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Problem Solving : Set A

Bronze:

a] $y = x^4 - \frac{20}{3}x^3 + 14x^2 + 6x - 4$

$\frac{dy}{dx} = 4x^3 - 20x^2 + 28x + 6$

$\frac{d^2y}{dx^2} = 12x^2 - 40x + 28$

b] $12x^2 - 40x + 28 \leq 0$

$3x^2 - 10x + 7 \leq 0$

$(3x - 7)(x - 1)$

$x = \frac{7}{3}$ or $x = 1$

$1 \leq x \leq \frac{7}{3} \longrightarrow \left[1, \frac{7}{3}\right)$

Hence it is concave.

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Silver:

$y = 4x^3 \ln x^2$

let $u = 4x^3$ $\qquad$ $v = \ln x^2$

$\frac{du}{dx} = 12x^2$ $\qquad$ $\frac{dv}{dx} = \frac{2x}{x^2} \Longrightarrow \frac{2}{x}$

$\frac{dy}{dx} = 12x^2 \ln x^2 + \frac{2}{x} \times 4x^3$

$\frac{dy}{dx} = 12x^2 \ln x^2 + 8x^2$

let $u = 12x^2$ $\qquad$ $v = \ln x^2$

$\frac{du}{dx} = 24x$ $\qquad$ $\frac{dv}{dx} = \frac{2}{x}$

Problem Solving : Set A

Silver Cont:

$$\frac{d^2y}{dx^2} = 12x^2 \times \frac{2}{x} + 24x\ln x^2 + 16x$$

$$\frac{d^2y}{dx^2} = 24x + 24x\ln x^2 + 16x$$

$$\frac{d^2y}{dx^2} = 40x + 24x\ln x^2 \geq 0$$

$$\ln x^2 \geq -\frac{5}{3}$$

$$x^2 \geq e^{-5/3}$$

$$x \geq (e^{-5/3})^{1/2}$$

$$x \geq e^{-5/6}$$

Hence $\frac{d^2y}{dx^2} \geq 0$ when $x \geq e^{-5/6}$ and the curve is convex.

Gold

$$f(x) = ax^4 + 2x^3 + 3x^2 - 10x + 5$$

$$f'(x) = 4ax^3 + 6x^2 + 6x - 10$$

$$f''(x) = 12ax^2 + 12x + 6$$

$$= 2ax^2 + 2x + 1 \geq 0$$

$$b^2 - 4ac = (4) - 4(2x)(1)$$

$$= 2a = \cancel{0}\,1$$

$$a = \frac{1}{2}$$

Problem Solving : Set B

Bronze:

a) $x^2 + y = (2-x)(y-1)$

$x^2 + y = 2y - 2 - xy + x$

$x^2 - x + y - 2y + 2 + xy = 0$

$x^2 - x - y + xy + 2 = 0$

let $u = x$ $\quad v = y$

$\frac{du}{dx} = 1 \quad \frac{dv}{dy} = \frac{dy}{dx}$

$\frac{du}{dv} = y + x\frac{dy}{dx}$

$= 2x - 1 - \frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$-\frac{dy}{dx} + x\frac{dy}{dx} = -2x + 1 - y$

$\frac{dy}{dx}(x-1) = -2x + 1 - y$

$\frac{dy}{dx} = \frac{-2x + 1 - y}{x - 1}$

$\frac{dy}{dx} = \frac{2x + y - 1}{1 - x}$

b) when $x = 0$

$x^2 + y = (2-x)(y-1)$

$0^2 + y = (2-0)(y-1)$

$y = 2y - 2$

$2 = y$ so $(0, 2)$

Sub $(0,2)$ from $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2(0) + 2 - 1}{1 - 0}$

$\frac{dy}{dx} = 1$

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Problem Solving : Set B

Bronze cont:

b] So equation to normal

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 2 = -1(x - 0)$$

$$y = -x + 2$$

Silver

a] $x = 2\sin t - 1$ $y = 2\cos t + 1$

$\frac{dx}{dt} = 2\cos t$ $\frac{dy}{dt} = -2\sin t$

$$\frac{dy}{dx} = \frac{-2\sin t}{2\cos t}$$

$$\frac{dy}{dx} = -\tan t$$

$$\left(\frac{x+1}{2}\right)^2 + \left(\frac{y-3}{2}\right)^2 = 1$$

$$(x+1)^2 + (y-3)^2 = 2^2$$

So $C(-1, 3)$ $r = 2$

b) when $x = 0$

$$(y-3)^2 = 3$$

$$y = \sqrt{3} + 3$$

So $B(0, 3 + \sqrt{3})$

c) at $x = 0$

$$0 = 2\sin t - 1$$

$$\sin t = \frac{1}{2}$$

$$t = 30$$

$$\frac{dy}{dx} = -\tan(30)$$

$$\frac{dy}{dx} = -\frac{\sqrt{3}}{3}$$

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Problem Solving Set B

Silver Cont:

c) $y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 3 - \sqrt{3} = \frac{3}{\sqrt{3}}(x - 0)$

$y = \frac{3x}{\sqrt{3}} + 3 + \sqrt{3}$

$\sqrt{3}y = 3x + 3\sqrt{3} + 3$

Sub $y = 0$

$\sqrt{3}(0) = 3x + 3\sqrt{3} + 3$

$3x = -3\sqrt{3} - \sqrt{3}$

$x = -\sqrt{3} - 1$

$x = -(\sqrt{3} + 1)$

$\text{Area} = \frac{1}{2} \times b \times h$

$\text{Area} = \frac{1}{2}(\sqrt{3} + 1)(\sqrt{3} + 3)$

$\text{Area} = \frac{3\sqrt{3} + 3 + 3 + \sqrt{3}}{2}$

~~Ar~~ $\text{Area} = \frac{4\sqrt{3} + 6}{2}$

$\text{Area} = 2\sqrt{3} + 3$

Gold

$\cos y + \sin x = 1$

$-\sin y \frac{dy}{dx} + \cos x = 0$

$y = x \longrightarrow \frac{dy}{dx} = 1$

(Parallel = Same gradient)

Problem Solving Set B

Gold Cont:

$-\sin y + \cos x = 0$

$\cos x = \sin y \qquad\qquad \sin x = 1 - \cos y$

Using $\sin^2 x + \cos^2 x = 1$

$(1 - \cos y)^2 + \sin^2 y = 1$

$1 - 2\cos y + \underbrace{\cos^2 y + \sin^2 y}_{1} = 1$

$1 - 2\cos y = 1 - 1$

$2\cos y = 1$

$\cos y = \frac{1}{2} \qquad\qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \frac{\pi}{3}, -\frac{\pi}{3}$

Sub y into $\cos x = \sin y$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$y = x + c$

$\frac{\pi}{3} = \frac{\pi}{6} + c \qquad \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

$c = \frac{\pi}{6}$

$y = x + \frac{\pi}{6}$

$-\frac{\pi}{3} = \frac{5\pi}{6} + c \qquad \left(\frac{5\pi}{6}, -\frac{\pi}{3}\right)$

$c = -\frac{7\pi}{6}$

$y = x - \frac{7\pi}{6}$

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