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9.5 The Quotient Rule

1a] $f(x) = \frac{4x+3}{2x-9}$

$u = 4x+3$ $\quad v = 2x-9$

$\frac{du}{dx} = 4$ $\quad \frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{4(2x-9) - 2(4x+3)}{(2x-9)^2}$$

$$= \frac{\cancel{8x} - 36 - \cancel{8x} - 6}{(2x-9)^2}$$

$$= \frac{-42}{(2x-9)^2}$$

b] $f(x) = \frac{6x^2}{3-5x}$

$u = 6x^2$ $\quad v = 3-5x$

$\frac{du}{dx} = 12x$ $\quad \frac{dv}{dx} = -5$

$$\frac{dy}{dx} = \frac{(3-5x)12x - 6x^2(-5)}{(3-5x)^2}$$

$$= \frac{36x - 60x^2 + 30x^2}{(3-5x)^2}$$

$$= \frac{-30x^2 + 36x}{(3-5x)^2}$$

$$= \frac{6x(6-5x)}{(3-5x)^2}$$

c] $f(x) = \frac{1-8x}{(3x^3+2)^2}$

The Quotient Rule — 9.5

c] Cont.

$u = 1 - 8x$ $\qquad v = (3x^3 + 2)^2$

$\frac{du}{dx} = -8$ $\qquad \frac{dv}{dx} = 9x^6 + 12x^3 + 4$

$\frac{dv}{dx} = 54x^5 + 36x^2$

$$\frac{dy}{dx} = \frac{-8(3x^3+2)^2 - (54x^5+36x^2)(1-8x)}{(3x^3+2)^4}$$

$$= \frac{-8(3x^3+2)^2 + (54x^5+36x^2)(-1+8x)}{(3x^3+2)^4}$$

$$= \frac{-8(3x^3+2)^2 + 18x^2(8x-1)(3x^3+2)}{(3x^3+2)^4}$$

$$\frac{dy}{dx} = \frac{-8(3x^3+2)^2 + 18x^2(8x-1)(3x^3+2)}{(3x^3+2)^4}$$

d] $f(x) = \frac{7x^2 - 9x}{\sqrt{x^2+6}}$

$u = 7x^2 - 9x$ $\qquad v = (x^2+6)^{1/2}$

$\frac{du}{dx} = 14x - 9$ $\qquad \frac{dv}{dx} = \frac{1}{2}(x^2+6)^{-1/2} \times 2x$

$= x(x^2+6)^{-1/2}$

$$\frac{dy}{dx} = \frac{(14x-9)(x^2+6)^{1/2} - (7x^2-9x)(x(x^2+6)^{-1/2})}{(x^2+6)}$$

$$= \frac{(14x-9)(x^2+6)^{1/2} - (7x^3-9x^2)(x^2+6)^{-1/2}}{(x^2+6)}$$

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1d] Cont.

$$= \frac{(x^2+6)^{-1/2}\left((14x-9)(x^2+6) - 7x^3 + 9x^2\right)}{(x^2+6)}$$

$$= \frac{(14x-9)(x^2+6) - 7x^3 + 9x^2}{(x^2+6)^{1/2}(x^2+6)}$$

$$= \frac{14x^3 + 84x - 9x^2 - 54 - 7x^3 + 9x^2}{(x^2+6)^{3/2}}$$

$$= \frac{7x^3 + 84x - 54}{(x^2+6)^{3/2}}$$

2a] $y = \frac{4x^2}{\cos 2x}$

$u = 4x^2$ $\quad v = \cos 2x$

$\frac{du}{dx} = 8x$ $\quad \frac{dv}{dx} = 2(-\sin 2x)$

$= -2\sin 2x$

$$\frac{dy}{dx} = \frac{8x(\cos 2x) - (-2\sin 2x)(4x^2)}{\cos^2 2x}$$

$$= \frac{8x\cos 2x + 8x^2 \sin 2x}{\cos^2 2x}$$

$$= \frac{8x(\cos 2x + x\sin 2x)}{\cos^2 2x}$$

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2b) $y = \dfrac{x - e^{-x}}{\ln x}$

