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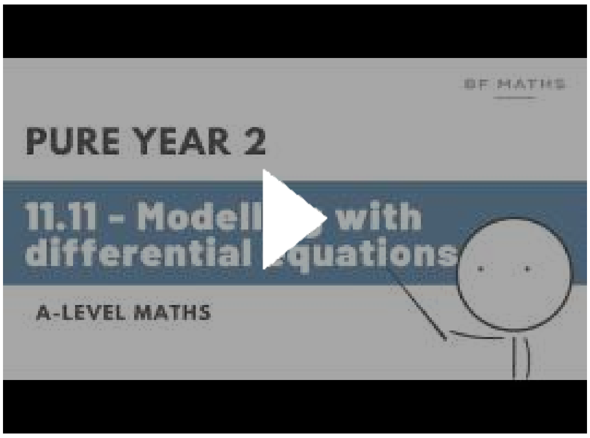
Thank you for using BF Maths for your maths revision!

If you need help on this chapter:
[A-Level Maths | Pure Year 2 | 11.11 - Modelling with differential equations Walkthrough | Edexcel]



11.11 - Modelling with differential equations

1a) $\frac{dP}{dt} = 3P^{\frac{1}{3}}$

$\int \frac{1}{P^{\frac{1}{3}}} dP = \int 3 \, dt$

$\frac{3}{2}P^{\frac{2}{3}} = 3t + c$

1b) When $t=0$, $P=1000$

$\frac{3}{2}(1000)^{\frac{2}{3}} = 3(0) + c$

$c = 150$

1c) $\frac{3}{2}P^{\frac{2}{3}} = 3t + 150$

$P^{\frac{2}{3}} = \frac{2}{3}(3t + 150)$

$P^{\frac{2}{3}} = 2t + 100$

$P = (2t + 100)^{\frac{3}{2}}$

1d) When $P = 2500$,

$2500 = (2t + 100)^{\frac{3}{2}}$

$2500^{\frac{2}{3}} = 2t + 100$

$t = \frac{2500^{\frac{2}{3}} - 100}{2} = 42.1$ years

11.11 - Modelling with differential equations

2ai) $\frac{dV}{dt}$ is the rate of change of the value of a laptop over time.

2aii) negative sign means the value of laptop decreases over time.

2aiii) The rate of change is directly proportional to the value of laptop.

2b) $\frac{dV}{dt} = -kV$

$$\int \frac{1}{V}\,dV = \int -k\,dt$$

$$\ln V = -kt + c$$

When $t=0$, $V = V_0$

$$\ln V_0 = 0 + c \Rightarrow c = \ln V_0$$

$$\ln V = -kt + \ln V_0$$

$$\cancel{e}^{\ln V} = e^{-kt + \ln V_0}$$

$$V = e^{-kt} \times \cancel{e}^{\ln V_0}$$

$$\underline{\underline{V = V_0 e^{-kt}}}$$

2c) $V_0 = \underline{\underline{800}}$

2d) $V = 800e^{-kt}$

When $t=4$, $V=550$

$$550 = 800e^{-k(4)}$$

$$\frac{11}{16} = e^{-4k}$$

$$-4k = \ln\left(\frac{11}{16}\right)$$

$$k = \frac{\ln\left(\frac{11}{16}\right)}{-4} = \underline{\underline{0.094}}$$

11.11 - Modelling with differential equations

3a) $\frac{dP}{dt} = kP$

$\int \frac{1}{P} dP = \int k dt$

$\ln P = kt + c$

$e^{\ln P} = e^{kt+c}$

$P = e^{kt} \times e^{c}$

Let $A = e^{c}$

$P = Ae^{kt}$

When $t=0$, $P=1600$

$\Rightarrow A = 1600$

$\Rightarrow P = 1600e^{kt}$

When $t=50$, $P = 2600$

$2600 = 1600e^{50k}$

$\ln\left(\frac{2600}{1600}\right) = 50k$

$$k = \frac{1}{50}\ln\left(\frac{13}{8}\right)$$

3b) When $P = 7800 \leftarrow (2600 \times 3)$

$7800 = 1600e^{\frac{1}{50}\ln\left(\frac{13}{8}\right)t}$

$\ln\left(\frac{7800}{1600}\right) = \frac{1}{50}\ln\left(\frac{13}{8}\right)t$

$t = \ln\left(\frac{7800}{1600}\right) \div \left(\frac{1}{50}\ln\left(\frac{13}{8}\right)\right) = 163.14\ldots$

$\therefore$ In Year 2063 $(1900 + 163 = 2063)$

3c) The model is not accurate when t is large.

11.11 - Modelling with differential equations

4a) $\frac{dT}{dt}$ is the rate of change of temperature of hot soup over time.

$-k$ represents the constant of direct proportionality

$(T-20)$ represents the difference in temperature between the soup and the room.

