Author: Mr Fan

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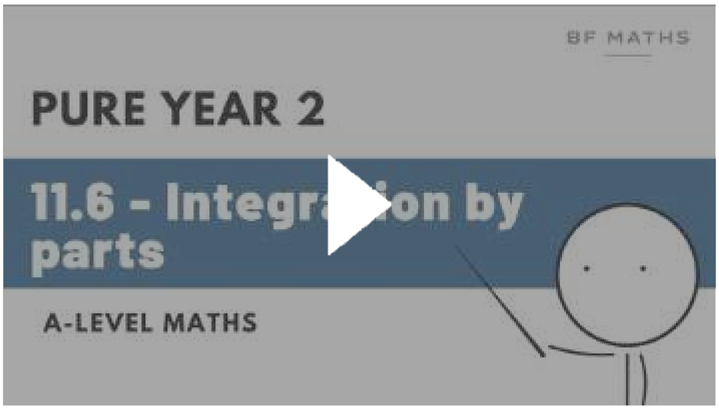
If you need help on this chapter:
A-Level Maths | Pure Year 2 | 11.6 - Integration by parts Walkthrough | Edexcel



11.6 - Integration by parts (IBP)

1a) $\int x\cos x\,dx$ (ILATE)

$u = x \xrightarrow{\text{diff}} \frac{du}{dx} = 1$

$\frac{dv}{dx} = \cos x \xrightarrow{\text{Int}} v = \sin x$

$$\int x\cos x\,dx = x(\sin x) - \int \sin x(1)\,dx$$

$$= x\sin x - (-\cos x) + c$$

$$= \underline{\underline{x\sin x + \cos x + c}}$$

1b) $\int x\sec^2 x\,dx$ (ILATE)

$u = x$ $\quad \frac{dv}{dx} = \sec^2 x$ use FB

$u' = 1$ $\quad v = \tan x$

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x(1)\,dx$$

FB

$$= \underline{\underline{x\tan x - \ln|\sec x|^{*} + c}}$$

(book solution has $\ln|\cos x|$, $-\ln|\sec x| = -\ln(\cos x)^{-1} = \ln|\cos x|$)

1c) $\int xe^{2x}\,dx$ (ILATE)

$u = x$ $\quad v' = e^{2x}$

$u' = 1$ $\quad v = \frac{1}{2}e^{2x}$

$$\int xe^{2x}\,dx = x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x} \times 1\,dx$$

$$= \underline{\underline{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c}}$$

1d) $\int x\,\mathrm{cosec}^2 4x\,dx$ (ILATE)

$u = x$ $\quad \frac{dv}{dx} = \mathrm{cosec}^2 4x$ FB

$u' = 1$ $\quad v = -\frac{1}{4}\cot 4x$

$$\int x\,\mathrm{cosec}^2 4x\,dx = x\left(-\frac{1}{4}\cot 4x\right) - \int \left(-\frac{1}{4}\cot 4x\right)(1)\,dx$$

$$= -\frac{1}{4}\cot 4x + \frac{1}{4}\int \cot 4x\,dx$$

FB

$$= -\frac{1}{4}\cot 4x + \frac{1}{4} \times \frac{1}{4}\ln|\sin 4x| + c$$

$$= \underline{\underline{-\frac{1}{4}\cot 4x + \frac{1}{16}\ln|\sin 4x| + c}}$$

11.6 - Integration by parts (IBP)

1e) $\int x e^{-4x}\,dx$ (ILATE)

$u = x$ $\quad$ $v' = e^{-4x}$

$u' = 1$ $\quad$ $v = -\frac{1}{4}e^{-4x}$

$$\int x e^{-4x}\,dx = x\left(-\frac{1}{4}e^{-4x}\right) - \int -\frac{1}{4}e^{-4x}(1)\,dx$$

$$= -\frac{1}{4}e^{-4x} + \frac{1}{4}\int e^{-4x}\,dx$$

$$= -\frac{1}{4}e^{-4x} - \frac{1}{16}e^{-4x} + C$$

1f) $\int x\sin 5x\,dx$ (ILATE)

