Author: <u>Blinzy Fernandes</u>

This step-by-step solution guide has been created by **Blinzy Fernandes** for educational purposes. While we have made every effort to ensure the accuracy of the information presented, it is possible that there may be errors or omissions. We encourage users to critically evaluate and verify the content. BF Maths and the author cannot be held responsible for any errors or inaccuracies in this guide.

If you find any mistakes or have any suggestions for improvements, please contact us at bfmathshello@gmail.com. Your feedback is invaluable in helping us maintain the quality and accuracy of our resources. Please specify *<u>which exercise</u> and <u>which question</u>* in the email.

Thank you for using BF Maths for your maths revision!

11.1 Integrating Standard Functions

1a) $e^x + 2\sin x$

$= \int e^x + 2\sin x \; dx$

$= \int e^x + 2\int \sin x \; dx$

$= e^x + 2(-\cos x)$

$= e^x - 2\cos x + c$

b) $4\cos x - \dfrac{3}{x^2}$

$\int 4\cos x - \dfrac{3}{x^2} \; dx$

$= 4\int \cos x - 3\int x^{-2}$

$= 4(\sin x) - \dfrac{3x^{-1}}{-1} + c$

$= 4 \cancel{\cos} \sin x + 3x^{-1} + c$

c) $\dfrac{5}{x} - 2e^x$

$= \int \dfrac{5}{x} - 2e^x \; dx$

$= 5\int \dfrac{1}{x} \, dx - 2\int e^x \; dx$

$= 5(\ln|x|) - 2(e^x)$

$= 5(\ln|x|) - 2e^x + c$

d) $\sqrt{x} + \dfrac{3}{4}\sin x$

$= \int \sqrt{x} + \dfrac{3}{4}\sin x \; dx$

$= \int x^{1/2} + \dfrac{3}{4}\int \sin x \; dx$

11.1 Integrating Standard Functions

1d] Cont.

$= \frac{2}{3} x^{3/2} + \frac{3}{4}(-\cos x)$

$= \frac{2}{3} x^{3/2} - \frac{3}{4}\cos x + c$

e] $11x^2 + \frac{1}{2}e^x - \frac{6}{x}$

$= \int 11x^2 + \frac{1}{2}e^x - \frac{6}{x}\, dx$

$= 11\int x^2 + \frac{1}{2}\int e^x - 6\int \frac{1}{x}\, dx$

$= \frac{11}{3}x^3 + \frac{1}{2}e^x - 6(\ln|x|)$

$= \frac{11}{3}x^3 + \frac{1}{2}e^x - 6\ln|x| + c$

f] $3\cos x - \frac{1}{\sqrt{x}}$

$= \int 3\cos x - x^{-1/2}\, dx$

$= 3\int \cos x - \int x^{-1/2}\, dx$

$= 3\sin x - 2x^{1/2} + c$

$= 3\sin x - 2\sqrt{x} + c$

2a] $\int \frac{1}{\sin^2 x}\, dx$

$= \int \operatorname{cosec}^2 x\, dx$

$= -\cot x + c$

11.1 Integrating Standard Functions

2b) $\int \frac{\sin x}{\cos^2 x}\, dx$

$= \int \tan x \sec x \; dx$

$= \int \sec x \tan x \, dx$

$= \sec x + c$

c) $\int \frac{\cos x}{\sin^2 x}\, dx$

$= \int \cot x \operatorname{cosec} x \; dx$

$= -\operatorname{cosec} x + c$

d) $\cancel{\int \frac{4}{\cos^2 x}}$ $\int \frac{4}{\cos^2 x}\, dx$

$= 4\int \sec^2 x \; dx$

$= 4\tan x + c$

e) $\int \frac{4}{1-\cos^2 x}\, dx$

$= \int \frac{4}{\sin^2 x}\, dx$

$= 4\int \operatorname{cosec}^2 x \; dx$

$= -4\cot x + c$

f) $\int \frac{8\cos x}{\cos^2 x - 1}\, dx$

$= \int \frac{8\cos x}{-\sin^2 x}\, dx$

$= \int -8\cot x \operatorname{cosec}^2 x \; dx$

$= -8\int \cot x \operatorname{cosec}^2 x \, dx$

$= 8\operatorname{cosec} x + c$

11.1 Integrating standard functions

3a) $\int \sec x(\sec x + \tan x)\,dx$

$= \int \sec^2 x + \int \sec x \tan x \quad dx$

$= \tan x + \sec x + c$

b) $\int \frac{7x^2 - 5}{x}\,dx$

$= \int \frac{7x^2}{x} - \int \frac{5}{x} \quad dx$

$= \int 7x - 5\int \frac{1}{x}\,dx$

$= \frac{7}{2}x^2 - 5\ln|x| + c$

c) $\int \operatorname{cosec}^2 x\,(3e^x \sin^2 x - 5)$

$= \int \operatorname{cosec}^2 x \sin^2 x\, 3e^x - 5\operatorname{cosec}^2 x\,dx$

