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8.4 Points of Intersection

1a] $x = t - 3$ — ① $y = t + 4$ — ③

$t = x + 3$ — ②

Sub ② in ③

$y = x + 3 + 4$ $(-7, 0), (0, 7)$

$y = x + 7$

when $x = 0$, $y = 7$

when $y = 0$, $x = -7$

b] $x = 3t$ — ① $y = 3(t+2)(t-1)$ — ③

$\frac{x}{3} = t$ — ②

Sub ② in ③

$y = 3\left(\frac{x}{3} + 2\right)\left(\frac{x}{3} - 1\right)$

$y = 3\left(\frac{x^2}{9} - \frac{x}{3} + \frac{2x}{3} - 2\right)$

$y = 3\left(\frac{x^2}{9} + \frac{x}{3} - 2\right)$

$y = \frac{x^2}{3} + x - 6$ $(-6, 0), (3, 0)$

when $x = 0$, $y = -6$ $(0, -6)$

when $y = 0$, $x = 3$ and -6

c] $x = 2t^2 - 2$ — ②   , $y = t^2 - 4t$ — ③

$\frac{x+2}{2} = t^2$

Sub ① in ③

$\sqrt{\frac{x+2}{2}} = t$ — ①

$y = \frac{x+2}{2} - 4\left(\sqrt{\frac{x+2}{2}}\right)$

Square both sides

8.4 Points of Intersection

1c] Cont.

$y^2 = \left(\frac{x+2}{2}\right)^2 - 4^2\left(\sqrt{\frac{x+2}{2}}\right)^2$

$y^2 = \frac{x^2+4x+4}{4} - 16\left(\frac{x+2}{2}\right)$

$y^2 = \frac{x^2+4x+4}{4} - 8x - 16$

$4y^2 = x^2+4x+4 - 32x - 64$

$4y^2 = x^2 - 28x - 60$

When $y=0$, $x = 30, -2$ $\quad (-2,0)$ $(30,0)$ ~~(0,-3)~~ ~~(0,5)~~

$x = 2t^2 - 2$ when $x = 0$

$t = 1$ or -1

$y = 1^2 - 4(1) \Rightarrow -3$ $\quad (0,-3)$

$y = (-1)^2 - 4(-1) \Rightarrow 5$ $\quad (0,5)$

2a] $x = \sin 2t + 1$, $y = 2\tan^2 t$

When $y = 0$

$2\tan^2 t = 0$

$\tan^2 t = 0$

$\tan t = 0$

$t = 0$ —— ①

Sub ① in x

$x = \sin 2(0) + 1$

$x = 1$

Coordinates $= (1,0)$

When $x = 0$

$\sin 2t + 1 = 0$

$\sin 2t = -1$

$2t = -\frac{\pi}{2}$

$t = \frac{\pi}{4}$ —— ③

Sub ③ into y

$y = 2\tan^2\left(-\frac{\pi}{4}\right)$

$= 2(1)$

$y = 2$

$(0,2)$

$\therefore$ Coordinates are $(1,0)$ & $(0,2)$

8.4 Points of Intersection

2b] $x = 2t$, $y = \cos t$

When $x = 0$, $t = 0$

$y = \cos(0) \Rightarrow y = 1$

So coordinate $= (0, 1)$

When $y = 0$

$\cos t = 0$ $\qquad -\pi \le t \le \pi$

$t = -\frac{\pi}{2}, \frac{\pi}{2}$

$t = -\frac{\pi}{2}$ $\qquad t = \frac{\pi}{2}$

$\cancel{x = 2\left(\frac{-\pi}{2}\right)}$ $x = 2\left(\frac{-\pi}{2}\right)$ $\qquad x = 2\left(\frac{\pi}{2}\right)$

$x = -\pi$ $\qquad x = \pi$

So, the coordinates are $(-\pi, 0)$ & $(\pi, 0)$

x-axis : $(-\pi, 0)$ & $(\pi, 0)$

y-axis : $(0, 1)$

c] $x = e^t - 3$, $y = e^{2t} - 2$ $\qquad t > 0$

When $y = 0$

$e^{2t} - 2 = 0$

$e^{2t} = 2$

$\cancel{\ln} e^{2t} = \ln 2$

$2t = \ln 2$

$t = \frac{\ln 2}{2}$ $\qquad$ Sub t value in x

So, $x = e^{\frac{\ln 2}{2}} - 3$

$x = \sqrt{2} - 3$

$\therefore$ The coordinates are $(\sqrt{2} - 3, 0)$

8.4 Points of Intersection

2c] Cont.