$u = x$ $\quad$ $v' = \sin 5x$

$u' = 1$ $\quad$ $v = -\frac{1}{5}\cos 5x$

$$\int x\sin 5x\,dx = x\left(-\frac{1}{5}\cos 5x\right) - \int -\frac{1}{5}\cos 5x \times 1\,dx$$

$$= -\frac{1}{5}x\cos 5x + \frac{1}{5}\int \cos 5x\,dx$$

$$= -\frac{1}{5}x\cos 5x + \frac{1}{25}\sin 5x + C$$

2a) $\int x\ln x\,dx$ (ILATE)

$u = \ln x$ $\quad$ $v' = x$

$u' = \frac{1}{x}$ $\quad$ $v = \frac{x^2}{2}$

$$\int x\ln x\,dx = \ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\left(\frac{1}{x}\right)dx$$

$$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

2b) $\int x^3\ln x\,dx$ (ILATE)

$u = \ln x$ $\quad$ $v' = x^3$

$u' = \frac{1}{x}$ $\quad$ $v = \frac{x^4}{4}$

$$\int x^3\ln x\,dx = \ln x\left(\frac{x^4}{4}\right) - \int \frac{x^4}{4} \times \frac{1}{x}\,dx$$

$$= \frac{x^4}{4}\ln x - \int \frac{x^3}{4}\,dx$$

$$= \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

11.6 - Integration by parts (IBP)

2c) $\int \ln x \, dx$

This one is a special case. See my video for explanation

$u = \ln x \qquad v' = 1$

$u' = \frac{1}{x} \qquad v = x$

$$\int \ln x \, dx = \ln x (x) - \int x \times \frac{1}{x} dx$$

$$= x\ln x - x + c$$

3a) $\int x^2 \sin 2x \, dx$ (ILATE)

$u = x^2 \qquad v' = \sin 2x$

$u' = 2x \qquad v = -\frac{1}{2}\cos 2x$

$$\int \text{''} \, dx = x^2\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x \,(2x)\, dx$$

$$= -\frac{1}{2}x^2\cos 2x + \int x\cos 2x \, dx$$

IBP again (ILATE)

$u = x \qquad v' = \cos 2x$

$u' = 1 \qquad v = \frac{1}{2}\sin 2x$

$$\int \text{''} \, dx = x\left(\frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\sin 2x \, dx$$

$$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x$$

$$= -\frac{1}{2}x^2\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$$

3b) $\int x^2 e^{-3x} \, dx$ (ILATE)

$u = x^2 \qquad v' = e^{-3x}$

$u' = 2x \qquad v = -\frac{1}{3}e^{-3x}$

$$\int \text{''} \, dx = x^2\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}(2x)\, dx$$

$$= -\frac{1}{3}x^2 e^{-3x} + \frac{2}{3}\int x e^{-3x} \, dx$$

IBP again (ILATE)

$u = x \qquad v' = e^{-3x}$

$u' = 1 \qquad v = -\frac{1}{3}e^{-3x}$

$$\int \text{''} \, dx = x\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}(1)\, dx$$

$$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}$$

$$= \frac{-1}{3}x^2 e^{-3x} + \frac{2}{3}\left(-\frac{1}{3}x e^{-3x} - \frac{1}{9}e^{-3x}\right) + c$$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} - \frac{2}{27}e^{-3x} + c$$

11.6 - Integration by parts (IBP)

3c) $\int 2x^2(1+x)^3\,dx$

$u=(1+x)^3 \qquad v'=2x^2$

$u'=3(1+x)^2 \qquad v=\frac{2x^3}{3}$

$\int uv'\,dx=(1+x)^3\left(\frac{2x^3}{3}\right)-\int\frac{2x^3}{3}\times 3(1+x)^2dx$