$u = x - e^{-x}$ $\qquad v = \ln x$

$\dfrac{du}{dx} = 1 + e^{-x}$ $\qquad \dfrac{dv}{dx} = \dfrac{1}{x}$

$$\frac{dy}{dx} = \frac{\ln x(1 + e^{-x}) - \left(\frac{1}{x}\right)(x - e^{-x})}{(\ln x)^2}$$

$$= \frac{\ln x + \ln x e^{-x} - 1 + \frac{e^{-x}}{x}}{(\ln x)^2}$$

$$= \frac{\ln x + e^{-x}\ln x + \frac{1}{x}e^{-x} - 1}{(\ln x)^2}$$

c) $y = \dfrac{e^x}{\cos^2 x}$

$u = e^x$ $\qquad v = \cos^2 x$

$\dfrac{du}{dx} = e^x$ $\qquad = (\cos x)^2$

$\dfrac{dv}{dx} = 2(\cos x)(-\sin x)$

$= -2\cos x \sin x$

$$\frac{dy}{dx} = \frac{e^x \cos^2 x - e^x(-2\cos x \sin x)}{\cos^4 x}$$

$$= \frac{e^x \cos^2 x + 2e^x \cos x \sin x}{\cos^4 x}$$

9.5 The Quotient Rule

2c] Cont.

$$\frac{dy}{dx} = \frac{e^x \cos x (\cos x + 2\sin x)}{\cos^4 x}$$

$$= \frac{e^x (\cos x + 2\sin x)}{\cos^3 x}$$

d] $y = \dfrac{5 - e^{-x}}{\sin 4x}$

$u = 5 - e^{-x}$ $\qquad$ $v = \sin 4x$

$\dfrac{du}{dx} = e^{-x}$ $\qquad$ $\dfrac{dv}{dx} = 4\cos 4x$

$$\frac{dy}{dx} = \frac{e^{-x}\sin 4x - 4\cos 4x (5 - e^{-x})}{(\sin 4x)^2}$$

$$= \frac{e^{-x}\sin 4x - 4(5 - e^{-x})\cos 4x}{\sin^2 4x}$$

3] $y = \dfrac{8 - 2x}{4x + 1}$

$u = 8 - 2x$ $\qquad$ $v = 4x + 1$

$\dfrac{du}{dx} = -2$ $\qquad$ $\dfrac{dv}{dx} = 4$

$$\frac{dy}{dx} = \frac{-2(4x+1) - 4(8 - 2x)}{(4x+1)^2}$$

$$= \frac{-8x - 2 - 32 + 8x}{(4x+1)^2}$$

9.5 The Quotient Rule

3] Cont.

$$\frac{dy}{dx} = \frac{-34}{(4x+1)^2}$$

Sub $x = \frac{1}{2}$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-34}{(4(\frac{1}{2})+1)^2} \Rightarrow \frac{-34}{9}$$

4a] $y = \frac{x^2 - 6x}{e^{2x}}$

$u = x^2 - 6x \qquad v = e^{2x}$

$\frac{du}{dx} = 2x - 6 \qquad \frac{dv}{dx} = 2e^{2x}$

$$\frac{dy}{dx} = \frac{e^{2x}(2x-6) - 2e^{2x}(x^2-6x)}{(e^{2x})^2}$$

$$= \frac{e^{2x}(2x - 6 - 2x^2 + 12x)}{(e^{2x})^2}$$

$$\frac{dy}{dx} = \frac{-2x^2 + 14x - 6}{e^{2x}}$$

b] Sub $x = 1$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2(1)^2 + 14(1) - 6}{e^{2(1)}}$$

$$= \frac{6}{e^2}$$

9.5 The Quotient Rule

4b] Cont.

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - \left(-\frac{5}{e^2}\right) = \frac{-e^2}{6}(x-1)$$

