Author: Naga Karthik

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10.2: Iteration

① $f(x) = x^2 - 8x + 3$

a) $f(x) = 0 \Rightarrow x^2 - 8x + 3 = 0 \Rightarrow x^2 = 8x - 3 = x = \sqrt{8x-3}$

b) $x^2 - 8x + 3 = 0 \Rightarrow (x^2 = 8x - 3) \div x \Rightarrow x = 8 - \dfrac{3}{x}$

c) $x^2 - 8x + 3 = 0 \Rightarrow 8x = x^2 + 3 \Rightarrow x = \dfrac{x^2+3}{8}$

② $f(x) = x^2 - 4x - 2 \qquad f(x) = 0$

a) $x^2 - 4x - 2 = 0 \Rightarrow x^2 = 4x + 2 \Rightarrow x = \sqrt{4x+2}$

b) $x_{n+1} = \sqrt{4x_n + 2}, \quad x_0 = 5$

$\Rightarrow x_1 = \sqrt{4x_0 + 2} = 4.690 \qquad \Rightarrow x_3 = \sqrt{4x_2 + 3} = 4.497$

$\Rightarrow x_2 = \sqrt{4x_1 + 2} = 4.556 \qquad \Rightarrow x_4 = \sqrt{4x_3 + 3} = 4.471$

③ $h(x) = -x^3 + 3x^2 - 1 \qquad h(x) = 0$

a) i) $-x^3 + 3x^2 - 1 = 0 \Rightarrow x^3 = 3x^2 - 1 \Rightarrow x = \sqrt[3]{3x^2 - 1}$

ii) $-x^3 + 3x^2 - 1 = 0 \Rightarrow 3x^2 = x^3 + 1 \Rightarrow x^2 = \dfrac{x^3+1}{3} \Rightarrow x = \sqrt{\dfrac{x^3+1}{3}}$

b) $x_{n+1} = \sqrt[3]{3x_n^2 - 1}, \quad x_0 = 1$

$x_1 = \sqrt[3]{3x_0^2 - 1} = 1.260 \qquad x_3 = \sqrt[3]{3x_2^2 - 1} = 1.843$

$x_2 = \sqrt[3]{3x_1^2 - 1} = 1.555 \qquad x_4 = \sqrt[3]{3x_3^2 - 1} = 2.094$

c) $x_{n+1} = \sqrt{\dfrac{x_n^3+1}{3}}, \quad x_0 = 1$

$x_1 = \sqrt{\dfrac{x_0^3+1}{3}} = 0.816 \qquad x_3 = \sqrt{\dfrac{x_2^3+1}{3}} = 0.676$

$x_2 = \sqrt{\dfrac{x_1^3+1}{3}} = 0.717 \qquad x_4 = \sqrt{\dfrac{x_3^3+1}{3}} = 0.660$

d) The iterations appear to be converging to be different numbers.

④ $f(x) = 2\sin x - \frac{1}{2}x^2$

a) $2\sin x - \frac{1}{2}x^2 = 0 \Rightarrow \frac{1}{2}x^2 = 2\sin x \Rightarrow x^2 = 4\sin x \Rightarrow x = \sqrt{4\sin x}$

b) $x_0 = 1.5 \qquad x_{n+1} = \sqrt{4\sin x_n}$

$x_1 = \sqrt{4\sin x_0} = 1.917$ $\qquad x_3 = \sqrt{4\sin x_2} = 1.943$

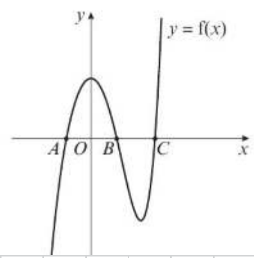
$x_2 = \sqrt{4\sin x_1} = 1.908$

c) $f(1.9335) = 2\sin(1.9335) - 0.5(1.9335)^2 = 0.0006... > 0$

$f(1.9345) = 2\sin(1.9345) - 0.5(1.9345)^2 = -0.0019... < 0$

Sign change implies atleast one root in interval, $a = 1.934$ (3dp)

⑤ $f(x) = x^5 - 5x^2 + 2$



a) $x^2 - x - 1 = 0 \qquad A\left(\frac{1-\sqrt{5}}{2}, 0\right) \quad C\left(\frac{1+\sqrt{5}}{2}, 0\right)$

b) $x^5 - 5x^2 + 2 = 0 \Rightarrow 5x^2 = x^5 + 2$

$\Rightarrow x^2 = \frac{x^5+2}{5} \Rightarrow x = \sqrt{\frac{x^5+2}{5}}$

c) $x_0 = 0.5$; $x_{n+1} = \sqrt{\frac{x_n^5+2}{5}}$

$x_1 = \sqrt{\frac{x_0^5+2}{5}} = 0.637 \qquad x_3 = \sqrt{\frac{x_2^5+2}{5}} = 0.650$

