - 0\right]$

$= \underline{\underline{\frac{\pi^2}{4} - 2}}$

3d) $\%$ error $= \dfrac{\text{Estimated} - \text{Actual}}{\text{Actual}} \times 100$

$= \dfrac{0.436 - \left(\frac{\pi^2}{4} - 2\right)}{\frac{\pi^2}{4} - 2} \times 100$

$= \underline{\underline{-6.72\%}}$ (3 sf)

11.9 - The trapezium rule

4a) $y = \frac{x-15}{(2x+3)(x-4)}$

x	0	0.5	1	1.5	2	2.5	3
y	1.25	1.038	0.933	0.9	0.929	1.041	1.333

4b) Area $\approx \frac{0.5}{2}\left[1.25 + 2(1.038 + 0.933 + 0.9 + 0.929 + 1.041) + 1.333\right]$

$= \underline{\underline{3.07}}$ (2 dp)

4c) $\frac{x-15}{(2x+3)(x-4)} = \frac{A}{2x+3} + \frac{B}{x-4}$

$x - 15 = A(x-4) + B(2x+3)$

$x=4$: $4 - 15 = B(2\times4+3) \Rightarrow B = -1$

$x = -\frac{3}{2}$: $-\frac{3}{2} - 15 = A(-\frac{3}{2} - 4) \Rightarrow A = 3$

$$\int_0^3 \frac{x-15}{(2x+3)(x-4)}\,dx = \int_0^3 \frac{3}{2x+3} - \frac{1}{x-4}\,dx$$

$$= \left[3\times\frac{1}{2}\ln|2x+3| - \ln|x-4|\right]_0^3$$

$$= \left[\frac{3}{2}\ln 9 - \ln 1\right] - \left[\frac{3}{2}\ln 3 - \ln 4\right]$$

$$= \frac{3}{2}\times 2\ln 3 - \frac{3}{2}\ln 3 + \ln 4$$

$$= \frac{3}{2}\ln 3 + \ln 4$$

$$= \ln(3^{\frac{3}{2}}\times 4) = \underline{\underline{\ln(12\sqrt{3})}}$$

4d) $\%\ \text{error} = \frac{\text{Estimated} - \text{Actual}}{\text{Actual}}\times 100$

$$= \frac{3.07 - \ln(12\sqrt{3})}{\ln(12\sqrt{3})}\times 100$$

$$= \underline{\underline{1.18\%}}\ (3\text{sf})$$

11.9 - The trapezium rule

5a) $y = \sin^2 x \cos^3 x$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	0.1155	0.1768	0.0478	0

5b) $I \approx \frac{\frac{\pi}{8}}{2}\left[0 + 2(0.1155 + 0.1768 + 0.0478) + 0\right]$

$= \underline{\underline{0.1336}}$

5c) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

u	0	$\frac{\pi}{2}$
x	0	1

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx = \int_0^1 u^2 \times \cos^{\cancel{3}\,2} x \times \frac{du}{\cancel{\cos x}}$$

$$= \int_0^1 u^2 \cos^2 x \, du$$

$$= \int_0^1 u^2 (1 - \sin^2 x) \, du$$

$$= \int_0^1 u^2 (1 - u^2) \, du$$

$$= \int_0^1 u^2 - u^4 \, du$$

$$= \left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^1$$

$$= \left[\frac{1}{3} - \frac{1}{5}\right] - \left[0 - 0\right]$$

$$= \underline{\underline{\frac{2}{15}}}$$

11.9 - The trapezium rule

6a) $y = \frac{1}{4}x^2 \ln x + 1$

x	1	1.25	1.5	1.75	2
y	1	1.0872	1.2281	1.4285	1.6931

6b) $R \approx \frac{0.25}{2}\left[1 + 2(1.0872 + 1.2281 + 1.4285) + 1.6931\right]$

$= \underline{\underline{1.2726}}$ (4 dp)

6c) $\int_1^2 \left(\frac{1}{4}x^2 \ln x + 1\right) dx$

$= \int_1^2 \frac{1}{4}x^2 \ln x + \left[x\right]_1^2$

IBP (ILATE)

$u = \ln x \qquad v' = \frac{1}{4}x^2$

$u' = \frac{1}{x} \qquad v = \frac{1}{12}x^3$

$\int \frac{1}{4}x^2 \ln x \, dx = \ln x\left(\frac{1}{12}x^3\right) - \int \frac{1}{12}x^3 \times \frac{1}{x} \, dx$

$= \frac{1}{12}x^3 \ln x - \int \frac{1}{12}x^2 \, dx$

$= \frac{1}{12}x^3 \ln x - \frac{1}{36}x^3$

$= \left[\frac{1}{12}x^3 \ln x - \frac{1}{36}x^3\right]_1^2 + (2 - 1)$

$= \left(\frac{1}{12}(2)^3 \ln 2 - \frac{1}{36}(2)^3\right) - \left(\frac{1}{12}(1)^3 \ln(1) - \frac{1}{36}\right) + 1$

$= \frac{2}{3}\ln 2 - \frac{2}{9} + \frac{1}{36} + 1$

$= \underline{\underline{\frac{2}{3}\ln 2 + \frac{29}{36}}}$