$=\frac{2x^3}{3}(1+x)^3-2\int \underbrace{x^3(1+x)^2}_{\text{IBP again}}dx$

$u=(1+x)^2 \qquad v'=x^3$

$u'=2(1+x) \qquad v=\frac{x^4}{4}$

$\int uv'\,dx=\frac{x^4}{4}(1+x)^2-\int\frac{x^4}{4}\times 2(1+x)dx$

$=\frac{x^4}{4}(1+x)^2-\frac{1}{2}\int x^4(1+x)dx$

$=\frac{x^4}{4}(1+x)^2-\frac{1}{2}\int x^4+x^5dx$

$=\frac{x^4}{4}(1+x)^2-\frac{1}{2}\left(\frac{x^5}{5}+\frac{x^6}{6}\right)$

$=\frac{x^4}{4}(1+x)^2-\frac{x^5}{10}-\frac{x^6}{12}$

don't forget

$=\frac{2x^3}{3}(1+x)^3-2\left(\frac{x^4}{4}(1+x)^2-\frac{x^5}{10}-\frac{x^6}{12}\right)+C$

$=\frac{2x^3}{3}(1+x)^3-\frac{x^4}{2}(1+x)^2+\frac{x^5}{5}+\frac{x^6}{6}+C$ *

* I have verified my answer is equivalent to the book's answer.

4a) $\int_0^{\frac{\pi}{4}} x\sin x\,dx$ (ILATE)

$u=x \qquad v'=\sin x$

$u'=1 \qquad v=-\cos x$

$\int uv'\,dx=\left[x(-\cos x)-\int(-\cos x)(1)\,dx\right]_0^{\frac{\pi}{4}}$

$=\left[-x\cos x+\sin x\right]_0^{\frac{\pi}{4}}$

$=\left[-\frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\right]-\left[0+\sin(0)\right]$

$=-\frac{\sqrt{2}}{8}\pi+\frac{\sqrt{2}}{2}$

4b) $\int_2^5 x\sqrt{6-x}\,dx$

$u=x \qquad v'=(6-x)^{\frac{1}{2}}$

$u'=1 \qquad v=-\frac{2}{3}(6-x)^{\frac{3}{2}}$

RCR

$y=(6-x)^{\frac{3}{2}}$

$y'=\frac{3}{2}(6-x)^{\frac{1}{2}}\times -1$

$y'=-\frac{3}{2}(6-x)^{\frac{1}{2}}$

$\int uv'\,dx=\left[x\left(-\frac{2}{3}(6-x)^{\frac{3}{2}}\right)-\int-\frac{2}{3}(6-x)^{\frac{3}{2}}(1)\,dx\right]_2^5$

$=\left[-\frac{2x}{3}(6-x)^{\frac{3}{2}}+\frac{2}{3}\int(6-x)^{\frac{3}{2}}dx\right]_2^5$

RCR

$y=(6-x)^{\frac{5}{2}}$

$y'=-\frac{5}{2}(6-x)^{\frac{3}{2}}$

$=\left[-\frac{2x}{3}(6-x)^{\frac{3}{2}}+\frac{2}{3}\times-\frac{2}{5}(6-x)^{\frac{5}{2}}\right]_2^5$

$=\left(-\frac{18}{5}\right)-\left(-\frac{96}{5}\right)=\frac{78}{5}$

11.6 - Integration by parts (IBP)

4c) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x \operatorname{Cosec}^2 x \, dx$ (ILATE)

$u = x$ $\quad$ $v' = \operatorname{Cosec}^2 x$

$u' = 1$ $\quad$ $v = -\operatorname{Cot} x$ $\quad$ FB

$\int \cdot \, dx = \left[x(-\cot x) - \int -\cot x \, (1) \, dx \right]_{\pi/4}^{3\pi/4}$

$= \left[-x\cot x + \ln|\sin x| \right]_{\pi/4}^{\frac{3\pi}{4}}$

$= \left(\frac{3\pi}{4} + \ln\frac{\sqrt{2}}{2} \right) - \left(-\frac{1}{4}\pi + \ln\frac{\sqrt{2}}{2} \right)$

$= \underline{\underline{\pi}}$

4d) $\int_0^{\frac{\pi}{6}} x \sec^2 2x \, dx$ (ILATE)

