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9.7 Parametric Differentiation

1a] $x = 4t - 1$ $\quad y = 2t^3$

$\frac{dx}{dt} = 4$ $\quad \frac{dy}{dt} = 6t^2$

$\frac{dy}{dx} = \frac{6t^2}{4} \Rightarrow \frac{3t^2}{2}$

b] $x = \frac{2}{t^2} \Rightarrow 2t^{-2}$ $\quad y = 4 - t^2$

$\frac{dy}{dt} = -2t$

$\frac{dx}{dt} = -4t^{-3}$

$\frac{dx}{dt} = \frac{-4}{t^3}$

$\frac{dy}{dx} = \frac{-2t}{\frac{-4}{t^3}} \Rightarrow \cancel{\frac{t^4}{3}} \quad \frac{t^4}{2}$

c] $x = 3t$ $\quad y = te^{4t}$

$\frac{dx}{dt} = 3$

$u = t$ $\quad v = e^{4t}$

$\frac{du}{dt} = 1$ $\quad \frac{dv}{dt} = 4e^{4t}$

$\frac{dy}{dt} = e^{4t} + 4te^{4t}$

$\frac{dy}{dx} = \frac{e^{4t} + 4te^{4t}}{3}$

d] $x = 4\sin t$ $\quad y = 5\tan t$

$\frac{dx}{dt} = 4(\cos t)$ $\quad \frac{dy}{dt} = 5\sec^2 t$

$\frac{dy}{dx} = \frac{5\sec^2 t}{4\cos t} \Rightarrow \frac{5}{4}\sec^3 t$

9.7 Parametric Differentiation

1e] $x = 4e^{3t} - 1$ $\qquad y = 2e^{t} + 4$

$\frac{dx}{dt} = 12e^{3t}$ $\qquad \frac{dy}{dt} = 2e^{t}$

$\frac{dy}{dx} = \frac{2e^{\cancel{t}}}{12e^{\cancel{3}t}} \Longrightarrow \frac{1}{6e^{2t}}$

b] $x = 2t^3 - 1$ $\qquad y = \ln t$

$\frac{dx}{dt} = 6t^2$ $\qquad \frac{dy}{dt} = \frac{1}{t}$

$\frac{dy}{dx} = \frac{\frac{1}{t}}{6t^2}$

$\frac{dy}{dx} = \frac{1}{t} \times \frac{1}{6t^2} \Longrightarrow \frac{1}{6t^3}$

2 a] $x = \tan t$ $\qquad y = \sec t$

$\frac{dx}{dt} = \sec^2 t$ $\qquad \frac{dy}{dt} = \sec t \tan t$

$\frac{dy}{dx} = \frac{\cancel{\sec t}\tan t}{\sec^{\cancel{2}} t} \Rightarrow \frac{\tan t}{\sec t}$

$\frac{dy}{dx} = \frac{\frac{\sin t}{\cancel{\cos t}}}{\frac{1}{\cancel{\cos t}}} \Rightarrow \sin t$

9.7 Parametric Differentiation

2b] $t = \frac{\pi}{4}$

Sub $t = \frac{\pi}{4}$ in x

$$x = \tan\left(\frac{\pi}{4}\right) \implies x = 1$$

Sub $t = \frac{\pi}{4}$ in y

$$y = \sec\left(\frac{\pi}{4}\right) \implies y = \sqrt{2}$$

Sub $t = \frac{\pi}{4}$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{2} = \frac{\sqrt{2}}{2}(x - 1)$$

$$y = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2} + \sqrt{2}$$

$$y = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}$$

3a] $x = 1 - t^2$ $\qquad$ $y = t^3 + 2t^2$

$\frac{dx}{dy} = -2t$ $\qquad$ $\frac{dy}{dt} = 3t^2 + 4t$

$$\frac{dy}{dx} = \frac{3t^2 + 4t}{-2t}$$

$$\frac{dy}{dx} = -\left(\frac{3t + 4}{2}\right)$$

9.7 Parametric Differentiation

3b) $t = -3$

Sub $t = -3$ in x

$x = 1 - (-3)^2 \implies x = -8$

Sub $t = -3$ in y

$y = (-3)^3 + 2(-3)^2 \implies y = -9$

Sub $t = -3$ in $\frac{dy}{dx}$

$\frac{dy}{dx} = -\left(\frac{3(-3)+4}{2}\right) \implies \frac{5}{2}$

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - (-9) = -\frac{2}{5}(x - (-8))$

