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9.3 The Chain Rule

1a] $y = (4-3x)^3$

$\frac{dy}{dx} = 3(4-3x)^2 \times (-3)$

$= -9(4-3x)^2$

b] $y = (6+5x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(6+5x)^{-1/2} \times (5)$

$= \frac{5}{2}(6+5x)^{-1/2}$

c] $y = (5+2x^3)^4$

$\frac{dy}{dx} = 4(5+2x^3)^3 \times 6x^2$

$= 24x^2(5+2x^3)^3$

d] $y = (7+3x^2)^{-5}$

$\frac{dy}{dx} = -5(7+3x^2)^{-6} \times (6x)$

$= -30x(7+3x^2)^{-6}$

e] $y = \frac{6}{\sqrt{4-5x}} \Rightarrow 6(4-5x)^{-1/2}$

$\frac{dy}{dx} = -3(4-5x)^{-3/2} \times (-5)$

$= 15(4-5x)^{-3/2}$

$= \frac{15}{(4-5x)^{3/2}}$

9.3 The Chain Rule

1b] $y = \frac{1}{6-x} = (6-x)^{-1}$

$\frac{dy}{dx} = -(6-x)^{-2} \times (-1)$

$= (6-x)^{-2}$

g] $y = \sqrt[3]{3+2x^4}$

$= (3+2x^4)^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}(3+2x^4)^{-2/3} \times 8x^3$

$= \frac{8}{3}x^3(3+2x^4)^{-2/3}$

h] $y = \sqrt{7x^3-4} \Rightarrow (7x^3-4)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(7x^3-4)^{-1/2} \times (21x^2)$

$= \frac{21x^2(7x^3-4)^{-1/2}}{2}$

2a] $y = \sin^3 x \Rightarrow (\sin x)^3$

$\frac{dy}{dx} = 3(\sin x)^2 \times \cos x$

$= 3\cos x \sin^2 x$

b] $y = \ln x^4$

$= 4\ln x$

$\frac{dy}{dx} = \frac{4}{x}$

9.3 The Chain Rule

2c] $y = e^{\cos 4x}$

let $u = \cos 4x$, $y = e^u$

$\frac{du}{dx} = -4\sin 4x$, $\frac{dy}{du} = e^u$

$\frac{dy}{dx} = (-4\sin 4x)\, e^{\cos 4x}$

d] $y = (\ln x)^2$

$\frac{dy}{dx} = 2(\ln x) \times \frac{1}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x}$

e] $y = \ln(\cos x)$

let $u = \cos x$, $y = \ln u$

$\frac{du}{dx} = -\sin x$, $\frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$

$\frac{dy}{dx} = -\tan x$

f] $y = \sin(e^{-3x})$

let $u = e^{-3x}$, $v = \sin u$

$\frac{du}{dx} = -3e^{-3x}$, $\frac{dv}{dx} = \cos u$

$\frac{dy}{dx} = -3e^{3x}\cos(e^{-3x})$

9.3 The Chain rule

3a] $f(x) = (3 - \ln 6x)^2$

$f'(x) = 2(3 - \ln 6x) \times \frac{-1}{x}$

$f'(x) = \frac{-2}{x}(3 - \ln 6x)$

b) $f'\left(\frac{e^2}{6}\right)$ $\qquad f(x) = (3 - \ln 6x)^2$

let $u = 3 - \ln 6x$

$\Rightarrow f(x) = u^2$

$f'(x) = 2u(u')$

$u = 3 - \ln 6x \qquad u' = -\frac{6}{6x} \Rightarrow \frac{-1}{x}$

$f'(x) = 2(3 - \ln(6x))\left(\frac{-1}{x}\right)$

$= \frac{-2(3 - \ln(6x))}{x}$

Now :- $f'\left(\frac{e^2}{6}\right) \Rightarrow \ln(6x)$

$= x = \frac{e^2}{6} \Rightarrow \ln\left(6 \times \frac{e^2}{6}\right)$

$= \ln e^2$

$= 2$

$f'\left(\frac{e^2}{6}\right) = \frac{-2(3-2)}{\frac{e^2}{6}} \Rightarrow \frac{-2(1)}{\frac{e^2}{6}} \Rightarrow \frac{-12}{e^2}$

9.3 The Chain Rule

4] $2y^2 - 4y = x$

$\frac{dx}{dy} = 4y - 4$

$\frac{dy}{dx} = \frac{1}{4y - 4}$ At point (6, 3)

