Author: Mr Fan

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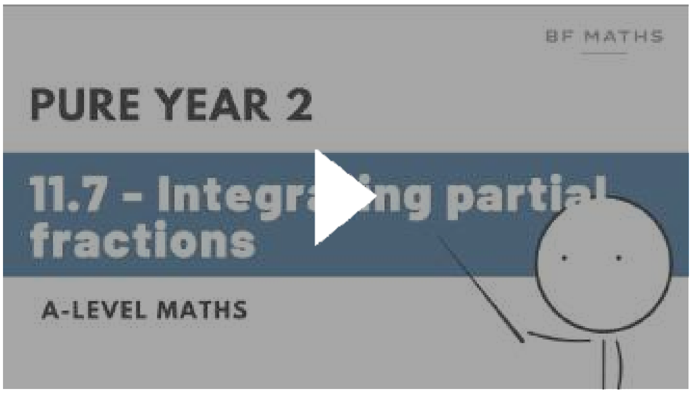
If you need help on this chapter:
A-Level Maths | Pure Year 2 | 11.7 - Integrating partial fractions Walkthrough | Edexcel



11.7 - Partial fractions

1a) $\dfrac{8x-4}{(x+2)(x-4)} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-4}$

$8x-4 = A(x-4) + B(x+2)$

$x=4$: $8(4)-4 = A(0) + B(4+2)$

$28 = 6B$

$B = \dfrac{14}{3}$

$x=-2$: $8(-2)-4 = A(-2-4) + B(0)$

$-20 = -6A$

$A = \dfrac{10}{3}$

1b) $\displaystyle\int \frac{8x-4}{(x+2)(x-4)}\,dx$

$= \displaystyle\int \frac{10}{3(x+2)} + \frac{14}{3(x-4)}\,dx$

$= \dfrac{10}{3}\displaystyle\int \frac{1}{x+2}\,dx + \frac{14}{3}\int \frac{1}{x-4}\,dx$

$= \dfrac{10}{3}\ln|x+2| + \dfrac{14}{3}\ln|x-4| + c$

2a) $\dfrac{4x+17}{(x+5)(x+2)} \equiv \dfrac{A}{x+5} + \dfrac{B}{x+2}$

$4x+17 = A(x+2) + B(x+5)$

$x=-2$: $4(-2)+17 = A(0) + B(-2+5)$

$9 = 3B \Rightarrow B = 3$

$x=-5$: $4(-5)+17 = A(-5+2) + B(0)$

$-3 = -3A \Rightarrow A = 1$

$\displaystyle\int \frac{1}{x+5} + \frac{3}{x+2}\,dx = \ln|x+5| + 3\ln|x+2| + c$

2b) $\dfrac{7x+6}{(x+3)(2-x)} \equiv \dfrac{A}{x+3} + \dfrac{B}{2-x}$

$7x+6 = A(2-x) + B(x+3)$

$x=2$: $7(2)+6 = A(0) + B(2+3)$

$20 = 5B \Rightarrow B = 4$

$x=-3$: $7(-3)+6 = A(2--3) + B(0)$

$-15 = 5A \Rightarrow A = -3$

$\displaystyle\int \frac{-3}{x+3} + \frac{4}{2-x}\,dx$

$= -3\ln|x+3| - 4\ln|2-x| + c$

2c) $\dfrac{x+17}{(x-1)(x+2)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+2}$

$x+17 = A(x+2) + B(x-1)$

$x=-2$: $-2+17 = A(0) + B(-2-1)$

$15 = -3B \Rightarrow B = -5$

$x=1$: $1+17 = A(1+2) + B(0)$

$18 = 3A \Rightarrow A = 6$

$\displaystyle\int \frac{6}{x-1} - \frac{5}{x+2}\,dx = 6\ln|x-1| - 5\ln|x+2| + c$

11.7 - Partial fractions

2d) $\dfrac{6}{(x-4)(2x-1)} \equiv \dfrac{A}{x-4} + \dfrac{B}{2x-1}$

$6 = A(2x-1) + B(x-4)$

$x=\frac{1}{2}$: $6 = A(0) + B(\frac{1}{2}-4)$

$B = \frac{-12}{7}$

$x=4$: $6 = A(2\times4-1) + B(0)$

$6 = 7A \Rightarrow A = \frac{6}{7}$

$\int \dfrac{6}{7(x-4)} - \dfrac{12}{7(2x-1)}\, dx$

$\frac{12}{7}\int \frac{1}{2x-1}\, dx$

RCR

Let $y = \ln|2x-1|$

$y' = \frac{1}{2x-1} \times 2$

$y' = \frac{2}{2x-1}$

$\int \frac{1}{2x-1}\, dx = \frac{1}{2}\ln|2x-1|$

$= \frac{6}{7}\ln|x-4| - \frac{12}{7} \times \frac{1}{2}\ln|2x-1| + c$

$= \frac{6}{7}\ln|x-4| - \frac{6}{7}\ln|2x-1| + c$

3a) $\dfrac{-2x^2+14}{(x+4)(2x-1)} = \dfrac{-2x^2+14}{2x^2+7x-4}$

$$\begin{array}{r|l} & -1 \\ \hline 2x^2+7x-4 & -2x^2 + 0x + 14 \\ - & -2x^2 - 7x + 4 \\ \hline & 7x + 10 \end{array}$$