$u = x$ $\quad$ $v' = \sec^2 2x$

$u' = 1$ $\quad$ $v = \frac{1}{2}\tan 2x$ $\quad$ FB

$\int \cdot \, dx = \left[x\left(\frac{1}{2}\tan 2x\right) - \int \frac{1}{2}\tan 2x \right]_0^{\pi/6}$ FB

$= \left[\frac{x}{2}(\tan 2x) - \frac{1}{4}\ln|\sec 2x| \right]_0^{\pi/6}$

$= \left(\frac{\pi}{12}\sqrt{3} - \frac{1}{4}\ln 2 \right) - \left(0 - \cancel{\frac{1}{4}\ln 1} \right)$

$= \underline{\underline{\frac{\pi}{12}\sqrt{3} - \frac{1}{4}\ln 2}}$

4e) $\int_0^{\pi/4} x^2 \operatorname{Cos} 4x \, dx$ (ILATE)

$u = x^2$ $\quad$ $v' = \operatorname{Cos} 4x$

$u' = 2x$ $\quad$ $v = \frac{1}{4}\sin 4x$

$\int \cdot \, dx = \left[x^2\left(\frac{1}{4}\sin 4x\right) - \int \frac{1}{4}\sin 4x \, (2x) \, dx \right]_0^{\pi/4}$

$= \left[\frac{1}{4}x^2 \sin 4x - \frac{1}{2}\int \underbrace{x \sin 4x \, dx}_{\text{IBP again (ILATE)}} \right]_0^{\pi/4}$

$u = x$ $\quad$ $v' = \sin 4x$

$u' = 1$ $\quad$ $v = -\frac{1}{4}\operatorname{Cos} 4x$

$\int \cdot \, dx = -\frac{1}{4}x\operatorname{Cos} 4x - \int -\frac{1}{4}\operatorname{Cos} 4x \, dx$

$= -\frac{1}{4}x\operatorname{Cos} 4x + \frac{1}{16}\sin 4x$

$= \left[\frac{1}{4}x^2 \sin 4x - \frac{1}{2}\left(-\frac{1}{4}x\operatorname{Cos} 4x + \frac{1}{16}\sin 4x \right) \right]_0^{\pi/4}$

$= \left[\frac{1}{4}x^2 \sin 4x + \frac{1}{8}x\operatorname{Cos} 4x - \frac{1}{32}\sin 4x \right]_0^{\pi/4}$

$= \left[0 - \frac{\pi}{32} - 0 \right] - \left[0 + 0 - 0 \right]$

$= \underline{\underline{-\frac{\pi}{32}}}$

11.6 - Integration by parts (IBP)

4f) $\int_0^3 x^3 e^{-x}\,dx$ (ILATE)

$u = x^3 \qquad v' = e^{-x}$

$u' = 3x^2 \qquad v = -e^{-x}$

$\int \cdots dx = \left[-x^3e^{-x} - \int -e^{-x}(3x^2)\,dx\right]_0^3$

$= \left[-x^3e^{-x} + 3\int \underbrace{x^2e^{-x}}\right]_0^3$

IBP again (ILATE)

$u = x^2 \qquad v' = e^{-x}$

$u' = 2x \qquad v = -e^{-x}$

$\int \cdots dx = -x^2e^{-x} - \int -e^{-x} \times 2x\,dx$

$= -x^2e^{-x} + \int \underbrace{2xe^{-x}}dx$

IBP again! (eyes rolling)

$u = 2x \qquad v' = e^{-x}$

$u' = 2 \qquad v = -e^{-x}$

$\int \cdots dx = -2xe^{-x} - \int -e^{-x}(2)dx$

$= -2xe^{-x} - 2e^{-x}$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x}$

$= \left[-x^3e^{-x} + 3(-x^2e^{-x} - 2xe^{-x} - 2e^{-x})\right]_0^3$

$= \left[-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x}\right]_0^3$

$= (-27e^{-3} - 27e^{-3} - 18e^{-3} - 6e^{-3}) -$

$(0 - 0 - 0 - 6e^0)$

$= \underline{\underline{-78e^{-3} + 6}}$

5) $\int_{\frac{1}{2}}^{2} 6x \ln 4x\,dx$ (ILATE)