When $x = 0$ → So, $y = e^{2(\ln 3)} - 2$

$e^t - 3 = 0$ — $y = e^{2\ln 3} - 2$

$e^t = 3$ — $y = e^{\ln 3^2} - 2$

$\cancel{\ln e}^t = \ln 3$ — $y = 9 - 2$

$t = \ln 3$ — $y = 7$

Coordinates $= (0, 7)$

x: axis $= (\sqrt{2} - 3, 0)$ and y axis: $(0, 7)$

3a] $x = t + 1$, $y = t + 3$

$x^2 + y^2 + 6x - 4 = 0$

$x^2 + 6x + y^2 - 4 = 0$ Centre $= (-3, 0)$

$(x+3)^2 - 9 + y^2 - 4 = 0$ radius $= \sqrt{13}$

$(x+3)^2 + y^2 = 13$

Sub x and y in the equation of a circle

$(t + 1 + 3)^2 + (t + 3)^2 = 13$

$(t + 4)^2 + (t + 3)^2 = 13$

$t^2 + 8t + 16 + t^2 + 6t + 9 = 13$

$2t^2 + 14t + 12 = 0$

$(t + 1)(t + 6) = 0$

$t = -1$, $t = -6$

b] Sub the values of t in x & y

When $t = -1$, When $t = -6$

$x = -1 + 1$ — $x = -6 + 1$

$x = 0$ — $x = -5$

$y = -1 + 3$ — $y = -6 + 3$

$y = 2$ — $y = -3$

$(0, 2)$ — $(-5, -3)$

So, the coordinates are $(0, 2)$, $(-5, -3)$

8.4 Points of Intersection

4a] $x = p(1+2t)$, $y = p(1-2t)^2$

point $(6,0)$

$6 = p(1+2t)$ $\qquad 0 = p(1-2t)^2$ — ②

$\frac{6}{1+2t} = p$ — ①

Sub ① in ②

$\frac{6}{1+2t}(1-2t)^2 = 0$

$\frac{6}{1+2t}(1-4t+4t^2) = 0$

$6(1-4t+4t^2) = 0$

$6 - 24t + 24t^2 = 0$

$t = \frac{1}{2}$

Sub $t = \frac{1}{2}$ in x

$x = p\left(1+2\left(\frac{1}{2}\right)\right)$

$6 = p(2)$

$\underline{\underline{p = 3}}$

b] $x = qt^2 - 2q$, $y = q(t-1)(t-3)^2$

$\left(\frac{7}{2}, 0\right)$ $\qquad 0 = q(t-1)(t-3)^2$ — ②

$\frac{7}{2} = qt^2 - 2q$ $\qquad$ Sub ① and ② together

$\frac{7}{2} = q(t^2-2)$

$\frac{7}{2(t^2-2)} = q$ — ①

$0 = \frac{7}{2(t^2-2)}(t-1)(t-3)^2$

$0 = 7(t-1)(t^2-6t+9)$

8.4 Points of Intersection

4b) Cont.

$$0 = 7(t^3 - 6t^2 + 9t - t^2 + 6t - 9)$$
$$0 = 7(t^3 - 7t^2 + 15t - 9)$$
$$0 = 7t^3 - 49t^2 + 105t - 63$$

let $f(t) = 1$

$$f(1) = 7(1)^3 - 49(1)^2 + 105(1) - 63$$
$$= 0$$

So $(t-1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & 7t^2 - 42t + 63 \\ \hline t-1 & 7t^3 - 49t^2 + 105t - 63 \\ & \cancel{7t^3} - 7t^2 \\ & - \quad + \\ \hline & \cancel{-42t^2} + 105t - 63 \\ & -42t^2 + 42t \\ & + \quad - \\ \hline & 63t - 63 \\ & 63t - 63 \\ & - \quad + \\ \hline & 0 \end{array}$$

$$(t-1)(7t^2 - 42t + 63)$$
$$(t-1)(t-3)^2$$

$t = 1$ and $t = 3$

Since $t > 1$, we take $t = 3$

$$\frac{7}{2} = q(3^2) - 2q$$

$$\frac{7}{2} = 9q - 2q$$

$$q = \frac{1}{2}$$

8.4 Points of Intersection

5a] $x = e^{2t}$ — (4) $\quad y = 4e^{t} - 4$ — (2)