$$y + \frac{5}{e^2} = \frac{-e^2}{6}(x-1)$$

5] $y = \dfrac{\sin^2 x}{e^{2x}}$

$u = \sin^2 x$
$= (\sin x)^2$

$\dfrac{du}{dx} = 2(\sin x)\cos x$
$= 2\sin x \cos x$

$v = e^{2x}$

$\dfrac{dv}{dx} = 2e^{2x}$

$$\frac{dy}{dx} = \frac{e^{2x}(2\sin x(\cos x)) - \sin^2 x\,(2e^{2x})}{e^{4x}}$$

$$= \frac{2\sin x\,(e^{2x}\cos x - \sin x\, e^{2x})}{e^{4x}}$$

$$= \frac{2\sin x e^{2x}(\cos x - \sin x)}{e^{4x}}$$

$$= \frac{2\sin x(\cos x - \sin x)}{e^{2x}}$$

9.5 The Quotient Rule

6] $y = \dfrac{e^{4x}}{x(x-5)}$

$u = e^{4x}$ $\qquad$ $v = x(x-5)$

$\dfrac{du}{dx} = 4e^{4x}$ $\qquad$ $= x^2 - 5x$

$\dfrac{dv}{dx} = 2x - 5$

$$\frac{dy}{dx} = \frac{x^2 - 5x(4e^{4x}) - (2x-5)e^{4x}}{(x^2-5x)^2}$$

$$= \frac{e^{4x}(4x^2 - 20x - 2x + 5)}{x^2(x-5)^2}$$

$$= \frac{e^{4x}(4x^2 - 22x + 5)}{x^2(x-5)^2}$$

7] $x = \dfrac{\cos 2y}{ey}$ $\qquad$ $y = \dfrac{\pi}{8}$

Sub $y = \dfrac{\pi}{8}$ in x

$$x = \frac{\cos 2\left(\frac{\pi}{8}\right)}{e^{\pi/8}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{e^{\pi/8}} \quad \Longrightarrow \quad \frac{\sqrt{2}}{2e^{\pi/8}}$$

$$x = \frac{\cos 2y}{ey}$$

9.5 The Quotient Rule

7] Cont.

$u = \cos 2y$ $\qquad v = e^y$

$\frac{du}{dy} = 2(-\sin 2y)$ $\qquad \frac{dv}{dy} = e^y$

$= -2\sin 2y$

$$\frac{dx}{dy} = \frac{e^y(-2\sin 2y) - e^y(\cos 2y)}{e^{2y}}$$

$$= \frac{-2e^y \sin 2y - e^y \cos 2y}{e^{2y}}$$

$$= \frac{e^y(-2\sin 2y - \cos 2y)}{e^{2y}}$$

$$= \frac{-2\sin 2y - \cos 2y}{e^y}$$

Sub $y = \frac{\pi}{8}$ in $\frac{dx}{dy}$

$$\frac{dx}{dy} = \frac{-2\sin 2\left(\frac{\pi}{8}\right) - \cos 2\left(\frac{\pi}{8}\right)}{e^{\pi/8}}$$

$$= \frac{-2\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}}{e^{\pi/8}}$$

$$= \frac{-\sqrt{2} - \frac{\sqrt{2}}{2}}{e^{\pi/8}}$$

9.5 The Quotient Rule

7] Cont.

$$\frac{dx}{dy} = \frac{-\frac{3\sqrt{2}}{2}}{e^{\pi/8}}$$

$$= \frac{-3\sqrt{2}}{2e^{\pi/8}}$$

$$\frac{dy}{dx} = -\frac{2e^{\pi/8}}{3\sqrt{2}}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{8} = -\frac{2e^{\pi/8}}{3\sqrt{2}}\left(x - \frac{\sqrt{2}}{2e^{\pi/8}}\right)$$

$$y - \frac{\pi}{8} = -\frac{2xe^{\pi/8}}{3\sqrt{2}} + \frac{\sqrt{2}}{3\sqrt{2}}$$

$$3\sqrt{2}y - \frac{3\sqrt{2}\pi}{8} = -2xe^{\pi/8} + \sqrt{2} \quad (\div \sqrt{2})$$

$$3y - \frac{3\pi}{8} = -\frac{2xe^{\pi/8}}{\sqrt{2}} + 1$$

$$= 3y + \sqrt{2}e^{\pi/8}x - 1 - \frac{3\pi}{8} = 0$$

9.5 The Quotient Rule

8 a] $y = f(x)$ $\quad f(x) = \dfrac{\cos^2 x}{e^x}$

$u = \cos^2 x$
$= (\cos x)^2$
$\dfrac{du}{dx} = 2(-\sin x)(\cos x)$
$= -2\sin x \cos x$

$v = e^x$
$\dfrac{dv}{dx} = e^x$

$$\frac{dy}{dx} = \frac{-2e^x \sin x \cos x - e^x \cos^2 x}{e^{2x}}$$

$$= \frac{e^x(-2\sin x \cos x - \cos^2 x)}{e^{2x}}$$

$$= \frac{\cos x(-2\sin x - \cos x)}{e^x}$$

When $\dfrac{dy}{dx} = 0$

$$\frac{\cos x(-2\sin x - \cos x)}{e^x} = 0$$

$$\cos x(-2\sin x - \cos x) = 0$$

So $\cos x = 0$ OR $-2\sin x - \cos x = 0$

$-2\sin x = \cos x$

$\tan x = \dfrac{-1}{2}$

So either $\cos x = 0$ or $\tan x = \dfrac{-1}{2}$

9.5 The Quotient Rule

8b] $\cos x = 0$ $\qquad$ $\tan x = \frac{-1}{2}$

$x = \pm$

$x = -0.46, \pi - 0.46$

$x = -0.46, 2.68$

So $x = -1.57, 1.57, -0.46, 2.68$

So A : $x = -1.57$

B : $x = -0.46$

C : $x = 1.57$

D : $x = 2.68$