$u = \ln 4x \qquad v' = 6x$

$u' = \frac{1}{4x}(4) = \frac{1}{x} \qquad v = \frac{6x^2}{2} = 3x^2$

$\int \cdots dx = \left[\ln 4x(3x^2) - \int 3x^2\left(\frac{1}{x}\right)dx\right]_{\frac{1}{2}}^2$

$= \left[3x^2\ln 4x - \int 3x\,dx\right]_{1/2}^2$

$= \left[3x^2\ln 4x - \frac{3x^2}{2}\right]_{\frac{1}{2}}^2$

$= (12\ln 8 - 6) - \left(\frac{3}{4}\ln 2 - \frac{3}{8}\right)$

$\ln 8 = \ln(2^3) = 3\ln 2$

$= 36\ln 2 - 6 - \frac{3}{4}\ln 2 + \frac{3}{8}$

$= \underline{\underline{\frac{141}{4}\ln 2 - \frac{45}{8}}}$

6) $\int_1^2 x^2e^{2x}\,dx$ (ILATE)

$u = x^2 \qquad v' = e^{2x}$

$u' = 2x \qquad v = \frac{1}{2}e^{2x}$

$\int \cdots dx = \left[x^2 \times \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}(2x)\right]_1^2$

$= \left[\frac{1}{2}x^2e^{2x} - \int \underbrace{xe^{2x}}\right]_1^2$

IBP again (ILATE)

$u = x \qquad v' = e^{2x}$

$u' = 1 \qquad v = \frac{1}{2}e^{2x}$

$\int \cdots dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}(1)dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$

$= \left[\frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)\right]_1^2$

$= \left[2e^4 - e^4 + \frac{1}{4}e^4\right] - \left[\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right]$

$= \frac{5}{4}e^4 - \frac{1}{4}e^2 = \underline{\underline{\frac{e^2}{4}(5e^2 - 1)}}$

11.6 - Integration by parts (IBP)

7a) $\int x \sin \pi x \, dx$ (ILATE)

$u = x$ $\quad$ $v' = \sin(\pi x)$

$u' = 1$ $\quad$ $v = \frac{-1}{\pi}\cos(\pi x)$

$$\int \text{''} \, dx = x\left(-\frac{1}{\pi}\cos(\pi x)\right) - \int -\frac{1}{\pi}\cos(\pi x)(1)\,dx$$

$$= -\frac{x}{\pi}\cos(\pi x) + \frac{1}{\pi}\int \cos \pi x \, dx$$

$$= \frac{-x}{\pi}\cos(\pi x) + \frac{1}{\pi} \times \frac{1}{\pi}\sin(\pi x) + c$$

$$= \underline{\underline{\frac{-x}{\pi}\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x) + c}}$$

7b) $\int_{3/2}^{5/2} x^2 \cos(\pi x)\,dx$ (ILATE)

$u = x^2$ $\quad$ $v' = \cos(\pi x)$

$u' = 2x$ $\quad$ $v = \frac{1}{\pi}\sin(\pi x)$

$$\int \text{''} \, dx = \left[x^2\left(\frac{1}{\pi}\sin \pi x\right) - \int \frac{1}{\pi}\sin(\pi x)(2x)\,dx\right]_{3/2}^{5/2}$$

$$= \left[\frac{x^2}{\pi}\sin(\pi x) - \frac{2}{\pi}\int \underbrace{x \sin \pi x \, dx}_{\text{use (a)}}\right]_{3/2}^{5/2}$$

$$= \left[\frac{x^2}{\pi}\sin(\pi x) - \frac{2}{\pi}\left(\frac{-x}{\pi}\cos(\pi x) + \frac{1}{\pi^2}\sin(\pi x)\right)\right]_{3/2}^{5/2}$$

