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9.1 Differentiating Sinx and Cosx

1a) $y = 6\sin x$

$\frac{dy}{dx} = 6\cos x$

b) $y = -3\sin\frac{1}{2}x$

$\frac{dy}{dx} = -3\cos\frac{1}{2}x \times \left(\frac{1}{2}\right)$

$= -\frac{3}{2}\cos\frac{1}{2}x$

c) $y = \frac{1}{3}\sin 5x$

$\frac{dy}{dx} = \frac{1}{3}\cos 5x \times 5$

$= \frac{5}{3}\cos 5x$

d) $y = \cos 4x$

$\frac{dy}{dx} = -\sin 4x \times (4)$

$= -4\sin 4x$

e) $y = 5\cos 6x$

$\frac{dy}{dx} = 5(-\sin 6x) \times 6$

$= -30\sin 6x$

9.1 Differentiating Sin x and Cos x

1 b]
$$y = -\frac{1}{2}\cos 7x$$
$$\frac{dy}{dx} = -\frac{1}{2}(-\sin 7x) \times 7$$
$$= \frac{7}{2}\sin 7x$$

2 a]
$$f(x) = 4\sin x - 3\cos x$$
$$f'(x) = 4\cos x - 3(-\sin x)$$
$$= 4\cos x + 3\sin x$$

b]
$$f(x) = \frac{1}{2}\sin 2x + 4\cos x - 6\sin \frac{1}{4}x$$
$$f'(x) = \frac{1}{2}(\cos 2x) \times 2 + 4(-\sin x) - 6\left(\frac{1}{4}\cos \frac{1}{4}x\right)$$
$$= \cos 2x - 4\sin x - \frac{3}{2}\cos \frac{1}{4}x$$

c]
$$f(x) = 5x^{3/2} - \frac{4}{x^2} - 6\sin \frac{1}{3}x$$
$$= 5x^{3/2} - 4x^{-2} - 6\sin \frac{1}{3}x$$
$$f'(x) = 5\left(\frac{3}{2}\right)x^{1/2} - 4(-2)x^{-3} - 6\cos \frac{1}{3}x \times \left(\frac{1}{3}\right)$$
$$= \frac{15}{2}x^{1/2} + \frac{8}{x^3} - 2\cos \frac{1}{3}x$$

9.1 Differentiating $\sin x$ and $\cos x$

2d] $f(x) = \dfrac{8}{\sqrt{x}} + 3\cos\dfrac{1}{6}x$

$= 8x^{-1/2} + 3\cos\dfrac{1}{6}x$

$f'(x) = 8\left(\dfrac{-1}{2}\right)x^{-3/2} + 3\left(-\sin\dfrac{1}{6}x\right) \times \left(\dfrac{1}{6}\right)$

$= -4x^{-3/2} - \dfrac{1}{2}\sin\dfrac{1}{6}x$

$= \dfrac{-4}{x^{3/2}} - \dfrac{1}{2}\sin\dfrac{1}{6}x$

3] $f(x) = 5\sin x - 3\cos x$

$f'(x) = 5\cos x - 3(-\sin x)$

$= 5\cos x + 3\sin x$

When $f'(x) = 0$

$5\cos x + 3\sin x = 0$

$5\cos x = -3\sin x$

$\dfrac{5}{-3} = \dfrac{\sin x}{\cos x}$

$\tan x = \dfrac{-5}{3}$

$x = \tan^{-1}\left(-\dfrac{5}{3}\right)$

$x = -59.0$, $x = 180 - 59.0$

So $x = -59.0°, 121.0°$

9.1 Differentiating Sin x and Cos x

4] $f(x) = x - \cos 2x$

$f'(x) = 1 - 2(-\sin 2x)$

$= 1 + 2\sin 2x$

Sub $x = \frac{\pi}{4}$

$f'(x) = 1 + 2\sin 2\left(\frac{\pi}{4}\right)$

$= 1 + 2$

$= 3$

$y - y_1 = m(x - x_1)$

$y - \frac{\pi}{4} = 3\left(x - \frac{\pi}{4}\right)$

$y = 3x - \frac{3\pi}{4} + \frac{\pi}{4}$

$y = 3x - \frac{\pi}{2}$

5] $f(x) = 6\sin x - 3\cos 2x$

$f'(x) = 6(\cos x) - 3(-2\sin 2x)$

$= 6\cos x + 6\sin 2x$

* Using $\sin 2x = 2\sin x \cos x$

$= 6\cos x + 6(2\sin x \cos x)$

$= 6\cos x + 12\sin x \cos x$

9.1 Differentiating Sin x and Cos x

5] Cont.