$= \int \frac{1}{\sin^2 x} \times \sin^2 x\; 3e^x - 5\int \operatorname{cosec}^2 x \quad dx$

$= 3\int e^x - 5\int \operatorname{cosec}^2 x\,dx$

$= 3e^x + 5\cot x + c$

d) $\int \frac{x^2 - x + 1}{x}\,dx$

$= \int \frac{x^2}{x} - \int \frac{x}{x} + \int \frac{1}{x}\,dx$

$= \int x - \int 1 + \int \frac{1}{x}\,dx$

$= \frac{x^2}{2} - x + \ln|x| + c$

11.1 Integrating Standard functions

4a) $\int_{\ln 5}^{\ln 20} e^{x} \, dx$

$= \left[e^{x}\right]_{\ln 5}^{\ln 20}$

$= e^{\ln 20} - e^{\ln 5}$

$= 15$

b) $\int_{2}^{5} \frac{1 - 3x^3}{x}$

$= \int_{2}^{5} \frac{1}{x} - \frac{3x^3}{\cancel{x}} \, dx$

$= \int \frac{1}{x} - 3\int x^2 \, dx$

$= \left[\ln|x| - x^3\right]_2^5$

$= \left[\ln 5 - (5)^3\right] - \left[\ln 2 - (2)^3\right]$

$= \ln 5 - 125 - \ln 2 + 8$

$= \ln \frac{5}{2} - 117$

c) $\int_{0}^{\frac{\pi}{3}} \left(\frac{\sin x}{\cos^2 x} + 4e^{x}\right) dx$

$= \int \frac{\sin x}{\cos^2 x} + \int 4e^{x} \, dx$

$= \int \tan x \sec x + 4\int e^{x} \, dx$

$= \sec x + 4e^{x}$

$= \left[\sec x + 4e^{x}\right]_0^{\pi/3}$

11.1 Integrating Standard functions

4c) Cont.

$$= \left(\sec\frac{\pi}{3} + 4e^{\pi/3}\right) - \left(\sec 0 + 4e^{0}\right)$$

$$= 2 + 4e^{\pi/3} - 1 - 4$$

$$= 4e^{\pi/3} - 3$$

d) $$\int_0^{\pi/4} \sec^2 x\,(4 - \cos x \tan x)\,dx$$

$$= \int 4\sec^2 x - \cos x \sec^2 x \tan x \,dx$$

$$= \int 4\sec^2 x - \sec x \tan x \,dx$$

$$= 4\int \sec^2 x - \int \tan x \sec x \,dx$$

$$= 4\tan x - \sec x$$

$$= \left[4\tan x - \sec x\right]_0^{\pi/4}$$

$$= \left[4\tan\left(\frac{\pi}{4}\right) - \sec\left(\frac{\pi}{4}\right)\right] - \left[4\tan(0) - \sec(0)\right]$$

$$= \left[4 - \sqrt{2}\right] - \left[-1\right]$$

$$= 5 - \sqrt{2}$$

e) $$\int_{\ln 2}^{\ln 6} \left(e^{x} + e^{2x}\right) dx$$

$$= \int e^{x} + \int e^{2x}\,dx$$

$$= e^{x} + \frac{1}{2}e^{2x}$$

$$= \left[e^{x} + \frac{1}{2}e^{2x}\right]_{\ln 2}^{\ln 6}$$

$$= \left[\cancel{e}^{\cancel{\ln} 6} + \frac{1}{2}e^{2\ln 6}\right] - \left[\cancel{e}^{\cancel{\ln} 2} + \frac{1}{2}e^{2\ln 2}\right]$$

11·1 Integrating Standard functions

4e] Cont.