$$= \left[\frac{x^2}{\pi}\sin(\pi x) + \frac{2x}{\pi^2}\cos(\pi x) - \frac{2}{\pi^3}\sin(\pi x)\right]_{3/2}^{5/2}$$

$$= \left[\frac{25}{4\pi} \times 1 + \frac{5}{\pi^2}(0) - \frac{2}{\pi^3}(1)\right] - \left[\frac{9}{4\pi}(-1) + \frac{3}{\pi^2}(0) - \frac{2}{\pi^3}(-1)\right]$$

$$= \frac{25}{4\pi} - \frac{2}{\pi^3} + \frac{9}{4\pi} - \frac{2}{\pi^3}$$

$$= \underline{\underline{\frac{17}{2\pi} - \frac{4}{\pi^3}}}$$

11.6 - Integration by parts (IBP)

8) $\int_{\frac{\pi}{4}}^{\pi/2} 2x^2 \operatorname{cosec}^2 x \cot x \, dx$

$u = 2x^2$ $\quad v' = \operatorname{cosec}^2 x \cot x$

$u' = 4x$

(RCR) $y = \operatorname{cosec}^2 x$

$\frac{dy}{dx} = 2\operatorname{cosec} x(-\operatorname{cosec} x \cot x)$

$= -2\operatorname{cosec}^2 x \cot x$

$v = -\frac{1}{2}\operatorname{cosec}^2 x$

$$\int .. \, dx = \left[2x^2\left(-\tfrac{1}{2}\operatorname{cosec}^2 x\right) - \int -\tfrac{1}{2}\operatorname{cosec}^2 x\,(4x)\,dx\right]_{\pi/4}^{\pi/2}$$

$$= \left[-x^2\operatorname{cosec}^2 x + 2\underbrace{\int x\operatorname{cosec}^2 x\,dx}_{\text{IBP}}\right]_{\pi/4}^{\pi/2}$$

$u = x$ $\quad v' = \operatorname{cosec}^2 x$

$u' = 1$ $\quad v = -\cot x$ FB

$\int .. \, dx = x(-\cot x) - \int -\cot x \, dx$

$= -x\cot x + \ln|\sin x|$

$$= \left[-x^2\operatorname{cosec}^2 x + 2(-x\cot x + \ln|\sin x|)\right]_{\pi/4}^{\pi/2}$$

$\cot(\frac{\pi}{2}) = 0$, use $\cot x$ graph, calculator gives maths error.

$$= \left[-\frac{\pi^2}{4}(1) + 0 + 2\ln 1\right] - \left[-\frac{\pi^2}{16}(2) - \frac{\pi}{2}(1) + 2\ln\left(\frac{\sqrt{2}}{2}\right)\right]$$

$$= -\frac{\pi^2}{4} + \frac{\pi^2}{8} + \frac{\pi}{2} - 2\ln\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\pi}{2} - \frac{\pi^2}{8} - 2\ln\left(\frac{\sqrt{2}}{2}\right)$$

$2\ln\left(\frac{\sqrt{2}}{2}\right) = 2\ln\left(\frac{2^{\frac{1}{2}}}{2^1}\right) = 2\ln\left(2^{-\frac{1}{2}}\right)$

$= 2 \times -\frac{1}{2}\ln 2$

$= -\ln 2$

$$= \frac{\pi}{2} - \frac{\pi^2}{8} + \ln 2$$

11.6 - Integration by parts (IBP)

9a) $\int x\cos 2x\,dx$ (ILATE)

$u = x$ $v' = \cos 2x$

$u' = 1$ $v = \frac{1}{2}\sin 2x$

$\int \text{''}\,dx = x\left(\frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\sin(2x)(1)\,dx$

$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

9b) $\int x\sin^2 x\,dx$ $\rightarrow$ $\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$= \int x\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)dx$

$= \int \frac{1}{2}x - \frac{1}{2}x\cos 2x\,dx$

$= \frac{1}{4}x^2 - \frac{1}{2}\int x\cos 2x\,dx$

↓ from (a)

$= \frac{1}{4}x^2 - \frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right) + c$

$= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$