$y = x - 1$ — (1)

Sub (1) and (2) together

$x - 1 = 4e^{t} - 4$

$x = 4e^{t} - 3$ — (3)

Sub (3) and (4) together

$e^{2t} = 4e^{t} - 3$

$e^{2t} - 4e^{t} + 3 = 0$ $\quad$ let $\cancel{\$}e^{2t} = z^2$

$z^2 - 4z + 3 = 0$ $\quad e^{t} = z$

$z = 3 \qquad z = 1$

$e^{t} = \cancel{\#}3 \qquad e^{t} = 1$

$t\,\cancel{\ln e} = \ln 3 \qquad t\,\cancel{\ln e} = \ln 1$

$t = \ln 3$ or $t = 0$

b] When $t = \ln 3$

$x = e^{2\ln 3}$ $\qquad y = 4e^{\ln 3} - 4$

$x = (e^{\ln 3})^2$ $\qquad y = 4(3) - 4$

$x = 3^2$ $\qquad y = 8$

$x = 9$ $\qquad \underline{(9, 8)}$

When $t = 0$ $\qquad y = 4e^{0} - 4$

$x = e^{2(0)}$ $\qquad y = 0$

$x = 1$ $\qquad \underline{(1, 0)}$

6a] $x = \cos t$ $\qquad y = 2\sin 2t + 1$

$y = 1$

$1 = 2\sin 2t + 1$

$2\sin(2t) = 0$

$\sin 2t = 0$

$2t = \pi, 2\pi$ $\qquad \frac{\pi}{4} < t < \frac{3\pi}{2}$

$t = \frac{\pi}{2}, \pi$

$\frac{\pi}{2} < 2t < 3\pi$

8.4 Points of Intersection

6a) Cont.

Sub $t = \frac{\pi}{2}$ in x

$x = \cos\left(\frac{\pi}{2}\right)$

$x = 0$

Sub $t = \pi$ in x

$x = \cos(\pi)$

$x = -1$

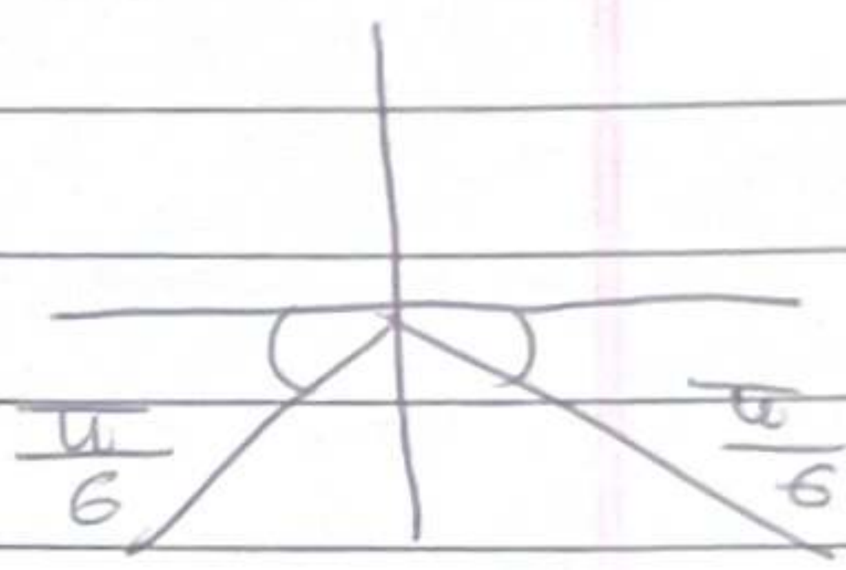
So the points are $(0,1)$ and $(-1,1)$

b) Since it cuts the x-axis

$y = 0$, $2\sin(2t) + 1 = 0$

$2\sin 2t = -1$

$\sin 2t = -\frac{1}{2}$



$2t = -\frac{\pi}{6}$

$2t = \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = \frac{7\pi}{12}, \frac{11\pi}{12}$