$= -1 + \dfrac{7x+10}{2x^2+7x-4}$

$= -1 + \dfrac{7x+10}{(x+4)(2x-1)}$

3b) $\dfrac{7x+10}{(x+4)(2x-1)} \equiv \dfrac{A}{x+4} + \dfrac{B}{2x-1}$

$7x+10 = A(2x-1) + B(x+4)$

$x=\frac{1}{2}$: $7(\frac{1}{2})+10 = A(0) + B(\frac{1}{2}+4)$

$B = 3$

$x=-4$: $7(-4)+10 = A(2\times-4-1) + B(0)$

$-18 = -9A$

$A = 2$

$f(x) = \dfrac{2}{x+4} + \dfrac{3}{2x-1} - 1$

3c) $\int \dfrac{-2x^2+14}{(x+4)(2x-1)}\, dx = \int \dfrac{2}{x+4} + \dfrac{3}{2x-1} - 1\, dx$

$= 2\ln|x+4| + \frac{3}{2}\ln|2x-1| - x + c$

again $\int \frac{3}{2x-1}\, dx$

$= 3\int \frac{1}{2x-1}\, dx$

Let $y = \ln|2x-1|$

$y' = \frac{1}{2x-1} \times 2 = \frac{2}{2x-1}$

$\int \frac{3}{2x-1}\, dx = 3 \times \frac{1}{2}\ln|2x-1| = \frac{3}{2}\ln|2x-1|$

11.7 - Partial fractions

4a) $\frac{16x^2-37x+17}{(4-x)(2x-3)^2} \equiv \frac{A}{4-x} + \frac{B}{2x-3} + \frac{C}{(2x-3)^2}$

$16x^2-37x+17 = A(2x-3)^2 + B(4-x)(2x-3) + C(4-x)$

$x=4$: $16(4)^2-37(4)+17 = A(2\times4-3)^2 + B(0) + C(0)$

$125 = 25A$

$A = 5$

$x=\frac{3}{2}$: $16(\frac{3}{2})^2-37(\frac{3}{2})+17 = A(0)+B(0)+C(4-\frac{3}{2})$

$-2.5 = 2.5C$

$C=-1$

$x=0$: $16(0)^2-37(0)+17 = 5(-3)^2 + B(4)(-3) + (-1)(4)$

$17 = 45 - 12B - 4$

$12B = 24$

$B = 2$

$\Rightarrow \frac{16x^2-37x+17}{(4-x)(2x-3)^2} \equiv \frac{5}{4-x} + \frac{2}{2x-3} - \frac{1}{(2x-3)^2}$

4b) $\int \frac{5}{4-x} + \frac{2}{2x-3} - \frac{1}{(2x-3)^2}\,dx$

$= 5\int\frac{1}{4-x}dx + 2\int\frac{1}{2x-3}dx - \int(2x-3)^{-2}dx$

Let $y=(2x-3)^{-1}$

$y' = -1(2x-3)^{-2}\times 2$

$= -2(2x-3)^{-2}$

$\int dx = -\frac{1}{2}(2x-3)^{-1}$

$= -5\ln|4-x| + 2\times\frac{1}{2}\ln|2x-3| - -\frac{1}{2}(2x-3)^{-1} + c$

$= -5\ln|4-x| + \ln|2x-3| + \frac{1}{2}(2x-3)^{-1} + c$

11.7 - Partial fractions

5a) $\dfrac{3x+15}{(x-1)(x+2)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+2}$

$3x+15 = A(x+2) + B(x-1)$

$x=-2: \; 3(-2)+15 = A(0) + B(-3)$

$B = -3$

$x=1: \; 3(1)+15 = A(3) + B(0)$

$A = 6$

$\dfrac{3x+15}{(x-1)(x+2)} = \dfrac{6}{x-1} - \dfrac{3}{x+2}$

5b) $\displaystyle\int_2^3 f(x)\,dx = \int_2^3 \frac{6}{x-1} - \frac{3}{x+2}\,dx$

$= \Big[6\ln|x-1| - 3\ln|x+2|\Big]_2^3$

$= (6\ln 2 - 3\ln 5) - (6\ln 1 - 3\ln 4)$

$= \ln(2^6) - \ln(5^3) + \ln(4^3)$

$= \ln(2^6 \div 5^3 \times 4^3) = \ln\left(\dfrac{4096}{125}\right)$

11.7 - Partial fractions

6a) $\dfrac{2x^2-5x+5}{2x^2-5x+2}$

Improper fractions
$\rightarrow$ Algebraic division

$$\begin{array}{r} 1 \\ 2x^2-5x+2 \overline{) \, 2x^2-5x+5} \\ -) \; 2x^2-5x+2 \\ \hline 3 \end{array}$$