4b) $\frac{dT}{dt} = -k(T-20)$

$$\int \frac{1}{T-20}\,dT = \int -k\,dt$$

$$\ln(T-20) = -kt + c$$

$$T-20 = e^{-kt+c}$$

$$T-20 = e^{-kt} \times e^{c}$$

Let $A = e^{c}$

$$T-20 = Ae^{-kt}$$

$$T = Ae^{-kt} + 20$$

When $t=0$, $T=75$

$$75 = Ae^{0} + 20$$

$$A = 55$$

$$\Rightarrow T = 55e^{-kt} + 20$$

When $t=5$, $T=65$

$$65 = 55e^{-5k} + 20$$

$$\frac{45}{55} = e^{-5k}$$

$$\ln\left(\frac{45}{55}\right) = -5k$$

$$k = \frac{\ln\left(\frac{45}{55}\right)}{-5} = 0.04$$

$$T = 55e^{-0.04t} + 20$$

When $T=50$,

$$50 = 55e^{-0.04t} + 20$$

$$\frac{30}{55} = e^{-0.04t}$$

$$t = \frac{\ln\left(\frac{30}{55}\right)}{-0.04} = \underline{\underline{15.2\text{ mins}}}$$

11.11 - Modelling with differential equations

5a) Overall $\frac{dV}{dt} = 100\pi - 3\pi h$ ← net = "In" − "Out"

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$V = \pi r^2 h$
$V = \pi (30)^2 h$ – Cylinder
$V = 900\pi h$
$\frac{dV}{dh} = 900\pi$

$\frac{dh}{dt} = (100\pi - 3\pi h) \times \frac{dh}{dV}$

$\frac{dh}{dV} = \frac{1}{900\pi}$

$\frac{dh}{dt} = (100\cancel{\pi} - 3\cancel{\pi}h) \times \frac{1}{900\cancel{\pi}}$

$$\underline{\underline{900\frac{dh}{dt} = 100 - 3h}}$$

5b) $\int \frac{900}{100-3h}\,dh = \int 1\,dt$

RCR
Let $y = \ln|100-3h|$
$\frac{dy}{dh} = \frac{1}{100-3h} \times -3 = \frac{-3}{100-3h}$
$\therefore \int \cdots dh = -300\ln|100-3h|$

$-300\ln|100-3h| = t + c$

When $t=0$, $h=20$

$-300\ln 40 = c$

$\Rightarrow -300\ln|100-3h| = t - 300\ln 40$

$t = 300\ln 40 - 300\ln|100-3h|$

When $h = 30$,

$t = 300\ln 40 - 300\ln|100-3(30)|$

$= 300\ln 40 - 300\ln 10$

exact value.

$= 300\ln\left(\frac{40}{10}\right) = \underline{\underline{300\ln 4 \text{ seconds}}}$

11.11 - Modelling with differential equations

6a) $\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA}$ ← If you are struggling to construct differential equations, watch P2 Chp9.10(2) video.

$\frac{dr}{dt} = -k\sin\left(\frac{t}{4\pi}\right) \times \frac{dr}{dA}$ → $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = -k\sin\left(\frac{t}{4\pi}\right) \times \frac{1}{2\pi r}$ ← $\frac{dr}{dA} = \frac{1}{2\pi r}$

$$\underline{\underline{\frac{dr}{dt} = \frac{-k}{2\pi r}\sin\left(\frac{t}{4\pi}\right)}}$$

6b) $\int 2\pi r \, dr = \int -k\sin\left(\frac{t}{4\pi}\right) dt$

$\frac{2\pi r^2}{2} = -k\int \sin\left(\frac{t}{4\pi}\right) dt$ → eg. $\int \sin(5t)\,dt = -\frac{1}{5}\cos 5t$

$\pi r^2 = -k \times -4\pi\cos\left(\frac{t}{4\pi}\right) + c$

$\cancel{\pi} r^2 = 4k\cancel{\pi}\cos\left(\frac{t}{4\pi}\right) + c$

When $t = 0$, $r = 10$

$100 = 4k\cos(0) + c$

$c = 100 - 4k$

$\Rightarrow r^2 = 4k\cos\left(\frac{t}{4\pi}\right) + 100 - 4k$

When $t = 2\pi^2$, $r = 5$

$5^2 = 4k\cos\left(\frac{2\pi^2}{4\pi}\right) + 100 - 4k$

$25 = \cancel{4k\cos\left(\frac{1}{2}\pi\right)} + 100 - 4k$

0

$25 = 100 - 4k \Rightarrow k = \frac{75}{4}$

$\Rightarrow r^2 = 75\cos\left(\frac{t}{4\pi}\right) + 100 - 75$

$$\underline{\underline{r^2 = 75\cos\left(\frac{t}{4\pi}\right) + 25}}$$

6c) When $r = 0$,

$0 = 75\cos\left(\frac{t}{4\pi}\right) + 25$

$-\frac{25}{75} = \cos\left(\frac{t}{4\pi}\right)$

$\frac{t}{4\pi} = \cos^{-1}\left(\frac{-25}{75}\right)$

$t = 1.91 \times 4\pi = \underline{\underline{24\text{mins}}}$