When $t = \frac{7\pi}{12}$

$y = 2\sin 2\left(\frac{7\pi}{12}\right)$, $y = 0$

When $t = \frac{11\pi}{12}$

$y = 2\sin 2\left(\frac{11\pi}{12}\right)$, $y = 0$

Hence, proved.

8.4 Points of Intersection

6c] When $t = \frac{7\pi}{12}$, When $t = \frac{11\pi}{12}$

$x = \cos\left(\frac{7\pi}{12}\right)$ $x = \cos\left(\frac{11\pi}{12}\right)$

$x = \frac{-\sqrt{6}+\sqrt{2}}{4}$ $x = \frac{-\sqrt{6}-\sqrt{2}}{4}$

So, the coordinates are $\left(\frac{-\sqrt{6}+\sqrt{2}}{4}, 0\right)$ and $\left(\frac{-\sqrt{6}-\sqrt{2}}{4}, 0\right)$

7] $x = 4\sin t$, $y = 8\cos t$

When $t = \frac{\pi}{6}$

$y = 8\cos\left(\frac{\pi}{8}\right)$

$x = 4\sin\left(\frac{\pi}{6}\right)$

$y = 8 \cdot \frac{\sqrt{3}}{2}$

$x = 2$

$P(2, 4\sqrt{3})$ $y = 4\sqrt{3}$

When $t = \frac{3\pi}{2}$

$x = 4\sin\left(\frac{3\pi}{2}\right)$ $y = 8\cos\left(\frac{3\pi}{2}\right)$

$x = -4$, $y = 0$

$Q(-4, 0)$

$m = \frac{0-4\sqrt{3}}{-4-2} \Longrightarrow \frac{2\sqrt{3}}{3}$

$y - y_1 = m(x - x_1)$

$y - 4\sqrt{3} = \frac{2\sqrt{3}}{3}(x-2)$

$y = \frac{2\sqrt{3}}{3}x - \frac{4\sqrt{3}}{3} + 4\sqrt{3}$

$2\sqrt{3}x - 3y + 8\sqrt{3} = 0$

8.4 Points of Intersection

8a] $x = \ln(t+2)$ — ① , $y = \ln(t-1)$ — ②

$y = 2x - \ln 8$ — ③

Sub ① and ② in ③

$\ln(t-1) = 2\ln(t+2) - \ln 18$

$\ln(t-1) = 2\ln(t+2)^2 - \ln 18$

$\cancel{\ln}(t-1) = \cancel{\ln}\left(\frac{(t+2)^2}{18}\right)$

$t-1 = \frac{(t+2)^2}{18}$

$t-1 = \frac{t^2+4t+4}{18}$

$18t - 18 = t^2 + 4t + 4$

$t^2 + 4t - 18t + 4 + 18 = 0$

$t^2 - 14t + 22 = 0$

$t = 7 + 3\sqrt{3}$, $t = 7 - 3\sqrt{3}$

$a = 7$, $b = 3$

b] When $t = 7 + 3\sqrt{3}$

$x = \ln(7 + 3\sqrt{3} + 2)$, $y = \ln(7 + 3\sqrt{3} - 1)$

$x = \ln(9 + 3\sqrt{3})$, $y = \ln(6 + 3\sqrt{3})$

$(\ln(9 + 3\sqrt{3}), \ln(6 + 3\sqrt{3}))$

When $t = 7 - 3\sqrt{3}$

$x = \ln(7 - 3\sqrt{3} + 2)$, $y = \ln(7 - 3\sqrt{3} - 1)$

$x = \ln(9 - 3\sqrt{3})$, $y = \ln(6 - 3\sqrt{3})$

$(\ln(9 - 3\sqrt{3}), \ln(6 - 3\sqrt{3}))$