$y + 9 = -\frac{2}{5}(x + 8)$

$y = -\frac{2}{5}x - \frac{16}{5} - 9$

$y = -\frac{2}{5}x - \frac{61}{5}$

$5y = -2x - 61$

$2x + 5y + 61 = 0$

9.7 Parametric differentiation

4a] $x = 1 - \cos 2\theta$ $\qquad$ $y = 4\sin 2\theta$

$\frac{dx}{d\theta} = -2(-\sin 2\theta)$ $\qquad$ $\frac{dy}{d\theta} = 8(\cos 2\theta)$

$= 2\sin 2\theta$ $\qquad$ $= 8\cos 2\theta$

$$\frac{dy}{dx} = \frac{8\cos 2\theta}{2\sin 2\theta}$$

$$\frac{dy}{dx} = 4\cot 2\theta$$

b] When $\frac{dy}{dx} = 0$

$= 4\cot 2\theta = 0$

$= \cot 2\theta = 0$

* Cot is 0 at 90° and 270°

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Sub $\theta = \frac{\pi}{4}$ & $\frac{3\pi}{4}$ in x

$y = 4\sin 2\left(\frac{\pi}{4}\right)$, $y = 4\sin 2\left(\frac{3\pi}{4}\right)$

$y = 4$ $\qquad$ $y = -4$

9.7 Parametric differentiation

4b) Cont.

So, for $\theta = \frac{\pi}{4}$: $(1, 4)$

for $\theta = \frac{3\pi}{4}$: $(1, -4)$

So the coordinates are $(1, 4)$ & $(1, -4)$

5a) $x = 2\sec\theta$

$\frac{dx}{d\theta} = 2\sec\theta\tan\theta$

$y = 4\cos^2\theta$

$= 4(\cos\theta)^2$

$\frac{dy}{d\theta} = 8(\cos\theta(\sin\theta)(-1$

$\frac{dy}{d\theta} = -8\cos\theta\sin\theta$

$$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{2\sec\theta\tan\theta}$$

$$= \frac{-8\cos\theta\sin\theta}{2 \times \left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{-8\cos\theta\sin\theta}{\frac{2\sin\theta}{\cos^2\theta}}$$

$$= -8\cos\theta\cancel{\sin\theta} \times \frac{\cos^2\theta}{2\cancel{\sin\theta}}$$

$$= -4\cos^3\theta$$

9.7 Parametric Differentiation

5b] When $\theta = \frac{\pi}{3}$

Sub $\theta = \frac{\pi}{3}$ in x

$x = 2\sec\left(\frac{\pi}{3}\right) \qquad x \Rightarrow 4$

Sub $\theta = \frac{\pi}{3}$ in y

$y = 4\cos^2\left(\frac{\pi}{3}\right) \quad \Rightarrow \quad y = 1$

Sub $\theta = \frac{\pi}{3}$ in $\frac{dy}{dx}$

$\frac{dy}{dx} = -4\cos^3\left(\frac{\pi}{3}\right)$

$\frac{dy}{dx} = -\frac{1}{2}$

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 1 = +2(x - 4)$

$y = 2x - 8 + 1$

$y = 2x - 7$

6] a $x = (t-2)^2 \qquad\qquad y = t^2 + 4t$

$\frac{dx}{dt} = 2(t-2)(1) \qquad\qquad \frac{dy}{dt} = 2t + 4$

$= 2t - 4$

9.7 Parametric differentiation

6] Cont.

$\frac{dy}{dx} = \frac{2t+4}{2t-4} \Longrightarrow \frac{dy}{dx} = \frac{t+2}{t-2}$

Since the tangent line is parallel to
$y = 2x - 9$
So $\frac{dy}{dx} = 2$

$2 = \frac{t+2}{t-2}$

$2t - 4 = t + 2$

$t = 6$

Sub $t = 6$ in x
$x = (6-2)^2 \qquad x \Longrightarrow 16$

Sub $t = 6$ in y
$y = 6^2 + 4(6) \qquad y \Longrightarrow 60$

So the point l is $(16, 60)$

$y - y_1 = m(x - x_1)$

$y - 60 = 2(x - 16)$

$y - 60 = 2x - 32$

$y = 2x + 28$

9.7 Parametric differentiation

7 a] $x = -2\sec 2t$ $\quad\quad$ $y = 2\cot t$

$\frac{dx}{dt} = -2(2\sec 2t \tan 2t)$ $\quad\quad$ $\frac{dy}{dt} = -2\operatorname{cosec}^2 t$

$\frac{dx}{dt} = -4\sec 2t \tan 2t$

$\frac{dy}{dx} = \frac{\cancel{-}2\operatorname{cosec}^2 t}{\cancel{-}4\sec 2t \tan 2t}$