$= 1 + \dfrac{3}{2x^2-5x+2}$

$= 1 + \dfrac{3}{(x-2)(2x-1)}$

$\dfrac{3}{(x-2)(2x-1)} \equiv \dfrac{A}{x-2} + \dfrac{B}{2x-1}$

$3 = A(2x-1) + B(x-2)$

$x=2: \; 3 = A(3) \Rightarrow A = 1$

$x=\frac{1}{2}: \; 3 = B(-1.5) \Rightarrow B = -2$

$= 1 + \dfrac{1}{x-2} - \dfrac{2}{2x-1}$

$(A=1, B=-2, C=1)$

6b) $\displaystyle\int_3^5 f(x) = \int_3^5 1 + \frac{1}{x-2} - \frac{2}{2x-1} \, dx$

$= \left[x + \ln|x-2| - 2 \times \frac{1}{2}\ln|2x-1|\right]_3^5$

$= \left[x + \ln\left|\frac{x-2}{2x-1}\right|\right]_3^5$

$= 5 + \ln\left|\frac{3}{9}\right| - \left(3 + \ln\left|\frac{1}{5}\right|\right)$

$= 2 + \ln\left(\frac{3}{9} \div \frac{1}{5}\right) = \underline{\underline{2 + \ln\frac{5}{3}}}$

11.7 - Partial fractions

7a) $$\frac{12x+6}{(2x+1)^2(x+3)} \equiv \frac{A}{2x+1} + \frac{B}{(2x+1)^2} + \frac{C}{x+3}$$

$$12x+6 = A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$$

$x=-3$: $12(-3)+6 = C(-5)^2$

$$C = \frac{-6}{5}$$

$x=-\frac{1}{2}$: $12(-\frac{1}{2})+6 = B(-\frac{1}{2}+3)$

$$B = 0$$

$x=0$: $12(0)+6 = A(3) - \frac{6}{5}(1)^2$

$$A = \frac{12}{5}$$

$$\underline{\underline{\frac{12x+6}{(2x+1)^2(x+3)} = \frac{12}{5(2x+1)} - \frac{6}{5(x+3)}}}$$

7b) $$\int_1^2 f(x)\,dx = \int_1^2 \frac{12}{5(2x+1)} - \frac{6}{5(x+3)}\,dx$$

$$= \left[\frac{12}{5} \times \frac{1}{2}\ln|2x+1| - \frac{6}{5}\ln|x+3|\right]_1^2$$

$$= \left[\frac{6}{5}\ln|2x+1| - \frac{6}{5}\ln|x+3|\right]_1^2$$

$$= \left(\frac{6}{5}\ln 5 - \frac{6}{5}\ln 5\right) - \left(\frac{6}{5}\ln 3 - \frac{6}{5}\ln 4\right)$$

$$= \frac{6}{5}\ln 4 - \frac{6}{5}\ln 3$$

$$= \frac{6}{5}(\ln 4 - \ln 3)$$

$$= \underline{\underline{\frac{6}{5}\ln\frac{4}{3}}}$$

11.7 - Partial fractions

8a) $\dfrac{x^2+4}{x^2-1}$

Improper fraction

↳ Algebraic division

$$\begin{array}{r} 1 \\ x^2-1\;\big)\;x^2+4 \\ -)\;x^2-1 \\ \hline 5 \end{array}$$

$= 1 + \dfrac{5}{x^2-1}$

$= 1 + \dfrac{5}{(x-1)(x+1)}$

$\dfrac{5}{(x-1)(x+1)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$5 = A(x+1) + B(x-1)$

$x=1:\quad 5 = 2A \quad \Rightarrow A = \frac{5}{2}$

$x=-1:\quad 5 = -2B \quad \Rightarrow B = \frac{-5}{2}$

$= 1 + \dfrac{5}{2(x-1)} - \dfrac{5}{2(x+1)}$

$(A=1,\ B=-\frac{5}{2},\ C=\frac{5}{2})$

8b) $\int_2^4 f(x)\,dx = \int_2^4 1 + \dfrac{5}{2(x-1)} - \dfrac{5}{2(x+1)}\,dx$

$= \left[x + \frac{5}{2}\ln|x-1| - \frac{5}{2}\ln|x+1|\right]_2^4$

$= \left(4 + \frac{5}{2}\ln 3 - \frac{5}{2}\ln 5\right) - \left(2 + \frac{5}{2}\ln 1 - \frac{5}{2}\ln 3\right)$

$= 2 + \frac{5}{2}\ln 3 - \frac{5}{2}\ln 5 + \frac{5}{2}\ln 3$

$= 2 + \frac{5}{2}\ln(3 \div 5 \times 3)$

$= 2 + \frac{5}{2}\ln\frac{9}{5}$