Stationary point so $f'(x) = 0$

$$= 6\cos x + 12\sin x\cos x = 0$$
$$6\cos x\,(1 + 2\sin x) = 0$$

So $\cos x = 0$ or $1 + 2\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Sub x-values in $f(x)$

When $x = \frac{\pi}{2}$

$$\Rightarrow 6\sin\frac{\pi}{2} - 3\cos 2\left(\frac{\pi}{2}\right) \Rightarrow 9$$

When $x = \frac{3\pi}{2}$

$$\Rightarrow 6\sin\frac{3\pi}{2} - 3\cos 2\left(\frac{3\pi}{2}\right) \Rightarrow -3$$

When $x = \frac{7\pi}{6}$

$$\Rightarrow 6\sin\left(\frac{7\pi}{6}\right) - 3\cos 2\left(\frac{7\pi}{6}\right) \Rightarrow -\frac{9}{2}$$

When $x = \frac{11\pi}{6}$

$$\Rightarrow 6\sin\left(\frac{11\pi}{6}\right) - 3\cos 2\left(\frac{11\pi}{6}\right) \Rightarrow -\frac{9}{2}$$

So the coordinates are

$\left(\frac{\pi}{2}, 9\right), \left(\frac{7\pi}{6}, -\frac{9}{2}\right), \left(\frac{3\pi}{2}, -3\right), \left(\frac{11\pi}{6}, -\frac{9}{2}\right)$

9.1 Differentiating Sin x and Cos x

6) $g(x) = 6\sin 2x - 4\cos 2x$

$g'(x) = 6(2\cos 2x) - 4(-2\sin 2x)$

$= 12\cos 2x + 8\sin 2x$

Sub $x = \pi$ in $g'(x)$

$g'(x) = 12\cos 2\pi + 8\sin 2\pi$

$= 12$

So gradient is $-\frac{1}{12}$

$y - y_1 = m(x - x_1)$

$y - (-4) = -\frac{1}{12}(x - \pi)$

$y + 4 = -\frac{1}{12}(x - \pi)$

$y = -\frac{1x}{12} + \frac{\pi}{12} - 4$

7) $h(x) = 3\sin x - 4\cos x + \frac{1}{2}x^2$

let $y = h(x)$

$y = 3\sin x - 4\cos x + \frac{1}{2}x^2$

Sub $x = \frac{\pi}{2}$ in y

9.1 Differentiating $\sin x$ and $\cos x$

7] Cont:

$$y = 3\sin\left(\frac{\pi}{2}\right) - 4\cos\left(\frac{\pi}{2}\right) + \frac{1}{2}\left(\frac{\pi}{2}\right)^2$$

$$y = 3 + \frac{\pi^2}{8}$$

$$h(x) = 3\sin x - 4\cos x + \frac{1}{2}x^2$$

$$h'(x) = 3\cos x - 4(-\sin x) + \frac{1}{2}(2)x$$

$$h'(x) = 3\cos x + 4\sin x + x$$

Sub $x = \frac{\pi}{2}$ in $h'(x)$

$$h'(x) = 3\cos\frac{\pi}{2} + 4\sin\frac{\pi}{2} + \frac{\pi}{2}$$

$$= 4 + \frac{\pi}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - \left(3 + \frac{\pi^2}{8}\right) = \left(4 + \frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$$

(x8) $y - 3 - \frac{\pi^2}{8} = 4x - 2\pi + \frac{\pi x}{2} - \frac{\pi^2}{4}$

$= 8y - 24 - \pi^2 = 32x - 16\pi + 4\pi x - 2\pi^2$

$= 8y - 32x - 4\pi x = 24 - 16\pi - \pi^2$

$= 8y - (32 + 4\pi)x = 24 - 16\pi - \pi^2$

9.1 Differentiating $\sin x$ and $\cos x$

8] $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x}$$

$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left(\left(\frac{\cos h - 1}{h} \right) \cos h - \left(\frac{\sin h}{h} \right) \sin x \right)$$

Since, $\frac{\cos h - 1}{h} \longrightarrow 0$, and

$$\frac{\sin h}{h} \longrightarrow 1$$

$$= \lim_{h \to 0} 0(\cos x) - 1(\sin x)$$

So $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

$$= -\sin x$$

~~So~~ Hence the derivative of $\cos x$ is $-\sin x$.