$\frac{dy}{dx} = \frac{\operatorname{cosec}^2 t}{2\sec 2t \tan 2t}$

b] $A(-4, -2\sqrt{3})$

Sub $x = -4$ in x

$-4 = -2\sec 2t$

$\sec 2t = 2$

$2t = \frac{\pi}{3}$

$t = \frac{\pi}{6}$

Since $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

we must have $t = -\frac{\pi}{6}$

Sub $t = -\frac{\pi}{6}$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\operatorname{cosec}^2\left(-\frac{\pi}{6}\right)}{2\sec\left(2\times\left(-\frac{\pi}{6}\right)\right)\tan\left(2\times\left(-\frac{\pi}{6}\right)\right)}$$

9.7 Parametric differentiation

7b) Cont.

$$\frac{dy}{dx} = \frac{\operatorname{cosec}^2\left(-\frac{\pi}{6}\right)}{2\sec\left(-\frac{\pi}{3}\right)\tan\left(-\frac{\pi}{3}\right)}$$

$$\frac{dy}{dx} = \frac{4}{2(2)(-\sqrt{3})}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{3}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{3}}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2\sqrt{3}) = -\frac{\sqrt{3}}{3}\left(x - (-4)\right)$$

$$y + 2\sqrt{3} = -\frac{\sqrt{3}}{3}(x+4)$$

$$3y + 6\sqrt{3} = -\sqrt{3}x - 4\sqrt{3}$$

$$\sqrt{3}x + 3y + 10\sqrt{3} = 0$$

8a) $x = \frac{t}{1-t}$ $\qquad$ $y = \frac{1}{1+t^2}$

$u = t \qquad v = 1-t$

$\frac{du}{dt} = 1 \qquad \frac{dv}{dt} = -1$

$$\frac{du}{dv} = \frac{(1-t) - (-1)t}{(1-t)^2}$$

$$\frac{du}{dv} = \frac{1}{(1-t)^2}$$

$$y = (1+t^2)^{-1}$$

$$\frac{dy}{dt} = -(1+t^2)^{-2}(2t)$$

$$\frac{dy}{dt} = -2t(1+t^2)^{-2}$$

$$\frac{dy}{dt} = \frac{-2t}{(1+t^2)^2}$$

9.7 Parametric differentiation

8a) Cont.

So $\dfrac{dx}{dt} = \dfrac{1}{(1-t)^2}$

$$\frac{dy}{dx} = \frac{\frac{-2t}{(1+t^2)^2}}{\frac{1}{(1-t)^2}}$$

$$\frac{dy}{dx} = \frac{-2t}{(1+t^2)^2} \times (1-t)^2$$

$$\frac{dy}{dx} = \frac{-2t(1-t)^2}{(1+t^2)^2}$$

Sub $t = -\frac{1}{2}$ in $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-2\left(-\frac{1}{2}\right)\left(1-\left(-\frac{1}{2}\right)\right)^2}{\left(1+\left(-\frac{1}{2}\right)^2\right)^2}$$

$$\frac{dy}{dx} = \frac{36}{25}$$

Sub $t = -\frac{1}{2}$ in x

$$x = \frac{-\frac{1}{2}}{1-\left(-\frac{1}{2}\right)}$$

$$x = -\frac{1}{3}$$

9.7 Parametric differentiation

8a] Cont.

Sub $t = -\frac{1}{2}$ in y

$$y = \frac{1}{1+\left(-\frac{1}{2}\right)^2}$$

$$y = \frac{4}{5}$$

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - \frac{4}{5} = -\frac{25}{36}\left(x + \frac{1}{3}\right)$$

$$y = -\frac{25x}{36} - \frac{25}{108} + \frac{4}{5}$$

$$y = -\frac{25x}{36} + \frac{307}{540}$$

b] rearrange x

$$x = \frac{t}{1-t} \longrightarrow x(1-t) = t$$

$$x - tx = t$$

$$x = t + tx$$

$$x = t(1+x)$$

$$\frac{x}{1+x} = t$$

$$t = \frac{x}{1+x}$$

Sub t in y

$$y = \frac{1}{1+t^2}$$

9.7 Parametric differentiation

8b) Cont.

$$y = \frac{1}{1 + \left(\frac{x}{1+x}\right)^2}$$

$$= \frac{1}{1 + \frac{x^2}{(1+x)^2}}$$

$$= \frac{1}{\frac{(x+1)^2 + x^2}{(1+x)^2}}$$

$$= \frac{(1+x)^2}{(1+x)^2 + x^2}$$

$$= \frac{x^2 + 2x + 1}{x^2 + 2x + 1 + x^2}$$

$$= \frac{x^2 + 2x + 1}{2x^2 + 2x + 1}$$

Hence, proved!