$x_2 = \sqrt{\frac{x_1^5+2}{5}} = 0.649$

d) $f(0.6505) = 0.0007.... > 0$

$f(0.6515) = -0.0048.... < 0$

Sign change implies atleast one root in interval, so $a = 0.651$

⑥ $f(x) = e^{x+4} + 0.5x^2 - 10$

a) $f(-1.9) = e^{-1.9+4} + 0.5(-1.9)^2 - 10 = -0.0288... < 0$

$f(-1.8) = e^{-1.8+4} + 0.5(-1.8)^2 - 10 = 0.6450... > 0$

Sign change implies atleast one root in interval

b) $f(x)=0 \Rightarrow \ln e^{x+4} = \ln\left(10-\frac{1}{2}x^2\right) \Rightarrow x+4 = \ln\left(10-\frac{1}{2}x^2\right)$

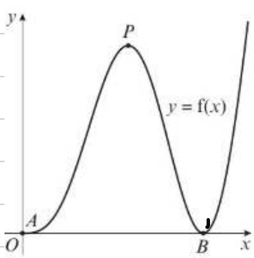
$\Rightarrow x = \ln\left(10-\frac{1}{2}x^2\right)-4$

c) $x_0=-3$ $\quad x_{n+1} = \ln\left(10-\frac{1}{2}x_n^2\right)-4$

$x_1 = \ln\left(10-0.5x_0^2\right)-4 = -2.295$ $\quad x_3 = \ln\left(10-0.5x_3^2\right)-4 = -1.921$

$x_2 = \ln\left(10-0.5x_1^2\right)-4 = -2.003$

d) $f(-1.89545) = -0.0002\ldots < 0$ $\quad$ Sign change implies at least
$f(-1.89535) = 0.0004\ldots > 0$ $\quad$ one root in interval, $a=-1.8954$

7) 

$f(x) = x\sin^2 x$

$(0,0)$, $(\pi,0)$

a) $x=0 \Rightarrow f(x) = 0\sin^2(0) = 0$
$x=\pi \Rightarrow f(x) = \pi\sin^2(\pi) = \pi \times 0 = 0$

b) $f'(x) =$ Product rule ; $u = x$ $\quad v = \sin^2 x$
$u' = 1$ $\quad v' = 2\cos 2x$

$\Rightarrow f'(x) = x(\sin x + 2x\cos x)$

c) $f'(1.8) = 0.18\left(\sin(0.18) + 2(0.18)\cos(0.18)\right) = 0.1518\ldots > 0$
$f'(1.9) = 0.19\left(\sin(0.19) + 2(0.19)\cos(0.19)\right) = -0.2670\ldots < 0$

Sign change implies at least one turning point in interval

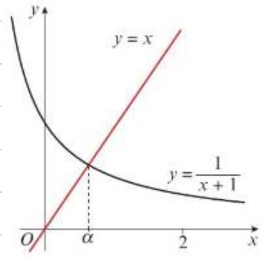
d) $\sin x(\sin x + 2x\cos x) = 0$ $\quad \sin x = 0$ or $2x\cos x = -\sin x$

$\cos x = \dfrac{-\sin x}{2x} \Rightarrow x = \arccos\left(\dfrac{-\sin x}{2x}\right)$

e) $x_0 = 0.8$ $\quad x_{n+1} = \arccos\left(\dfrac{-\sin x_n}{2x_n}\right)$

$x_1 = \cos^{-1}\left(\dfrac{-\sin x_0}{2x_0}\right) = 1.845$ $\quad x_2 = \cos^{-1}\left(\dfrac{-\sin x_1}{2x_1}\right) = 1.835$ $\quad x_3 = \cos^{-1}\left(\dfrac{-\sin x_2}{2x_2}\right) = 1.837$

⑧ a)



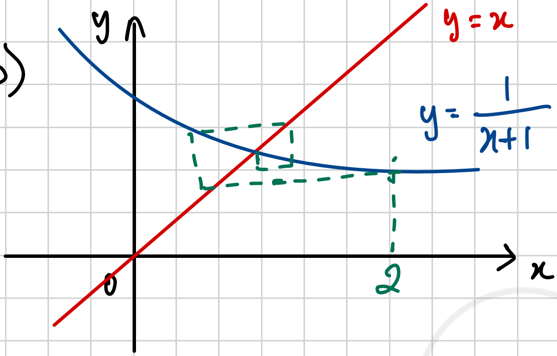
$$y = \frac{1}{x+1}$$

a) $f(0.5) = \frac{1}{0.5+1} = 0.1666.. > 0$

$f(1) = \frac{1}{-2} = -0.5 < 0$

sign change implies atleast one root in interval.

b)



The iteration will converge