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9.10 Rates of Change

1a] $S = 6x^2$ $\quad \frac{dx}{dt} = \frac{2}{3}$

$\frac{dS}{dt} = \frac{dx}{dt} * \frac{dS}{dx}$ $\longrightarrow$ $S = 6x^2$

$\frac{dS}{dx} = 12x$

$\frac{dS}{dt} = \frac{2}{3} \times 12x$

$\frac{dS}{dt} = 8x$

b] Sub $x = 3$ in $\frac{dS}{dt}$

$\frac{dS}{dt} = 8(3)$

$\frac{dS}{dt} = 24$

2] $y = x\cos 4x$ $\quad \frac{dx}{dt} = 3$

$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$ $\longrightarrow$ $y = x\cos 4x$

$u = x \quad v = \cos 4x$

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = -4\sin 4x$

$\frac{dy}{dx} = -4x\sin 4x + \cos 4x$

$\frac{dy}{dt} = 3 \times (-4x\sin 4x + \cos 4x)$

$\frac{dy}{dt} = -12x\sin 4x + 3\cos 4x$

Sub $x = \frac{5\pi}{16}$ in $\frac{dy}{dt}$

$\frac{dy}{dt} = -12\left(\frac{5\pi}{16}\right)\sin 4\left(\frac{5\pi}{16}\right) + 3\cos 4\left(\frac{5\pi}{16}\right)$

9.10 Rates of Change

2] Cont.

$$\frac{dy}{dt} = -\frac{15\pi}{4} \times \left(\frac{-1}{\sqrt{2}}\right) - \frac{3}{\sqrt{2}}$$

$$\frac{dy}{dt} = \frac{15\pi}{4\sqrt{2}} - \frac{3}{\sqrt{2}}$$

$$\frac{dy}{dt} = \frac{3}{4\sqrt{2}}\left(5\pi - 4\right)$$

3] $y = \frac{e^{2x}}{x^2}$ $\qquad \frac{dx}{dt} = -\frac{1}{2}$

$$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

$$\frac{dy}{dt} = \frac{-1}{2}\left(\frac{2xe^{2x} - 2e^{2x}}{x^3}\right)$$

$$\frac{dy}{dt} = \frac{2e^{2x} - 2xe^{2x}}{2x^3}$$

$y = \frac{e^{2x}}{x^2}$

$u = e^{2x}$ $\qquad v = x^2$

$\frac{du}{dx} = 2e^{2x}$ $\qquad \frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{2x^2e^{2x} - 2xe^{2x}}{x^4}$$

$$\frac{dy}{dx} = \frac{2xe^{2x} - 2e^{2x}}{x^3}$$

Sub $x = 4$

$$\frac{dy}{dt} = \frac{2e^{2(4)} - 2(4)e^{2(4)}}{2(4)^3}$$

$$\frac{dy}{dt} = \frac{2e^8 - 8e^8}{128}$$

$$\frac{dy}{dt} = -\frac{3e^8}{64}$$

4a] $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = 4\pi r^2$$

9.10 Rates of Change

4b] $\frac{dV}{dt} = \frac{dr}{dt} \times \frac{dV}{dr}$

$\frac{dV}{dt} = 3 \times 4\pi r^2$

$\frac{dV}{dt} = 12\pi r^2$

5a] $\frac{dP}{dt} = +KP$

b] $\frac{dP}{dt} = -KP$

The value of the constant K would be -ve.

6] $\frac{dI}{dt} = -K\sqrt{I}$

7a] $v = 23500e^{-0.25t}$, $t \geq 0$

when $t = 7$

$V = 23500\, e^{-0.25(7)}$

$V = £4083.69$

b] $\frac{dV}{dt} = -5875e^{-0.25t}$

Sub $t = 10$ in $\frac{dV}{dt}$

$\frac{dV}{dt} = -5875e^{-0.25(10)}$

$\frac{dV}{dt} = £-482.25$

9.10 Rates of change

8] a) $V = \frac{4}{3}\pi r^3$ — ②

$S = 4\pi r^2$

$\frac{S}{4\pi} = r^2$

$r = \sqrt{\frac{S}{4\pi}}$ — ①

Sub ① in ②

$V = \frac{4}{3}\pi \left(\sqrt{\frac{S}{4\pi}}\right)^3$

$V = \frac{4}{3}\pi \left(\frac{S^{1/2}}{(4\pi)^{1/2}}\right)^3$

$V = \frac{4}{3}\pi \left(\frac{S^{3/2}}{(4\pi)^{3/2}}\right)$

$V = \frac{4}{3}\pi \times \left(\frac{S^{3/2}}{8\pi^{3/2}}\right)$

$V = \frac{1}{6\sqrt{\pi}} S^{3/2}$

Hence, proved.

b] $V = \frac{1}{6\sqrt{\pi}} S^{3/2}$

$V = \frac{1}{6} \times \pi^{-1/2} \times S^{3/2}$

$\frac{dV}{dS} = \frac{3}{2} \times \frac{1}{6} \times \pi^{-1/2} \times S^{1/2}$

$\frac{dV}{dS} = \frac{1}{4}\sqrt{\frac{S}{\pi}}$

$\frac{dV}{dt} = \frac{dV}{dS} \times \frac{dS}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\sqrt{\frac{S}{\pi}} \times 4 \Longrightarrow \sqrt{\frac{S}{\pi}}$