$\frac{dy}{dx} = \frac{1}{4(3) - 4}$

$\frac{dy}{dx} = \frac{1}{8}$

5] $y = (2x - 3)^5$

x-coordinate $= 2$

$\frac{dy}{dx} = 5(2x - 3)^4 \times 2$

$\frac{dy}{dx} = 10(2x - 3)^4$

$\frac{dy}{dx} = 10(2(2) - 3)^4$

$\frac{dy}{dx} = 10$

$y = (2(2) - 3)^5$

$y = 1$

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 1 = -\frac{1}{10}(x - 2)$

$y = -\frac{x}{10} + \frac{1}{5} + 1$

9.3 The Chain Rule

5] Cont.

$$y = -\frac{x}{10} + \frac{6}{5}$$

$$x + 10y - 12 = 0$$

6] $f(x) = \frac{2}{(6-x)^3}$ $\quad x$-coordinate $= 4$

$$f(x) = \frac{3}{(6-4)^3}$$

$$f(x) = \frac{3}{8}$$

$$f'(x) = 3(6-x)^{-3}$$

$$f''(x) = -9(6-x)^{-4} \times (-1)$$

$$f'(x) = 9(6-x)^{-4}$$

$$f'(x) = 9(6-4)^{-4} \implies \frac{9}{16}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{8} = \frac{9}{16}(x - 4)$$

$$y = \frac{9x}{16} - \frac{9}{4} + \frac{3}{8}$$

$$y = \frac{9x}{16} - \frac{15}{8}$$

$$9x - 16y - 30 = 0$$

7] $f(x) = 3\sin^2 x - 2\cos^2 x$

x-coordinate $= -\frac{5\pi}{6}$

9.3 The Chain Rule

7) Cont.

$$f'(x) = 3(\sin x)^2 - 2(\cos x)^2$$
$$= 6(\sin x)(\cos x) - 4(\cos x)(-\sin x)$$
$$= 6\sin x \cos x + 4\sin x \cos x$$
$$= 10\sin x \cos x$$

$$f'(x) = 10\sin\left(\frac{-5\pi}{6}\right)\cos\left(-\frac{5\pi}{6}\right)$$

$$f'(x) = \frac{5\sqrt{3}}{2}$$

$$f(x) = 3\sin^2\left(\frac{-5\pi}{6}\right) - 2\cos^2\left(\frac{-5\pi}{6}\right)$$

$$f(x) = \frac{3}{4} - \frac{3}{2}$$

$$f(x) = -\frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y + \frac{3}{4} = \frac{5\sqrt{3}}{2}\left(x + \frac{5\pi}{6}\right)$$

$$y = \frac{5\sqrt{3}x}{2} + \frac{25\sqrt{3}\pi}{12} - \frac{3}{4}$$

$$y = \frac{5\sqrt{3}x}{2} - \frac{9 + 25\sqrt{3}\pi}{12}$$

$$12y = 30\sqrt{3}x - 9 + 25\sqrt{3}\pi$$

$$12y - 30\sqrt{3}x - 25\sqrt{3}\pi + 9 = 0$$

9.3 The Chain Rule

8] $x = -2\sin 4y$, $y = \frac{2\pi}{3}$

$$\frac{dx}{dy} = -2\sin 4y$$

$$\frac{dx}{dy} = -8\cos 4y$$

$$\frac{dy}{dx} = \frac{-1}{8\cos 4y}$$

$$\frac{dy}{dx} = \frac{-1}{8\cos 4\left(\frac{2\pi}{3}\right)}$$

$$\frac{dy}{dx} = \frac{1}{4}$$

$$x = -2\sin 4\left(\frac{2\pi}{3}\right)$$

$$x = -\sqrt{3}$$

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - \frac{2\pi}{3} = -4(x + \sqrt{3})$$

$$y = -4x - 4\sqrt{3} + \frac{2\pi}{3}$$

$$y = -4x + 2 - \frac{12\sqrt{3}\pi}{3}$$

$$y + 4x + 4\sqrt{3} - \frac{2\pi}{3} = 0$$

9.3 The Chain Rule

9] $y = 2^{x^4}$, points $(-1, 2)$

$\frac{dy}{dx} = 2^{x^4} \times \ln(2) \times 4x^3$

$\frac{dy}{dx} = (4\ln(2))\, 2^{x^4} \times x^3$

$\frac{dy}{dx} = (4\ln 2) \times 2^{1^4} \times 1^3$

$\frac{dy}{dx} = 8\ln 2$

$y - y_1 = m(x - x_1)$

$y - 2^1 = 8\ln 2\,(x + 1)$

$y = 8\ln 2\, x + 8\ln 2 + 2$

$(8\ln 2)x + y + 8\ln 2 - 2 = 0$