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9.2 Differentiating exponentials and logarithms

1a) $y = 6e^{-3x}$

$\frac{dy}{dx} = -18e^{-3x}$

b) $y = 5e^{2x} - 7e^{-x}$

$\frac{dy}{dx} = 10e^{2x} + 7e^{-x}$

c) $\frac{7}{e^{5x}}$

$y = 7e^{-5x}$

$\frac{dy}{dx} = \frac{-35}{e^{5x}}$

d) $\frac{(1-e^{2x})^2}{e^{2x}} \Rightarrow \frac{1 + e^{4x} - 2e^{2x}}{e^{2x}}$

$= \frac{1}{e^{2x}} + e^{2x} - 2$

$y = e^{-2x} + e^{2x} - 2$

Let $y = e^{-2x}$

$\frac{dy}{dx} = -2e^{-2x}$

Let $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$

So $\frac{dy}{dx} = 2e^{2x} - 2e^{-2x}$

9.2 Differentiating exponentials and logarithms

2a] $f(x) = \ln 2x$

$f'(x) = \frac{1}{x}$

b] $f(x) = \ln x^2$

$= \frac{1}{x} \times 2$

$= \frac{2}{x}$

c] $f(x) = \ln\left(\frac{7}{x}\right)$

$= \ln(7) - \ln(x)$

$= \frac{-1}{x}$

d] $f(x) = \ln(4x^3)$

$= \ln(4) + \ln(x^3)$

$= \frac{1}{x} \times 3$

$= \frac{3}{x}$

3a] $y = 3^x$

$\frac{dy}{dx} = 3^x(\ln 3)$

3b] $y = 5^{2x}$

$\frac{dy}{dx} = 5^{2x}(\ln 5^2)$

$= 5^{2x}(2\ln 5)$

c] $y = \left(\frac{2}{5}\right)^{4x}$

$\frac{dy}{dx} = \left(\frac{2}{5}\right)^{4x}\left(4\ln \frac{2}{5}\right)$

d] $y = 8^{-x}$

$\frac{dy}{dx} = 8^{-x}(-\ln 8)$

$= -\frac{\ln 8}{8^x}$

9.2 Differentiating exponentials and logarithm

4) $y = 4^x$ —— ①

Sub $x = \frac{-1}{2}$ in equation ①

$y = 4^{-1/2} \Rightarrow y = \frac{1}{2}$

$\frac{dy}{dx} = 4^x(\ln 4)$ —— ②

Sub $x = \frac{-1}{2}$ in equation ②

$\frac{dy}{dx} = 4^{-1/2}(\ln 4)$

$\frac{dy}{dx} = \frac{1}{2}(\ln 4)$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{2}(\ln 4)\left(x + \frac{1}{2}\right)$

$y = \frac{x}{2}\ln 4 + \frac{1}{4}\ln 4 + \frac{1}{2}$

$y = \frac{x}{2}\ln(2^2) + \frac{1}{4}(\ln(2^2)) + \frac{1}{2}$

$y = \frac{x(\cancel{2}\ln 2)}{\cancel{2}} + \frac{\cancel{2}\ln 2}{\cancel{4}2} + \frac{1}{2}$

$y = x\ln 2 + \frac{\ln 2}{2} + \frac{1}{2}$

$y = x\ln 2 + \frac{1}{2}(1 + \ln 2)$

9.2 Differentiating exponentials and logs

5] $f(x) = e^{-x} + 2\ln x^3$

$f'(x) = -e^{-x} + 2\left(\frac{1}{x}\right) \times 3$

$= -e^{-x} + \frac{6}{x}$

6] $y = 3e^{x} - \ln x^2$ —— ①

Sub $x = 1$ in equation ①

$y = 3e^{1} - \ln(1)^2$

$y = 3e - \ln 1$

$y = 3e$

$\frac{dy}{dx} = 3e^{x} - \frac{1}{x}(2)$

$\frac{dy}{dx} = 3e^{x} - \frac{2}{x}$ —— ②

Sub $x = 1$ in equation ②

$\frac{dy}{dx} = 3e^{1} - \frac{2}{1}$

$= 3e - 2$

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 3e = \frac{-1}{(3e-2)}(x - 1)$

9.2 Differentiating exponentials and logs

6] Cont.

$$y(3e-2) - 3e(3e-2) = -(x-1)$$

$$y(3e-2) - 9e^2 + 6e = -x+1$$

$$x + (3e-2)y - 9e^2 + 6x - 1 = 0$$

Hence, proved.

7] $f(x) = 4^x + 4^{-x}$

Let $y = f(x)$

$$y = 4^x + 4^{-x} \quad \text{——} \ ①$$

Sub $x = \frac{1}{2}$ in equation ①

$$y = 4^{1/2} + 4^{-1/2}$$

$$y = \frac{5}{2}$$

$$f'(x) = 4^x(\ln 4) - 4^{-x}(\ln 4^{-1})$$

$$= 4^x(\ln 4) - 4^{-x}(-\ln 4)$$

$$= 4^x(\ln 4) + \frac{\ln 4}{4x} \quad \text{——} \ ②$$

Sub $x = \frac{1}{2}$ in equation ②

9.2 Differentiating exponentials and logs

7] Cont.

$$f'(x) = 4^{1/2}(\ln 4) + \frac{\ln 4}{4^{1/2}}$$

$$= 2\ln 4 + \frac{\ln 4}{2}$$

$$= 2\ln(2^2) + \frac{\ln(2^2)}{2}$$

$$= 4\ln 2 + \ln 2$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{5}{2} = (4\ln 2 + \ln 2)\left(x - \frac{1}{2}\right)$$

$$y = (\ln 2^2 + \ln 2)\left(x - \frac{1}{2}\right) + \frac{5}{2}$$

$$y = (\ln 4 + \ln 2)\left(x - \frac{1}{2}\right) + \frac{5}{2}$$

$$y = \ln 8\left(x - \frac{1}{2}\right) + \frac{5}{2}$$

$$y = (\ln 8)x - \frac{1}{2}\ln 8 + \frac{5}{2}$$

$$y = (\ln 8)x + \frac{5}{2} - \frac{1}{2}\ln 8$$

Hence, proved.

9.2 Differentiating exponentials and logs

8] $f(x) = 9^{3x} + 3^{6x}$

a] $= (3^2)^{3x} + 6^{6x}$

$= 3^{6x} + 3^{6x}$

$= 6^{6x}$

$f(x) = 2(3^{6x})$

b] $f(x) = 2(3^{6x})$

$f'(x) = 2(3^{6x}(\ln 3^6))$

$f'(x) = 2(3^{6x}(6\ln 3))$

$f'(x) = 12(3^{6x}\ln 3)$

$f'(x) = (4\ln 3)(3 \times 3^{6x})$

$f'(x) = (4\ln 3)(3^{6x+1})$