$$= \left[6 + \frac{1}{2}\cancel{e^{\ln}} 6^2\right] - \left[2 + \frac{1}{2}\cancel{e^{\ln}} 2^2\right]$$

$$= \left[6 + \frac{36}{2}\right] - \left[2 + \frac{4}{2}\right]$$

$$= [6 + 18] - [4]$$

$$= 20$$

5] $\int_4^9 \frac{x(1+x)}{x^2}$

$$= \int \frac{x + x^2}{x^2}$$

$$= \int \frac{x}{x^2} + \int \frac{x^2}{x^2}$$

$$= \int \frac{1}{x} + \int 1$$

$$= \ln|x| + x$$

$$= \Big[\ln|x| + x\Big]_4^9$$

$$= [\ln 9 + 9] - [\ln 4 + 4]$$

$$= \ln 9 + 9 - \ln 4 - 4$$

$$= \ln \frac{9}{4} + 5$$

$$= \cancel{\ln \frac{9}{4}} \quad 5 + \ln \frac{9}{4}$$

11.1 Integrating Standard Functions

6] $\int_3^9 \frac{p}{x}\,dx = \ln 81$

$= p\int \frac{1}{x}\,dx$

$= p\ln|x|$

$= \left[p\ln|x|\right]_3^9$

$= [p\ln|9|] - [p\ln 3]$

$= p\ln 9 - p\ln 3 = \ln 81$

$= p(\ln 9 - \ln 3) = \ln 81$

$= p\left(\ln\left(\frac{9}{3}\right)\right) = \ln 81$

$= p\ln 3 = \ln 81$

$= p = \frac{\ln 81}{\ln 3}$

$= p = 4$

7] $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x(\cot^2 x - 3e^x\cos^2 x - 1)\,dx$

$= \int \sec^2 x\cot^2 x - \sec^2 x\, 3e^x\cos^2 x - \sec^2 x\,dx$

$= \int \frac{1}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\cos^2 x} \times \cos^2 x \times 3e^x - \sec^2 x\,dx$

$= \int \operatorname{cosec}^2 x - 3e^x - \sec^2 x\,dx$

$= \int \operatorname{cosec}^2 x - 3\int e^x - \int \sec^2 x\,dx$

$= -\cot x - 3e^x - \tan x$

$= \left[-\cot x - 3e^x - \tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

11.1 Integrating Standard Functions

7] Cont.

$= \left[-\cot\left(\frac{\pi}{3}\right) - 3e^{\pi/3} - \tan\left(\frac{\pi}{3}\right)\right] - \left[-\cot\left(\frac{\pi}{6}\right) - 3e^{\pi/6} - \tan\left(\frac{\pi}{6}\right)\right]$

$= \left[-\frac{\sqrt{3}}{3} - 3e^{\pi/3} - \sqrt{3}\right] - \left[-\sqrt{3} - 3e^{\pi/6} - \frac{\sqrt{3}}{3}\right]$

$= \cancel{-\frac{\sqrt{3}}{3}} - 3e^{\pi/3} \cancel{- \sqrt{3}} \cancel{+ \sqrt{3}} + 3e^{\pi/6} \cancel{+ \frac{\sqrt{3}}{3}}$

$= -3e^{\pi/3} + 3e^{\pi/6}$

$= 3\left(e^{\pi/6} - e^{\pi/3}\right)$

8] $\int_{2a}^{5a} \frac{2x+3}{x}\, dx = \ln 1000$

$= \int \frac{2x}{x} + \frac{3}{x}\, dx$

$= \int 2 + 3\int \frac{1}{x}\, dx$

$= 2x + 3\ln x$

$= \left[2x + 3\ln x\right]_{2a}^{5a}$

$= \left[2(5a) + 3\ln(5a)\right] - \left[2(2a) + 3\ln(2a)\right]$

$= 10a + 3\ln 5a - 4a - 3\ln 2a$

$= 6a + 3\ln 5a - 3\ln 2a = \ln 1000$

$= 6a + 3\left(\ln \frac{5\cancel{a}}{2\cancel{a}}\right) = \ln 1000$

11.1 Integrating Standard Functions

8) Cont.

$= 6a + 3\ln\frac{5}{2} = \ln 1000$

$= 6a = \ln 1000 - 3\ln\frac{5}{2}$

$= 6a = \ln 1000 - \ln\left(\frac{5}{2}\right)^3$

$= 6a = \ln 1000 - \ln\left(\frac{125}{8}\right)$

$= 6a = \ln\left(\frac{1000}{\frac{125}{8}}\right)$

$= 6a = \ln 64$

$= a = \frac{1}{6}\ln 64$

$= a = \ln(64)^{1/6}$

$= a = \underline{\underline{\ln 2}}$.